A study of Simple Harmonic Motion

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Abstract

In this paper we will be focusing our attention towards an oscillatory motion under a retarding force proportional to the amount of displacement from an equilibrium position which is also known as a 'Simple Harmonic Motion' .

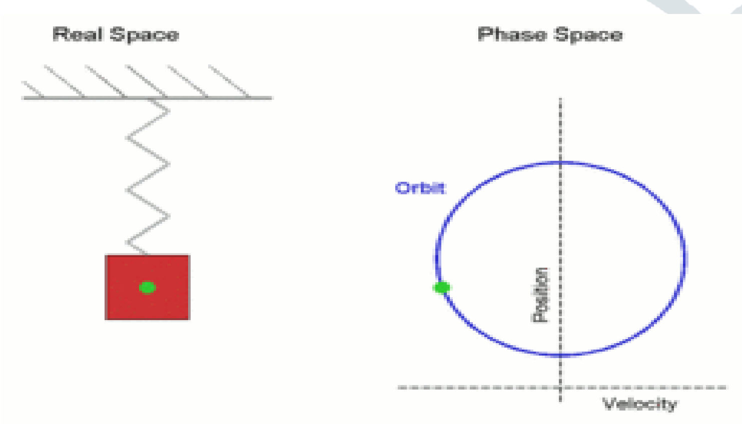
Simple harmonic motion provides a basis for the characterization of more complicated motions through the techniques of Fourier analysis (the definition can be restated as "the periodic motion of a body along a straight line, such that the acceleration is directed towards the center of the motion and also proportional to the displacement from that point").

Keywords Equilibrium, Displacement, Oscillation, Fourier analysis.

Introduction

The motion of a particle moving along a straight line with an acceleration whose direction is always towards a fixed point on the line and whose magnitude is proportional to the distance from the fixed point is called simple harmonic motion.

In the below shown diagram,



A simple harmonic oscillator, consisting of a weight attached to one end of a spring, is shown. The other end of the spring is connected to a rigid support such as a wall. If the system is left at rest at the equilibrium position then there is no net force acting on the mass. However, if the mass is displaced from the equilibrium position, the spring exerts a restoring elastic force that obeys Hooke's law.

Where F is the restoring elastic force exerted by the spring (in SI units: N), k is the spring constant (N·m−1), and x is the displacement from the equilibrium position (m).

Literature Review

Describing Vibration:- Also known as "the cyclical change in the position of an object as it moves alternately to one side and then back to it's initial position". . Vibration of rigid bodies can be rectilinear, rotational or a Combination of the two. Mathematically, Harmonic vibration of single degree of Freedom system can be given by the expression. U (t) = sin wt………… (1)

Where, u(t) = The position of the point with respect to time t. u(t) = maximum displacement of the Point from datum line. W = the circular frequency. T = time Sec. Cycle present. Wave form of Simple Harmonic Motion Describing the Velocity and acceleration at the vibration point can be determined by tacking the first And second derivative of equation. Velocity Acceleration From studding this equation one can see that the displacement Value u(t) and the acceleration value are at a maximum when the velocity is equal to Zero.

Equation methodology

Requirements

Table, Table stand, Measuring tape, Bungee cord, Eggs etc.

Hooks Procedure

We set up a hanging post on the edge of our lab table, where the length from top of the post to the floor was 244 cm. and then attached measuring tape, to calculate displacement .The bungee cord was tied to a knob 18 cm away from the vertical post in order to leave room for the bungee weight to bounce without interference.

In order to test the k value, we chose three different initial lengths of the bungee cord:

20 cm, 40 cm, and 60 cm. To test how the k value varies with added mass, we attached a 19.3 cm long hanging mass to the bungee cord that weighed either 100, 110, 120, 130, 150, 170 grams. To attach the hanging mass, we looped the knot three times around the top of the hanging mass and secured the weights with tape.

For each mass and length, we dropped the hanging mass from the top of the hanging Post, and recorded three jumps each.We took the readings from the bottom of the weight, and therefore, had to subtract the height of the weight (19.3 cm) from each Reading, in order to isolate the bungee cord. · We then took the average total length of Fall.

In order to calculate the displacement of the cord (x), we subtracted the initial length of The bungee cord from the total length of the fall (h). Results : Different observations on various length of above mentioned FIGURE A] Data table for the 20 cm bungee cord

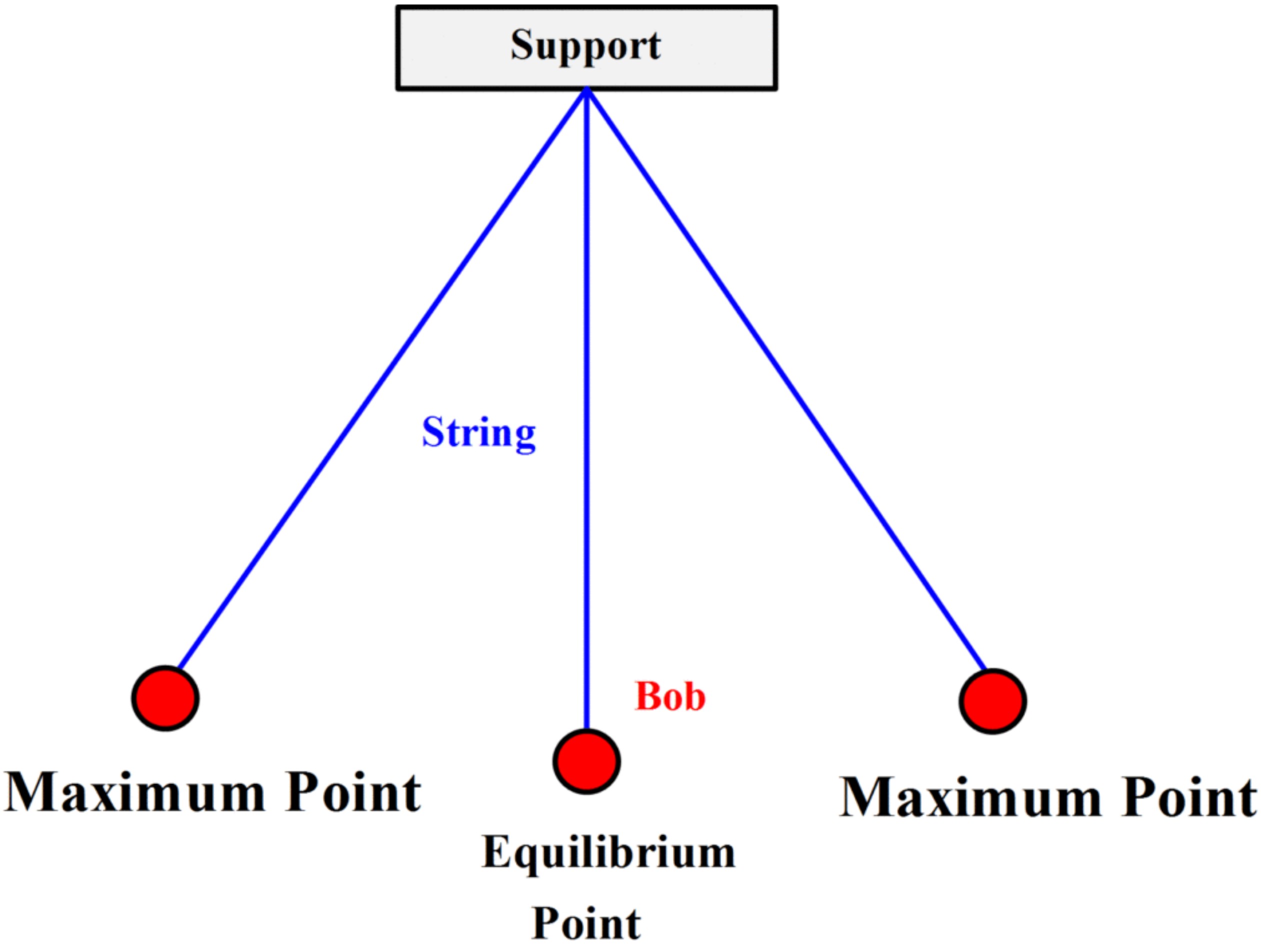
Mass m Length of total fall h Standard deviation for h Displacement x Standard Deviation for x K value (N/m) (g,± l g) (cm,±1cm) (cm,±1cm) ± 1 (N/m) 100 78 1 39 0 46 110 82 1 43 1 46 120 85 1 46 1 47 130 89 1 50 1 47 150 97 1 58 1 48 170 108 1 69 1 46.

Average k value: 47(N/m) B] Data table for the 40 cm bungee cord MASS Length of total Fall h Standard deviation for h Displacement x Standard deviation for x K value (N/m) (g,± l g) (cm,±1cm) (cm,±1cm) ± 1(N/m) 100 147 1 87 1 25 110 159 2 99 2 24 120 163 3 104 3 25 130 173 1 114 1 25 150 191 2 132 2 25 170 212 1 152 1 24. Average k value : 25(N/m) Results: In analyzing our data, we were able to show that k of the bungee cord Had an inverse relationship to the length of the cord.

The data shows the data for the bungee cord at the length of 60 cm.We did not have any mass measurements for this data table because a jump with Heavier masses would hit the floor. We averaged the heights and displacements from Our three trials, and calculated the standard deviations and the k values. The k values at This length were the lowest. We graphed the average k values for each length and the Lengths in order to get a better understanding between them.

The graph shows an inverse relationship between the k and the length of the bungee Cord. Hence, Our experiment proved that the spring constant and initial length of the Bungee cord are inversely proportional. Using this relationship, we can predict the final Stretch of a bungee cord by knowing the mass, initial height, and the length of the Bungee cord.Survey Data Collection Bungee jumping is after all a free fall jump, but in case one will Wonder how the “winners” will be decided , a panel of judges will assess the jumps Based on time taken to leave the ledge, distance forward of the leap , consistency of Body moves during the jump and style of jump. SR NO HEIGHT(in m) PLACE 1 206 Rio Grand Bridge, New Mexico 2 216 Bloukrans Bridge , South Africa 3 220 Verzasca Dam,Switzerland 4 233 Macau Tower , China

A Simple Pendulum



One type of simple harmonic oscillator is a simple pendulum. A simple pendulum is an object that has a small mass, which is suspended by a light wire or string.

When a simple pendulum is displaced from equilibrium, it swings in an arc. The length of the displacement is called the arc length and is identified as s. When displacement occurs, a restoring force is created that is in the direction towards the equilibrium position. This restoring force is directly proportional to the displacement.

Two factors affect the period of a simple pendulum, which is the time duration at which one oscillation takes place. One factor is the length of the string or wire, and the second factor is the acceleration due to gravity. The period T is nearly independent of amplitude and mass.

The differential equation which represents the motion of a simple pendulum is

$$\frac{d^2\theta}{dt^2} + \frac{g}{\ell}\sin\theta = 0 \qquad \textit{Eq. 1}$$

Where g is acceleration due to gravity, l is the length of the pendulum, and θ is the angular displacement.

'Force' equation derivation from eq. 1 :

Note that the path of the pendulum sweeps out an arc of a circle. The angle θ is measured in radians, and this is crucial for this formula. The blue arrow is the gravitational force acting on the bob, and the violet arrows are that same force resolved into components parallel and perpendicular to the bob's instantaneous motion. The direction of the bob's instantaneous velocity always points along the red axis, which is considered the tangential axis because its direction is always tangent to the circle. Consider Newton's second law,

F=ma

Where F is the sum of forces on the object, m is mass, and a is the acceleration. Because we are only concerned with changes in speed, and because the bob is forced to stay in a circular path, we apply Newton's equation to the tangential axis only. The short violet arrow represents the component of the gravitational force in the tangential axis, and trigonometry can be used to determine its magnitude. Thus,

$$F = -mg\sin\theta = ma, \qquad \text{so}$$
$$a = -g\sin\theta,$$

Where g is the acceleration due to gravity near the surface of the earth. The negative sign on the right hand side implies that θ and a always point in opposite directions. This makes sense because when a pendulum swings further to the left, we would expect it to accelerate back toward the right.

This linear acceleration a along the red axis can be related to the change in angle θ by the arc length formulas; s is arc length:

$$s = \ell\theta,$$
$$v = \frac{ds}{dt} = \ell\frac{d\theta}{dt},$$
$$a = \frac{d^2 s}{dt^2} = \ell\frac{d^2\theta}{dt^2},$$

thus:

$$\ell\frac{d^2\theta}{dt^2} = -g\sin\theta,$$
$$\frac{d^2\theta}{dt^2} + \frac{g}{\ell}\sin\theta = 0.$$

An apllication of SHM, Bunjee Jumping :

By using Hooke's Law, we were able to determine the "k" value of the relationship by determining the Force exerted on the bungee cord, measuring the distance the bungee cord Extends past equilibrium (x), and dividing the Force by the extension of the bungee cord. First, we hung the bungee cord from the stand and measured the L for experiment 1. We then, Attached the variable mass hanger to the bungee cord and dropped it from the top of the stand.

Table 1 (Experiment 1)
(We made sure to drop the mass from the top of the stand each time to limit uncertainties.)

L(m) +-. 01	X(m) +-. 01	M(kg)	F(N)	K+-. 01
0.8	0.16	0.05	0.49	3.063
0.8	0.25	0.075	0.735	2.94
0.8	0.35	0.1	0.98	2. 8
0.8	0.48	0.125	1.225	2.55
0.8	0.65	0.15	1.47	2.262
0.8	0.82	0.175	1.715	2.091
0.8	1.01	0.2	1.96	1.941
0.8	0.21	0.06	0.588	2.8
0.8	0.24	0.07	0.686	2.858
0.8	0.42	0.11	1.078	2.567

Holding a tape measure next to the mass as it decelerated, we were able to measure the extension Of the cord by recording the drop in slow-motion. We repeated this process a total of 10 times, each time altering the mass on the mass holder. For the second experiment, the same process was done except we altered the L and made it shorter. After collecting all of our "x" values, we were able to calculate the "k" values for each of the experiments by dividing the force (mass times gravity) by the measured "x" value.

Results:

The results of both of our experiments was that the "k" (bungee cord constant) values we found for each of the experiments were different, due to change in L. When we graphed this data, the line of best fit was obviously linear in nature, but the equation itself was not. The slope of our line was the "k" value, but the slope was not constant and changed instead.

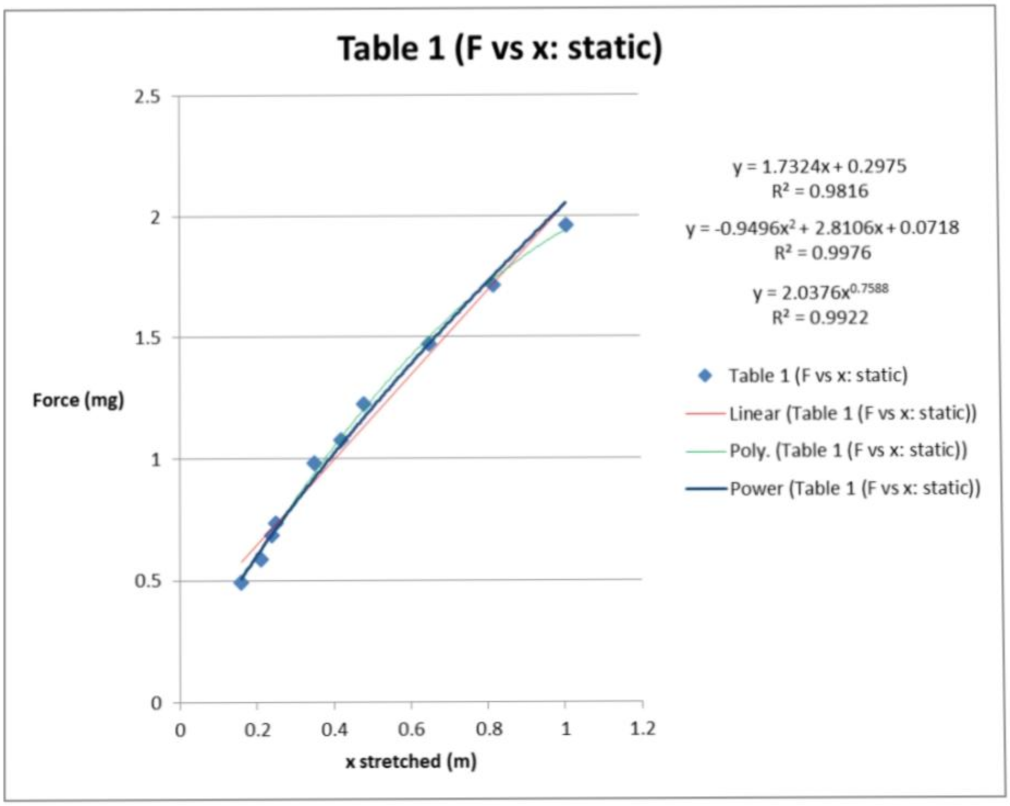


Table 2 (Experiment 2)

L(m) +-. 01	X(m) +-. 01	M(kg)	F(n)	K+-. 01
0.48	0.07	0.05	0.49	7
0.48	0.12	0.075	0.735	6.13
0.48	0.18	0.1	0.98	5.44
0.48	0.26	0.125	1.225	4.71
0.48	0.35	0.15	1.47	4.2
0.48	0.45	0.175	1.715	3.81
0.48	0.55	0.2	1.96	3.56
0.48	0.09	0.06	0.588	6.53
0.48	0.11	0.07	0.686	6.24
0.48	0.22	0.11	1.078	4.9

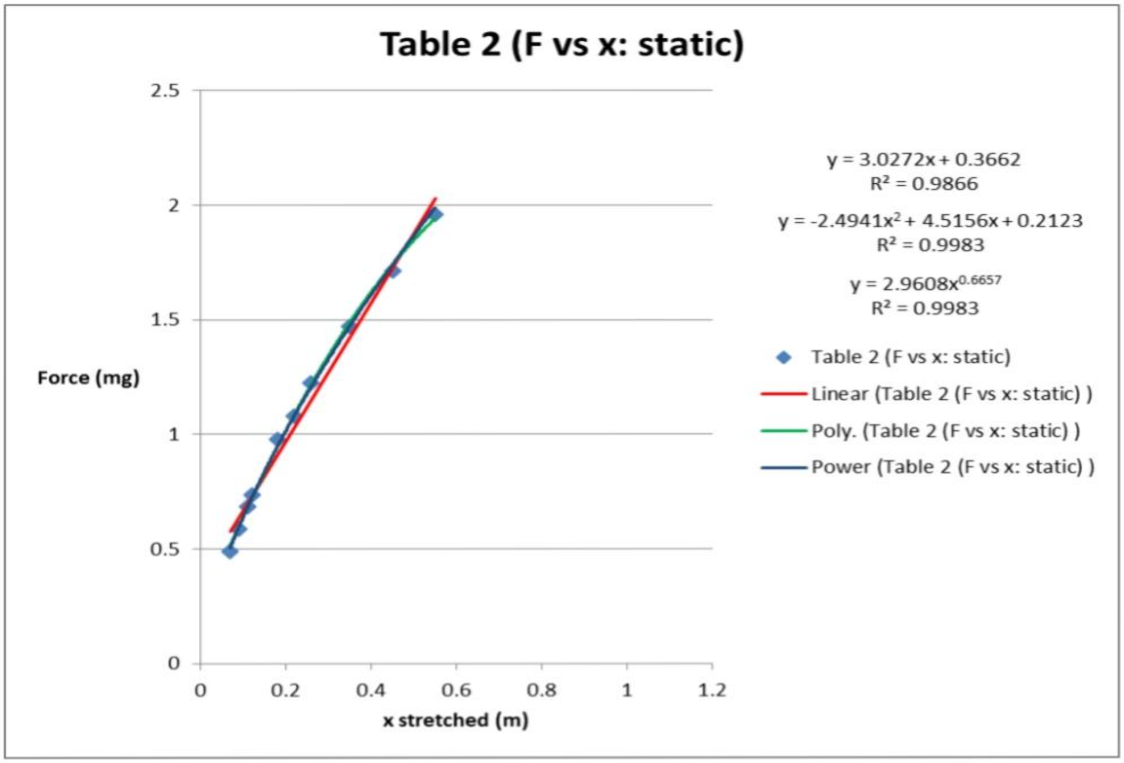
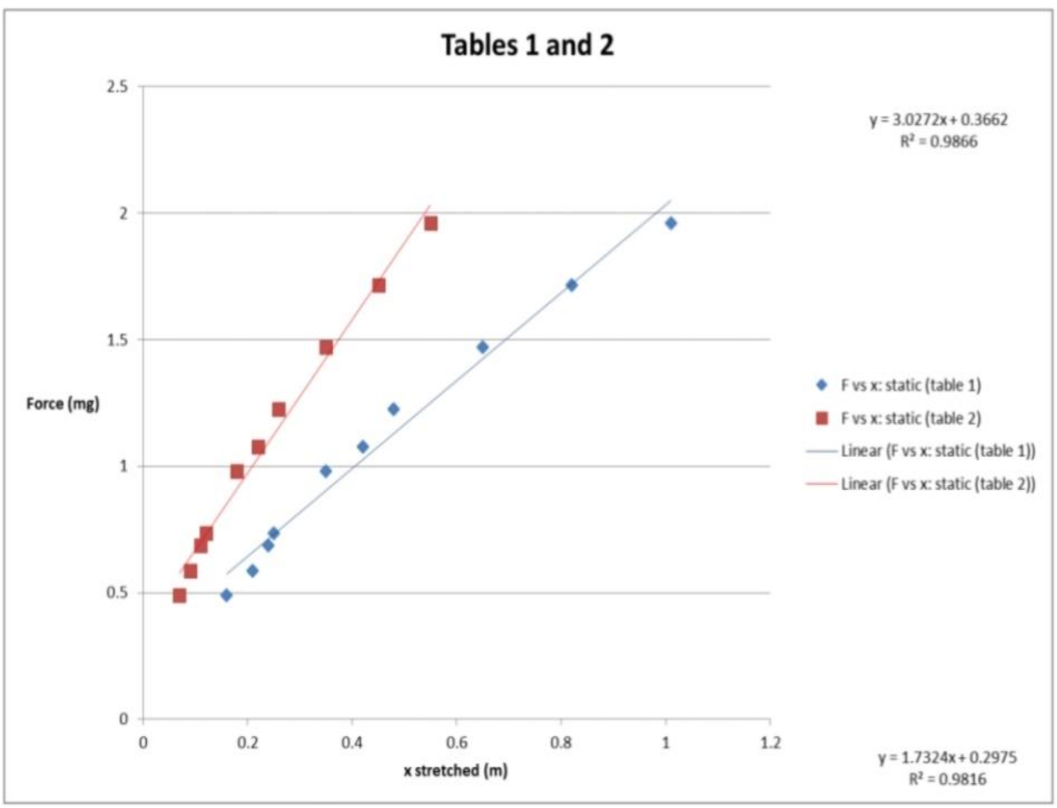


Table 2 (F vs x: static)
Force (mg)
x stretched (m)
y = 3.0272x + 0.3662
R² = 0.9866
y = -2.4941x² + 4.5156x + 0.2123
R² = 0.9983
y = 2.9608x^0.6657
R² = 0.9983
Table 2 (F vs x: static)
Linear (Table 2 (F vs x: static))
Poly. (Table 2 (F vs x: static))
Power (Table 2 (F vs x: static))


Tables 1 and 2
Force (mg)
x stretched (m)
y = 3.0272x + 0.3662
R² = 0.9866
F vs x: static (table 1)
F vs x: static (table 2)
Linear (F vs x: static (table 1))
Linear (F vs x: static (table 2))
y = 1.7324x + 0.2975
R² = 0.9816

Conclusion

We can calculate the periodic time value of an oscillating object, doing so with respect to it's Origin, by this method. Also, we can find out the amount of force required by an object to oscillate for a desired time or vice versa.

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