STUDY ON THE PROPERTIES OF TWO AND THREE DIMENSIONAL IMPRECISE NUMBERS

Sahalad Borgoyary

Department of Mathematics,
Central Institute of Technology Kokrajhar, Assam, India.

Abstract

To discuss the properties of two and three-dimensional imprecise numbers we use the definition of intersection and union of imprecise numbers. We want to prove that the properties of classical set theory are hold good in the definition of two and three-dimensional imprecise numbers. Here, intersection and the union of two imprecise numbers is defined by the maximum and the minimum operators.

***Keywords*-Reference function, imprecise number, Membership function, Membership value, Indicator function, Normal imprecise number, Two-dimensional imprecise number, Three-dimensional imprecise number.**

1. INTRODUCTION

Lofti A Zadeh 1965[1]was first introduced the theory of fuzzy set. In this theory classical set properties of intersection and unions are not satisfied. So, Baruah 2011[2],[3],[4] has identified that these properties are not hold good due to his complement definition and proposed new complement definition of fuzzy set with respect to membership function and the reference function. This new form is known as imprecise number. It is aninterval definable of fuzziness numbers, where both membership function and the complement are measured from the reference function. For the complement of fuzzy number membership function is the reference function from where membership value of the complementary of fuzzy number will be counted.

Set is the well-defined objects. So any two and three dimensional imprecise numbers is also like a classical set that can satisfies all the properties of classical set theory. So, the classical set theory properties occur under the set operations of intersection and union are proof in the definition of two and three-dimensional imprecise numbers with examples.Properties of universal laws, associativity laws and distributive laws are already proved in the article 2015[15]. Thus the remaining properties of classical settheories are undertaken to prove in this article.
Rest of this article is organized as follows-Section IIpreliminaries, Section III introduction definition of two-dimensional imprecise numbers and its complement along with prove of their properties,Section IV introduction definition of three dimensional imprecise numbers and their complement. Finally section V goes to the conclusion and the discussion.

II. PRELIMINARIES

2.1. ***Definition:***A two dimensional imprecise number

$$N_{XY} = [(\alpha_x, \alpha_y), (\beta_x, \beta_y), (\gamma_x, \gamma_y)]$$

isdivided into closed sub intervals with a partial element is presence in both the intervals. Where all the points in this interval are element of Cartesian product of two sets $X \times Y$and both the sets X and Y are the imprecise numbers.

2.2. Definition: For a two-dimensional imprecise number,

$N_{XY} = [(\alpha_x, \alpha_y), (\beta_x, \beta_y), (\gamma_x, \gamma_y)]$,the indicator function will be represented and defined by

$$\mu_{N_{XY}} = \begin{cases} \mu_{XY1}(x,y), & (\alpha_x, \alpha_y) \le (x,y) \le (\beta_x, \beta_y) \\ \mu_{XY2}(x,y), & (\beta_x, \beta_y) \le (x,y) \le (\gamma_x, \gamma_y) \\ 0, & otherwise \end{cases} \quad (1)$$

Such that $\mu_{XY1}(\alpha_x, \alpha_y) = \mu_{XY2}(\gamma_x, \gamma_y) = 0$and $\mu_{XY1}(\beta_x, \beta_y) \ne 0, \mu_{XY2}(\beta_x, \beta_y) \ne 0$

where$\mu_{XY1}(x,y)$ is non-decreasing function over the closed interval $[(\alpha_x, \alpha_y), (\beta_x, \beta_y)]$ and $\mu_{XY2}(x,y)$ is non-increasing over the closed interval, $[(\beta_x, \beta_y), (\gamma_x, \gamma_y)]$.

Two dimensional imprecise numbers can be characterized by,

$$\{\mu_{XY1}(x,y), \mu_{XY2}(x,y) : (x,y) \in X \times Y\},$$

where$\mu_{XY1}(x,y)$ and $\mu_{XY2}(x,y)$ are called membership function and the reference function of the indicator function $\mu_{N_{XY}}$ defined above and the membership value is measured from the reference function, then

$$(\mu_{XY1}(x,y) - \mu_{XY2}(x,y)) = (x_1 - x_2) \times (y_1 - y_2) \quad (2)$$

iscalled the membership value of the indicator function.

Where,$\mu_{XY1}(x,y) = (x_1, y_1)$ and $\mu_{XY2}(x,y) = (x_2, y_2)$respectively.

The collection of all such elements is calledtwo dimensional imprecise set.

If $A(\mu_{XY}(x,y)) = \{\mu_{XY1}(x,y), \mu_{XY2}(x,y): (x,y) \in X \times Y\}$ and
$B(\mu_{XY}(x,y)) = \{\mu_{XY3}(x,y), \mu_{XY4}(x,y): (x,y) \in X \times Y\}$

Then, intersection and union of two-dimensional imprecise numbers are defined by,

$$A(\mu_{XY}(x,y)) \cap B(\mu_{XY}(x,y)) = \left\{ \begin{matrix} min(\mu_{XY1}(x,y), \mu_{XY3}(x,y)), \\ max(\mu_{XY2}(x,y), \mu_{XY4}(x,y)): (x,y) \in X \times Y \end{matrix} \right\} (3)$$

$$A(\mu_{XY}(x,y)) \cup B(\mu_{XY}(x,y)) = \left\{ \begin{matrix} max(\mu_{XY1}(x,y), \mu_{XY3}(x,y)), \\ min(\mu_{XY2}(x,y), \mu_{XY4}(x,y)): (x,y) \in X \times Y \end{matrix} \right\} (4)$$

III. PROPERTIES OF TWO-DIMENSIONAL IMPRECISE NUMBERS

In the definition of Baruah [3] , imprecise number is defined over the real line. It is along the X-axis. This case is studied if the effect of fuzziness over the physical significance is along the x-axis or the real line when all other remaining axes are already fully membership. In practical such a standard problemis limited. So, to study more practical problems it may be introduced two-dimensional imprecise numbers. Two-dimensional numberis expressible in XY-plane. Here, imprecise number is defined in the two-dimensional form such a way that full membership along the x-axis and the y-axis are considered one and the other axes are already fully membership.

Based on the intersection and the union definitionof two imprecise numbers the followingclassical set theory properties can be proposed for the two dimensional imprecise numbers.

If $A(\mu_{XY}(x,y)) = \{\mu_{XY1}(x,y), \mu_{XY2}(x,y): (x,y) \in X \times Y\}$ and
$B(\mu_{XY}(x,y)) = \{\mu_{XY3}(x,y), \mu_{XY4}(x,y): (x,y) \in X \times Y\}$, then

3.1. Property(Idempotence Law)

(i) $A((\mu_{XY}(x,y)) \cap A(\mu_{XY}(x,y)) = A((\mu_{XY}(x,y))$

(ii) $A((\mu_{XY}(x,y)) \cup A((\mu_{XY}(x,y)) = A((\mu_{XY}(x,y))$

Obviously the properties can be proved.

3.2. Property(Identity Law**)**

(i) $A((\mu_{XY}(x,y)) \cap \emptyset(\mu_{XY}(x,y)) = \emptyset((\mu_{XY}(x,y))$

(ii) $A((\mu_{XY}(x,y)) \cup \emptyset((\mu_{XY}(x,y)) = A((\mu_{XY}(x,y))$

(iii) $A((\mu_{XY}(x,y)) \cap X(\mu_{XY}(x,y)) = A((\mu_{XY}(x,y))$

(iv) $A((\mu_{XY}(x,y)) \cup X((\mu_{XY}(x,y)) = X((\mu_{XY}(x,y))$

Where $X((\mu_{XY}(x,y))$ is universal imprecise number and $\emptyset((\mu_{XY}(x,y))$ is null imprecise number.

To prove the property 3.2 (i) and 3.2 (ii), let us consider $A((\mu_{XY1}(x,y)) = \left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{3},\frac{1}{3}\right)\right\}$ and $\emptyset((\mu_{XY}(x,y)) = \left\{(0,0),\left(\frac{1}{3},\frac{1}{3}\right)\right\}$, be such that membership function of the imprecise number of $A((\mu_{XY1}(x,y))$ is $\left(\frac{1}{2},\frac{1}{2}\right)$ and is measured from the reference function, $\left(\frac{1}{3},\frac{1}{3}\right)$, where $\left(\frac{1}{2},\frac{1}{2}\right)$ and $\left(\frac{1}{3},\frac{1}{3}\right)$ are the half portion, one third portion of the two dimensional object respectively.

$\emptyset((\mu_{XY}(x,y)) = \left\{(0,0),\left(\frac{1}{3},\frac{1}{3}\right)\right\}$ is a null imprecise number measured from the one third potion of the two dimensional object. Here membership function is zero due to null .Then

(i) **Proof:** $A((\mu_{XY1}(x,y)) \cap \emptyset((\mu_{XY}(x,y)) = \left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{3},\frac{1}{3}\right)\right\} \cap \left\{(0,0),\left(\frac{1}{3},\frac{1}{3}\right)\right\}$

$$= \left\{ \begin{matrix} \left(\min\left(\frac{1}{2},0\right), \min\left(\frac{1}{2},0\right)\right), \\ \left(max\left(\frac{1}{3},\frac{1}{3}\right), max\left(\frac{1}{3},\frac{1}{3}\right)\right) \end{matrix} \right\} = \left\{(0,0),\left(\frac{1}{3},\frac{1}{3}\right)\right\} = \emptyset((\mu_{XY1}(x,y))$$

Hence proved

Similarly property 3.2(ii) can be proved.

To prove the property 3.2 (iii) and 3.2 (iv), let us consider $A((\mu_{XY1}(x,y)) = \left\{\left(\frac{1}{4},\frac{1}{4}\right),\left(\frac{1}{5},\frac{1}{5}\right)\right\}$ and $X((\mu_{XY}(x,y)) = \left\{\left(\frac{3}{4},\frac{3}{4}\right),\left(\frac{1}{5},\frac{1}{5}\right)\right\}$, be such that membership function of the imprecise number of $A((\mu_{XY1}(x,y))$ is $\left(\frac{1}{4},\frac{1}{4}\right)$ and measured from the reference function, $\left(\frac{1}{5},\frac{1}{5}\right)$, where $\left(\frac{1}{4},\frac{1}{4}\right)$ and $\left(\frac{1}{5},\frac{1}{5}\right)$ are the one fourth portion, one fifth portion of the two dimensional object respectively.

$X((\mu_{XY}(x,y)) = \left\{\left(\frac{3}{4},\frac{3}{4}\right),\left(\frac{1}{5},\frac{1}{5}\right)\right\}$ is the universal imprecise number measured from the one fifth potion of the two dimensional object. Here membership function is three fourth portion of the two dimensional object and is greater than the membership value of $A((\mu_{XY1}(x,y))$. Then

(ii) **Proof:** $A((\mu_{XY1}(x,y)) \cap \emptyset((\mu_{XY}(x,y)) = \left\{\left(\frac{1}{4},\frac{1}{4}\right),\left(\frac{1}{5},\frac{1}{5}\right)\right\} \cap \left\{\left(\frac{3}{4},\frac{3}{4}\right),\left(\frac{1}{5},\frac{1}{5}\right)\right\}$

$$=\left\{\begin{matrix}\left(\min\left(\frac{1}{4},\frac{3}{4}\right),\min\left(\frac{1}{4},\frac{3}{4}\right)\right),\\ \left(max\left(\frac{1}{5},\frac{1}{5}\right),max\left(\frac{1}{5},\frac{1}{5}\right)\right)\end{matrix}\right\}=\left\{\left(\frac{1}{4},\frac{1}{4}\right),\left(\frac{1}{5},\frac{1}{5}\right)\right\}=\emptyset((\mu_{XY1}(x,y))$$

Hence proved

(iii) **Proof:** $A((\mu_{XY1}(x,y))\cup\emptyset((\mu_{XY}(x,y))=\left\{\left(\frac{1}{4},\frac{1}{4}\right),\left(\frac{1}{5},\frac{1}{5}\right)\right\}\cup\left\{\left(\frac{3}{4},\frac{3}{4}\right),\left(\frac{1}{5},\frac{1}{5}\right)\right\}$

$$=\left\{\begin{matrix}\left(\max\left(\frac{1}{4},\frac{3}{4}\right),\max\left(\frac{1}{4},\frac{3}{4}\right)\right),\\ \left(min\left(\frac{1}{5},\frac{1}{5}\right),min\left(\frac{1}{5},\frac{1}{5}\right)\right)\end{matrix}\right\}=\left\{\left(\frac{3}{4},\frac{3}{4}\right),\left(\frac{1}{5},\frac{1}{5}\right)\right\}=X((\mu_{XY1}(x,y))$$

Hence prove

3.3. Property (Associatively Laws): If $A((\mu_{XY}(x,y))=\{\mu_{XY1}(x,y),\mu_{XY2}(x,y):(x,y)\in X\times Y\}$,
$B((\mu_{XY}(x,y))=\{\mu_{XY3}(x,y),\mu_{xy4}(x,y))(x,y)\in X\times Y\}$
and$C((\mu_{XY}(x,y))=\{\mu_{XY5}(x,y),\mu_{XY6}(x,y):(x,y)\in X\times Y\}$be two dimensional imprecise numbers, then

(i) $A(\mu_{XY}(x,y))\cup\left(B(\mu_{XY}(x,y))\cup C(\mu_{XY}(x,y))\right)=\left(A(\mu_{XY}(x,y))\cup B(\mu_{XY}(x,y))\right)\cup C(\mu_{XY}(x,y))$

(ii) $A(\mu_{XY}(x,y))\cap\left(B(\mu_{XY}(x,y))\cap C(\mu_{XY}(x,y))\right)=\left(A(\mu_{XY}(x,y))\cap B(\mu_{XY}(x,y))\right)\cap C(\mu_{XY}(x,y))$

To prove this property let us consider,$A((\mu_{XY}(x,y))=\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$, $B((\mu_{XY}(x,y))=\left\{\left(\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$and $C((\mu_{XY}(x,y))=\left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$, then

(i) **Proof:** $A(\mu_{XY}(x,y))\cup\left(B(\mu_{XY}(x,y))\cup C(\mu_{XY}(x,y))\right)$

$$=\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\cup\left(\left\{\left(\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\cup\left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\right)$$

$$=\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\cup\left\{\begin{matrix}\left(\max\left(\frac{1}{3},\frac{1}{6}\right),\max\left(\frac{1}{3},\frac{1}{6}\right)\right),\\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\}$$

$$=\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\cup\left\{\left(\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}=\left\{\begin{matrix}\left(\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right)\right),\\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\}$$

$$=\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$\left(A(\mu_{XY}(x,y))\cup B(\mu_{XY}(x,y))\right)\cup C(\mu_{XY}(x,y))=\left(\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\cup\left\{\left(\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\right)\cup\left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$=\left\{\begin{matrix}\left(\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right)\right),\\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\}\cup\left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}=\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\cup\left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$=\left\{\begin{matrix}\left(\max\left(\frac{1}{2},\frac{1}{6}\right),\max\left(\frac{1}{2},\frac{1}{6}\right)\right),\\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\}=\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$$

Hence proved

(ii) **Proof:** $A(\mu_{XY}(x,y))\cap\left(B(\mu_{XY}(x,y))\cap C(\mu_{XY}(x,y))\right)$

$$=\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\cap\left(\left\{\left(\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\cap\left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\right)$$

$$=\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\cap\left\{\begin{matrix}\left(\min\left(\frac{1}{3},\frac{1}{6}\right),\min\left(\frac{1}{3},\frac{1}{6}\right)\right),\\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\}=\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\cap\left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$=\left\{\begin{matrix}\left(\min\left(\frac{1}{2},\frac{1}{6}\right),\min\left(\frac{1}{2},\frac{1}{6}\right)\right),\\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\}=\left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$\left(A(\mu_{XY}(x,y)) \cap B(\mu_{XY}(x,y))\right) \cap C(\mu_{XY}(x,y)) = \left(\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left\{\left(\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\right) \cap \left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{matrix}\left(\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right)\right), \\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\} \cap \left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\} = \left\{\left(\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{matrix}\left(\min\left(\frac{1}{3},\frac{1}{6}\right),\min\left(\frac{1}{3},\frac{1}{6}\right)\right), \\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\} = \left\{\left(\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$$

Hence proved

3.4. Property (De Morgan's Law):

If $A((\mu_{XY}(x,y)) = \{\mu_{XY1}(x,y), \mu_{XY2}(x,y) : (x,y) \in X \times Y\}$,
$B((\mu_{XY}(x,y)) = \{\mu_{XY3}(x,y), \mu_{XY4}(x,y) : (x,y) \in X \times Y\}$be two dimensional imprecise numbers, then

(i) $\left(A(\mu_{XY}(x,y)) \cup B(\mu_{XY}(x,y))\right)^C = A(\mu_{XY}{}^C(x,y)) \cap B(\mu_{XY}{}^C(x,y))$

(ii) $\left(A(\mu_{XY}(x,y)) \cap B(\mu_{XY}(x,y))\right)^C = A(\mu_{XY}{}^C(x,y)) \cup B(\mu_{XY}{}^C(x,y))$

To prove this property let us take the above two dimensional imprecise numbers $A((\mu_{XY}(x,y)) = \left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$, $B((\mu_{XY}(x,y)) = \left\{\left(\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}$, then

(i) Proof: $\left(A(\mu_{XY}(x,y)) \cup B(\mu_{XY}(x,y))\right)^C = \left(\left\{\begin{matrix}\left(\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right)\right), \\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\}\right)^C$

$$= \left(\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C = \left\{(1,1),\left(\frac{1}{2},\frac{1}{2}\right)\right\}$$

$$A(\mu_{XY}{}^C(x,y)) \cap B(\mu_{XY}{}^C(x,y)) = \left(\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C \cap \left(\left\{\left(\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C$$

$$= \left\{(1,1),\left(\frac{1}{2},\frac{1}{2}\right)\right\} \cap \left\{(1,1),\left(\frac{1}{3},\frac{1}{3}\right)\right\} = \left\{\begin{matrix}(\min(1,1),\min(1,1)), \\ \left(max\left(\frac{1}{2},\frac{1}{3}\right),max\left(\frac{1}{2},\frac{1}{3}\right)\right)\end{matrix}\right\}$$

$$= \left\{(1,1),\left(\frac{1}{2},\frac{1}{2}\right)\right\}$$

Hence proved

(ii) Proof: $\left(A(\mu_{XY}(x,y)) \cap B(\mu_{XY}(x,y))\right)^C = \left(\left\{\begin{matrix}\left(\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right)\right), \\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{matrix}\right\}\right)^C$

$$= \left(\left\{\left(\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C = \left\{(1,1),\left(\frac{1}{3},\frac{1}{3}\right)\right\}$$

$$A(\mu_{XY}{}^C(x,y)) \cup B(\mu_{XY}{}^C(x,y)) = \left(\left\{\left(\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C \cup \left(\left\{\left(\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C$$

$$= \left\{(1,1),\left(\frac{1}{2},\frac{1}{2}\right)\right\} \cup \left\{(1,1),\left(\frac{1}{3},\frac{1}{3}\right)\right\} = \left\{\begin{matrix}(\max(1,1),\max(1,1)), \\ \left(min\left(\frac{1}{2},\frac{1}{3}\right),min\left(\frac{1}{2},\frac{1}{3}\right)\right)\end{matrix}\right\}$$

$$= \left\{(1,1),\left(\frac{1}{3},\frac{1}{3}\right)\right\}$$

Hence proved

Here, membership function of $A((\mu_{XY}(x,y))$ and $B((\mu_{XY}(x,y))$ are $\left(\frac{1}{2},\frac{1}{2}\right)$ and $\left(\frac{1}{3},\frac{1}{3}\right)$ and are measured from the reference function, $\left(\frac{1}{9},\frac{1}{9}\right)$. So the complement of $A((\mu_{XY}(x,y))$ and $B((\mu_{XY}(x,y))$ are measured from reference function $\left(\frac{1}{2},\frac{1}{2}\right)$ and $\left(\frac{1}{3},\frac{1}{3}\right)$ to till the highest membership function of the two dimensional imprecise function (1,1).

3.5. Applications of two dimensional Imprecise numbers

Two-dimensional imprecise number can be obtained as a applications in the field of economics. Assume that 60% of the effort of production of different crops of our country is done every year to fulfill the 75% needs or demandof our people. So, the production of crops should be increased to 85% so that demand can be fulfill hundred percent. In this case demand and the production situation can be expressed into two-dimensional imprecise number. Thus imprecise number is obtained in the following form,

$A((\mu_{XY}(x,y)) = \{(85\%\ ,100\%),(60\%,75\%)\} = \left\{\left(\frac{17}{20},1\right),\left(\frac{3}{5},\frac{3}{4}\right)\right\}$.

Here, membership value can be model in the following form,
$A\{\mu_{XY1}(x,y)),(\mu_{XY2}(x,y):(x,y)\in X\times Y\} = |x_1-x_2|,|y_1-y_2|$. Where $|x_1-x_2|$and $|y_1-y_2|$ are distinct behaviors. In case the behaviors of these are similar, then their product will be the membership function of the imprecise number. Otherwise membership function will be counted separately.
Thus, membership function of above demands and production problem is, $\left|\frac{3}{5}-\frac{17}{20}\right|,\left|1-\frac{3}{4}\right| = \left|\frac{-5}{20}\right|,\left|\frac{1}{4}\right| = \left|\frac{1}{4}\right|,\left|\frac{1}{4}\right|$. Which shows that 25% more effort of production and demand has to increase to fulfill the people need of our country.

IV.PRELIMINARY

It is mentioned in the definition of two dimensional imprecise numbers that the effecting parts of a body of fuzziness is along the two axes and all others are already fully membership. Roughly all the physical problems can be expressed in the three dimensions form. So to study effect of fuzziness in the body along the length, breadth and height it may be introduced three dimensional imprecise numbers. Three dimensional imprecise numbers is expressible in XYZ-solid geometry. Here, imprecise number is defined in the three-dimensional form in such a way that full membership along the x-axis, the y-axis and the z-axis respectively is considered membership value one.

4.1. Definition: A three dimensional imprecise number

$$N_{XYZ} = [(\alpha_x,\alpha_y\alpha_z),(\beta_x,\beta_y,\beta_y),(\gamma_x,\gamma_y,\gamma_z)]$$

Is divided into closed sub intervals with a partial element is presence in both the intervals. Where all the points in this interval are the elements of Cartesian product of two sets $X\times Y\times Z$and the sets X, Y and Z are imprecise numbers.

4.2. Definition: For a three-dimensional imprecise number,

$$N_{XYZ} = [(\alpha_x,\alpha_y\alpha_z),(\beta_x,\beta_y,\beta_y),(\gamma_x,\gamma_y,\gamma_z)],$$

the indicator function will be represented and defined by

$$\mu_{N_{XYZ}} = \begin{cases} \mu_{XYZ1}(x,y,z), & (\alpha_x,\alpha_y,\alpha_z) \le (x,y,z) \le (\beta_x,\beta_y,\beta_z) \\ \mu_{XYZ2}(x,y,z), & (\beta_x,\beta_y,\beta_z) \le (x,y,z) \le (\gamma_x,\gamma_y,\gamma_z) \\ 0 & otherwise \end{cases} \quad (5)$$

Such that, $\mu_{XYZ1}(\alpha_x,\alpha_y,\alpha_z) = \mu_{XYZ2}(\gamma_x,\gamma_y,\gamma_z) = (0,0,0)$
and$\mu_{XYZ1}(\beta_x,\beta_y,\beta_z) \ne 0, \mu_{XYZ2}(\beta_x,\beta_y,\beta_z) \ne 0$

where$\mu_{XYZ1}(x,y,z)$ is non-decreasing function over the closed interval $[(\alpha_x,\alpha_y,\alpha_z),(\beta_x,\beta_y,\beta_z)]$ and $\mu_{XYZ2}(x,y,z)$ is non-increasing over the closed interval, $[(\beta_x,\beta_y,\beta_z),(\gamma_x,\gamma_y,\gamma_z)]$.
Three-dimensional imprecise number would be characterized by,

$$\{\mu_{XYZ1}(x,y,z)),(\mu_{XYZ2}(x,y,z):(x,y,z)\in X\times Y\times Z\}$$

where$\mu_{XYZ1}(x,y,z)$ and $\mu_{XYZ2}(x,y,z)$ are called membership function and the reference function of the indicator function $\mu_{N_{XYZ}}$ defined above and the membership value is measured from the reference function, then

$$(\mu_{XYZ1}(x,y,z) - \mu_{XYZ2}(x,y,z)) = (x_1-x_2)\times(y_1-y_2)\times(z_1-z_2)(6)$$

is called the membership value of the indicator function.
Where$\mu_{XYZ1}(x,y,z) = (x_1,y_1,z_1)$and $\mu_{XYZ2}(x,y,z) = (x_2,y_2,z_2)$respectively.
The collection of all such elements is called three dimensional imprecise set.

Intersection and union of three-dimensional imprecise numbers defined by,
If $A(\mu_{XYZ}(x,y,z)) = \{\mu_{XYZ1}(x,y,z)),(\mu_{XYZ2}(x,y,z) : (x,y,z)\in X\times Y\times Z\}$
And $B(\mu_{XYZ}(x,y,z)) = \{\mu_{XYZ3}(x,y,z)),(\mu_{XYZ4}(x,y,z):(x,y,z)\in X\times Y\times Z\}$, then

$$A(\mu_{XYZ}(x,y,z)) \cap B(\mu_{XYZ}(x,y,z)) = \begin{Bmatrix} min(\mu_{XYZ1}(x,y,z),\mu_{XYZ3}(x,y,z)), \\ max(\mu_{XYZ2}(x,y,z),\mu_{XYZ4}(x,y,z)): \\ (x,y,z)\in X\times Y\times Z \end{Bmatrix} (7)$$

$$A(\mu_{XYZ}(x,y,z)) \cup B(\mu_{XYZ}(x,y,z)) = \begin{Bmatrix} max(\mu_{XYZ1}(x,y,z),\mu_{XYZ3}(x,y,z)), \\ min(\mu_{XYZ2}(x,y,z),\mu_{XYZ4}(x,y,z)): \\ (x,y,z)\in X\times Y\times Z \end{Bmatrix} (8)$$

V. PROPERTIES OF THREE-DIMENSIONAL IMPRECISE NUMBERS

Based on the above definition of intersection and union classical set theory properties can be proposed for the three dimensional imprecise numbers.

5.1. Property(Idempotence Law)

(i) $A((\mu_{XYZ}(x,y,z)) \cap A(\mu_{XYZ}(x,y,z)) = A((\mu_{XYZ}(x,y,z))$

and (ii) $((\mu_{XYZ}(x,y,z)) \cup A((\mu_{XYZ}(x,y,z)) = A((\mu_{XYZ}(x,y,z))$
Obviously the properties can be proved.

5.2. Property(Identity Law**)**

(i) $A((\mu_{XYZ}(x,y,z)) \cap \emptyset(\mu_{XYZ}(x,y,z)) = \emptyset((\mu_{XYZ}(x,y.z))$

(ii) $((\mu_{XYZ}(x,y,z)) \cup \emptyset((\mu_{XYZ}(x,y,z)) = A((\mu_{XYZ}(x,y,z))$

(iii) $A((\mu_{XYZ}(x,y,z)) \cap X(\mu_{XYZ}(x,y,z)) = A((\mu_{XYZ}(x,y,z))$

(iv)$((\mu_{XYZ}(x,y,z)) \cup X((\mu_{XYZ}(x,y,z)) = X((\mu_{XYZ}(x,y,z))$

Where,$X((\mu_{XYZ}(x,y,z))$ is universal set and $\emptyset((\mu_{XYZ}(x,y,z))$ is null set.

To prove 5.2. (i) and5.2.(ii) let us consider$A((\mu_{XYZ1}(x,y,z)) = \left\{\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\}$and $\emptyset((\mu_{XYZ2}(x,y,z)) = \left\{(0,0,0),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\}$, be such that membership function of three dimensional imprecise number of $A((\mu_{XYZ1}(x,y,z))$ is $\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right)$ and is measured from the reference function, $\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)$. Where $\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right)$ and $\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)$ are one seventh portion, one eighth portion of the three dimensional object respectively.

$\emptyset((\mu_{XYZ2}(x,y,z)) = \left\{(0,0,0),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\}$is a null imprecise number measured from the one eighth potion of the three dimensional object. Here membership function is zero due to null. Then,

(i) Proof:

$$A((\mu_{XYZ1}(x,y,z)) \cap \emptyset((\mu_{XYZ2}(x,y,z)) = \left\{\begin{array}{l}\left(\min\left(\frac{1}{7},0\right),\min\left(\frac{1}{7},0\right),min\left(\frac{1}{7},0\right)\right),\\ \left(max\left(\frac{1}{8},\frac{1}{8}\right),max\left(\frac{1}{8},\frac{1}{8}\right),max\left(\frac{1}{8},\frac{1}{8}\right)\right)\end{array}\right\}$$

$$= \left\{(0,0,0),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\} = \emptyset((\mu_{XYZ2}(x,y,z))$$

Hence proved

Similarly property 5.2(ii) can be proved.

To prove property 5.2.(iii) and 5.2.(iv) let,$A((\mu_{XYZ1}(x,y,z)) = \left\{\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\}$and $X((\mu_{XYZ2}(x,y,z)) = \left\{\left(\frac{2}{7},\frac{2}{7},\frac{2}{7}\right),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\}$, be such that membership function of three dimensional imprecise number of $A((\mu_{XYZ1}(x,y,z))$ is $\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right)$ and measured from the reference function, $\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)$. Where $\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right)$ and $\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)$ are one seventh portion, one eighth portion of the three dimensional object respectively.

$X((\mu_{XYZ2}(x,y,z)) = \left\{\left(\frac{2}{7},\frac{2}{7},\frac{2}{7}\right),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\}$is a universal set imprecise number measured from the one eighth potion of the three dimensional object. Here membership function is two third portion of the object and is greater than the membership value of $A((\mu_{XYZ1}(x,y,z))$. Then

(ii) Proof:

$$A((\mu_{XYZ1}(x,y,z)) \cap X((\mu_{XYZ2}(x,y,z)) = \left\{\begin{array}{l}\left(\min\left(\frac{1}{7},\frac{2}{7}\right),\min\left(\frac{1}{7},\frac{2}{7}\right),min\left(\frac{1}{7},\frac{2}{7}\right)\right),\\ \left(max\left(\frac{1}{8},\frac{1}{8}\right),max\left(\frac{1}{8},\frac{1}{8}\right),max\left(\frac{1}{8},\frac{1}{8}\right)\right)\end{array}\right\}$$

$$= \left\{\left(\frac{1}{7},\frac{1}{7},\frac{1}{7}\right),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\} = A((\mu_{XYZ1}(x,y,z))$$

Hence Prove

(iii) Proof:

$$A((\mu_{XYZ1}(x,y,z)) \cup X((\mu_{XYZ2}(x,y,z)) = \left\{\begin{array}{l}\left(\max\left(\frac{1}{7},\frac{2}{7}\right),\max\left(\frac{1}{7},\frac{2}{7}\right),max\left(\frac{1}{7},\frac{2}{7}\right)\right),\\ \left(min\left(\frac{1}{8},\frac{1}{8}\right),min\left(\frac{1}{8},\frac{1}{8}\right),min\left(\frac{1}{8},\frac{1}{8}\right)\right)\end{array}\right\}$$

$$= \left\{\left(\frac{2}{7},\frac{2}{7},\frac{2}{7}\right),\left(\frac{1}{8},\frac{1}{8},\frac{1}{8}\right)\right\} = X((\mu_{XYZ2}(x,y,z))$$

Hence proved

5.3. Property (Associatively Laws):

If $A((\mu_{XYZ}(x,y,z)) = \{\mu_{XYZ1}(x,y,z),\mu_{XYZ2}(x,y,z):(x,y,z) \in X \times Y \times Z\}$,
$B((\mu_{XYZ}(x,y,z)) = \{\mu_{XYZ3}(x,y,z),\mu_{XYZ4}(x,y,z):(x,y,z) \in X \times Y \times Z\}$
and$C((\mu_{XYZ}(x,y,z)) = \{\mu_{XYZ5}(x,y,z),\mu_{XYZ6}(x,y,z):(x,y,z) \in X \times Y \times Z\}$ be three dimensional imprecise number, then

(i) $A(\mu_{XYZ}(x,y,z)) \cup \left(B(\mu_{XYZ}(x,y,z)) \cup C(\mu_{XYZ}(x,y,z))\right) = \left(A(\mu_{XYZ}(x,y,z)) \cup B(\mu_{XYZ}(x,y,z))\right) \cup C(\mu_{XYZ}(x,y,z))$

(ii) $A(\mu_{XYZ}(x,y,z)) \cap \left(B(\mu_{XYZ}(x,y,z)) \cap C(\mu_{XYZ}(x,y,z))\right) = \left(A(\mu_{XYZ}(x,y,z)) \cap B(\mu_{XYZ}(x,y,z))\right) \cap C(\mu_{XYZ}(x,y,z))$

To prove this property let us consider, $A((\mu_{XYZ}(x,y,z)) = \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$, $B((\mu_{XYZ}(x,y,z)) = \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}andC((\mu_{XYZ}(x,y,z)) = \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$, then

(i) Proof: $A\left(\mu_{XYZ}(x,y,z)\right) \cup \left(B\left(\mu_{XYZ}(x,y,z)\right) \cup C\left(\mu_{XYZ}(x,y,z)\right)\right)$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cup \left(\left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cup \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cup \left\{\begin{array}{c}\left(\max\left(\frac{1}{3},\frac{1}{6}\right),\max\left(\frac{1}{3},\frac{1}{6}\right),\max\left(\frac{1}{3},\frac{1}{6}\right)\right),\\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{array}\right\}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cup \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{array}{c}\left(\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right)\right),\\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{array}\right\} = \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$\left(A\left(\mu_{XYZ}(x,y,z)\right) \cup B\left(\mu_{XYZ}(x,y,z)\right)\right) \cup C\left(\mu_{XYZ}(x,y,z)\right)$$

$$= \left(\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cup \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right) \cup \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{array}{c}\left(\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right)\right),\\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{array}\right\} \cup \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cup \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{array}{c}\left(\max\left(\frac{1}{2},\frac{1}{6}\right),\max\left(\frac{1}{2},\frac{1}{6}\right),\max\left(\frac{1}{2},\frac{1}{6}\right)\right),\\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)min\left(\frac{1}{9},\frac{1}{9}\right),\right)\end{array}\right\} = \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

Hence proved

(ii) Proof: $A\left(\mu_{XYZ}(x,y,z)\right) \cap \left(B\left(\mu_{XYZ}(x,y,z)\right) \cap C\left(\mu_{XYZ}(x,y,z)\right)\right)$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left(\left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left\{\begin{array}{c}\left(\min\left(\frac{1}{3},\frac{1}{6}\right),\min\left(\frac{1}{3},\frac{1}{6}\right),\min\left(\frac{1}{3},\frac{1}{6}\right)\right),\\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{array}\right\}$$

$$= \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{array}{c}\left(\min\left(\frac{1}{2},\frac{1}{6}\right),\min\left(\frac{1}{2},\frac{1}{6}\right),\min\left(\frac{1}{2},\frac{1}{6}\right)\right),\\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{array}\right\} = \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$\left(A\left(\mu_{XYZ}(x,y,z)\right) \cup B\left(\mu_{XYZ}(x,y,z)\right)\right) \cup C\left(\mu_{XYZ}(x,y,z)\right)$$

$$= \left(\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right) \cap \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{array}{c}\left(\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right)\right),\\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)\right)\end{array}\right\} \cap \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\} \cap \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

$$= \left\{\begin{array}{c}\left(\min\left(\frac{1}{3},\frac{1}{6}\right),\min\left(\frac{1}{3},\frac{1}{6}\right),\min\left(\frac{1}{3},\frac{1}{6}\right)\right),\\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)max\left(\frac{1}{9},\frac{1}{9}\right),\right)\end{array}\right\} = \left\{\left(\frac{1}{6},\frac{1}{6},\frac{1}{6}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$$

Hence proved

5.4. Property (De Morgan's Law):

If $A((\mu_{XYZ}(x,y,z)) = \{\mu_{XYZ1}(x,y,z), \mu_{XYZ2}(x,y,z): (x,y,z) \in X \times Y \times Z\}$,
$B((\mu_{XYZ}(x,y,z)) = \{\mu_{XYZ3}(x,y,z), \mu_{XYZ4}(x,y,z): (x,y,z) \in X \times Y \times Z\}$
be three dimensional imprecise number, then

(i) $\left(A\left(\mu_{XYZ}(x,y,z)\right) \cup B\left(\mu_{XYZ}(x,y,z)\right)\right)^C = A\left({\mu_{XYZ}}^C(x,y,z)\right) \cap B\left({\mu_{XYZ}}^C(x,y,z)\right)$

(ii) $\left(A\left(\mu_{XYZ}(x,y,z)\right) \cap B\left(\mu_{XYZ}(x,y,z)\right)\right)^C = A({\mu_{XYZ}}^C(x,y,z)) \cup B({\mu_{XYZ}}^C(x,y,z))$

To prove this property let us take the above three dimensional imprecise numbers $A((\mu_{XYZ}(x,y,z)) = \left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$, $B((\mu_{XYZ}(x,y,z)) = \left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}$, then

(i) **Proof:**

$$\left(A\left(\mu_{XYZ}(x,y,z)\right) \cup B\left(\mu_{XYZ}(x,y,z)\right)\right)^C = \left(\left\{\begin{matrix}\left(\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right),\max\left(\frac{1}{2},\frac{1}{3}\right)\right), \\ \left(min\left(\frac{1}{9},\frac{1}{9}\right),min\left(\frac{1}{9},\frac{1}{9}\right)min\left(\frac{1}{9},\frac{1}{9}\right),\right)\end{matrix}\right\}\right)^C$$

$$= \left(\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C = \left\{(1,1,1),\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\right\}$$

$$A\left({\mu_{XYZ}}^C(x,y,z)\right) \cap B\left({\mu_{XYZ}}^C(x,y,z)\right) = \left(\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C \cap \left(\left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C$$

$$= \left\{(1,1,1),\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\right\} \cap \left\{(1,1,1),\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right\}$$

$$= \left\{\begin{matrix}(\min(1,1),\min(1,1),\min(1,1)), \\ \left(max\left(\frac{1}{2},\frac{1}{3}\right),max\left(\frac{1}{2},\frac{1}{3}\right),max\left(\frac{1}{2},\frac{1}{3}\right)\right)\end{matrix}\right\} = \left\{(1,1,1),\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\right\}$$

Hence proved

(ii) Proof:

$$\left(A\left(\mu_{XYZ}(x,y,z)\right) \cap B\left(\mu_{XYZ}(x,y,z)\right)\right)^C = \left(\left\{\begin{matrix}\left(\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right),\min\left(\frac{1}{2},\frac{1}{3}\right)\right), \\ \left(max\left(\frac{1}{9},\frac{1}{9}\right),max\left(\frac{1}{9},\frac{1}{9}\right)max\left(\frac{1}{9},\frac{1}{9}\right),\right)\end{matrix}\right\}\right)^C$$

$$= \left(\left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C = \left\{(1,1,1),\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right\}$$

$$A\left({\mu_{XYZ}}^C(x,y,z)\right) \cap B\left({\mu_{XYZ}}^C(x,y,z)\right) = \left(\left\{\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C \cup \left(\left\{\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right),\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)\right\}\right)^C$$

$$= \left\{(1,1,1),\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\right\} \cup \left\{(1,1,1),\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right\}$$

$$= \left\{\begin{matrix}(\max(1,1),\text{miax}(1,1),\max(1,1)), \\ \left(min\left(\frac{1}{2},\frac{1}{3}\right),min\left(\frac{1}{2},\frac{1}{3}\right),min\left(\frac{1}{2},\frac{1}{3}\right)\right)\end{matrix}\right\} = \left\{(1,1,1),\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)\right\}$$

Hence proved

Here, membership function of $A((\mu_{XYZ}(x,y,z))$ and $B((\mu_{XYZ}(x,y,z))$ are $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ and $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ and are measured from the reference function, $\left(\frac{1}{9},\frac{1}{9},\frac{1}{9}\right)$. So the complement of $A((\mu_{XYZ}(x,y,z))$ and $B((\mu_{XYZ}(x,y,z))$ are measured from reference function $\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ and $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ to till the highest membership function of the three dimensional imprecise function,(1,1,1).

5.5. Applications of three dimensional Imprecise numbers

Three-dimensional imprecise number can be obtained as a applications in the field of economics. Assume that production, demand and service of our country is in such a way that at present 60% of the effort of production of different crops is done in every year to fulfill the 75% needs or demand of our peoples with service capacity of 65%. So, the production of crops should be increased up to 85% to fulfill the demand up to hundred percent with service effort 90%. In this case demand, production and service situation can be expressed into three-dimensional imprecise number. Thus imprecise numbers is obtained in the following form,

$$A\left(\mu_{XYZ}(x,y,z)\right) = \{(85\%, 100\%, 90\%), (60\%, 75\%, 65\%)\}$$

$$=\left\{\left(\frac{17}{20},1,\frac{9}{10}\right),\left(\frac{3}{5},\frac{3}{4},\frac{13}{20}\right)\right\}.$$

Here, membership value can be model in the following form,
$A\{\mu_{XYZ1}(x,y,z),\mu_{XYZ2}(x,y,z):(x,y,z)\in X\times Y\times Z\}=|x_1-x_2|,|y_1-y_2|,|z_1-z_2|.$
Where $|x_1-x_2|$, $|y_1-y_2|$ and $|z_1-z_2|$ are distinct behaviors. In case the behaviors of those variables are similar, then their product will be the membership function of the aboveimprecise number. Otherwise membership function will be counted separately.

Thus, membership value of production, demand and service problem is, $\left|\frac{3}{5}-\frac{17}{20}\right|,\left|1-\frac{3}{4}\right|\left|\frac{9}{10}-\frac{13}{20}\right|=\left|\frac{-5}{20}\right|,\left|\frac{1}{4}\right|=\left|\frac{1}{4}\right|,\left|\frac{1}{4}\right|,\left|\frac{1}{4}\right|$. Which shows that 25% more effort of production, demand and service has to increase to fulfill the people's need of our country.

VI. CONCLUSIONS

Real life problem is not always expressible into classical set from. It is expressible into a fuzzy or an imprecise number. Fully existence of their nature is depend on the dimension of object. So, we have discussed various situation of the imprecise numbers as the resultant of their properties with help of the definition of intersection and union of imprecise numbers. Operations of intersection and union of the imprecise numbers are expressed in term of maximum and minimum operators. And these are the tools to prove the above properties.

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