ROLE OF BODY ACCELERATION AND SLIP VELOCITY ON NON-NEWTONIAN PULSATILE FLOW OF BLOOD THROUGH A STENOSED ARTERY

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ABSTRACT-In the present analysis, the effect of external body acceleration and slip velocity on the non-Newtonian pulsatile flow of blood through a constricted blood vessel is discussed. The flow of blood in arteries is characterized as Bingham-Plastic fluid. The stenosis to be shown in the artery is taken to be overlapping stenoses. By applying a perturbation technique and expanding the axial velocity and shear stress in terms of Womersley frequency parameter α^2, the computable expression of axial velocity profile, shear stress at arterial wall, volumetric flow rate and apparent viscosity are obtained. The deviations of the flow characteristics under the influence of various parameters are presented graphically. It is observed that flow rate and velocity of blood escalate due to the influence of body acceleration and a velocity slip. To justify the validity of the model, comparisons are made with the existing results.

***KEYWORDS*-Bingham Plastic fluid, slip velocity, body acceleration, pulsatile flow, apparent viscosity, overlapping stenosis**

INTRODUCTION

Blood plays a vital role when it flows through a normal as well as diseased artery. In diseased artery, blood flow is often blocked due to abnormal tissue growth, called stenosis, in the arterial lumen. The cardiovascular diseases are one of the major reasons of the deaths in developed and developing countries. The blockage of blood flow in the coronary artery leads to angina pain and cardiac arrest. Abnormal flow of blood causes an unsteady pressure on the walls of arteries and the heart pumps the blood into the arteries in cyclic manner which creates the pulsatile flow of blood in the arteries.

Many times, external body acceleration or variations is applied to human body, for example, situation like rapid action of a tennis player, flying in an aircraft or a vibration therapy given to a heart patient etc. In the above situations, external body acceleration is applied to a particular part of the entire body which may disturbs the flow of blood and its normal functioning. When the body is exposed to these variations for a long time, it may cause some severe health problems. So the effect of the periodic body acceleration may play a significant role in the diagnosis and treatment of health problems.

To quantify the effect of periodic body acceleration on flow behaviour of blood, Elshehawey et al [1], El-shahed et al [2], Nagarani et al [3] and Biswas et al [4] studied the Newtonian and the non-Newtonian character of blood flow through normal and stenosed arteries taking no velocity slip condition. Mandal et al [5] discussed the generalized Power-law fluid model of nonlinear flow of blood through tapered vessel with time variant overlapping stenoses. Mishra et al [6] explained the effect of bell shaped stenosis on the non-Newtonian flow of blood and investigated the problem using analytical and numerical techniques. Blood is modeled as Herschel-Bulkley fluid. Taking blood as a Newtonian fluid, Verma et al [7], Biswas et al [8] and Srivastava et al [9 & 10] has done a fair amount of theoretical and experimental studies to illuminate the effect of stenosis on flow of blood through and beyond the constricted vessel. Sarojamma et al [11] studied the influence of body acceleration on the flow of blood through a catheterized artery with cosine shaped geometry of stenosis. It is noticed that flow resistance is considerably reduced on application of periodic body acceleration. Assuming the artery to be a cylindrical tube with time-dependent radius having mild stenosis, Varshney et al [12] developed a mathematical model for the pulsatile flow of blood through a stenosed tapered artery imposing external body acceleration and solved the equation of motion numerically using finite difference method. Bhatnagar et al [13] developed a mathematical model for analyzing the flow characteristics of blood through an atherosclerotic arterial segment taking the velocity slip condition at the arterial wall. Blood is considered as Herschel-Bulkley fluid flowing in a uniform right circular tube having a composite stenosis. Kakati et al [14] analyzed the Newtonian flow of blood to show the influence of body acceleration, porous medium and uniform magnetic field on flow behaviour of blood through tapered artery with mild stenosis. Ramana Reddy et al [15] studied the model of blood flow through a catheterized artery having mild overlapping stenosis and deciphered the governing equation and expressed the flow characteristics analytically. D. S. Sankar [16] analyzed pulsatile flow of blood by treating the blood in the core region as Casson fluid and the plasma in the peripheral layer as Newtonian fluid through mild stenosed narrow arteries. Bhatnagar et al [17 & 18] studied the slip effect on the Newtonian and non-Newtonian nature of blood flow through the non symmetric and overlapping stenosed artery.

The present analysis is proposed to analyze the effect of velocity slip on the pulsatile flow of blood through a narrow artery having overlapping stenoses under the application of periodic body acceleration. Non-Newtonian nature of blood is illustrated by Bingham-Plastic fluid model. It is believed that this investigation may aid the bio-engineers in the production and improvement of the artificial organs and the development of new diagnostic mechanism to treat cardiovascular diseases.

FORMULATION OF THE PROBLEM

Let us consider one dimensional fully developed, pulsatile, laminar and symmetric flow of blood through an artery in the presence of external periodic body acceleration. The artery is assumed to be rigid circular tube with overlapping stenoses inside its lumen. The geometry of overlapping stenoses in the lumen of artery is depicted (Srivastava et al [10]) in figure 1 as

$$\frac{R(z)}{R_0}=\begin{cases}1-\frac{32\delta}{L_0'^4}\left[\frac{11}{32}(z'-d')l_0'^3-\frac{47}{48}(z'-d')^2l_0'^2+(z'-d')^3l_0'-\frac{1}{3}(z'-d')^4\right], & d'\le z'\le d'+L_0' \\ 1, & otherwise\end{cases} \quad (1)$$

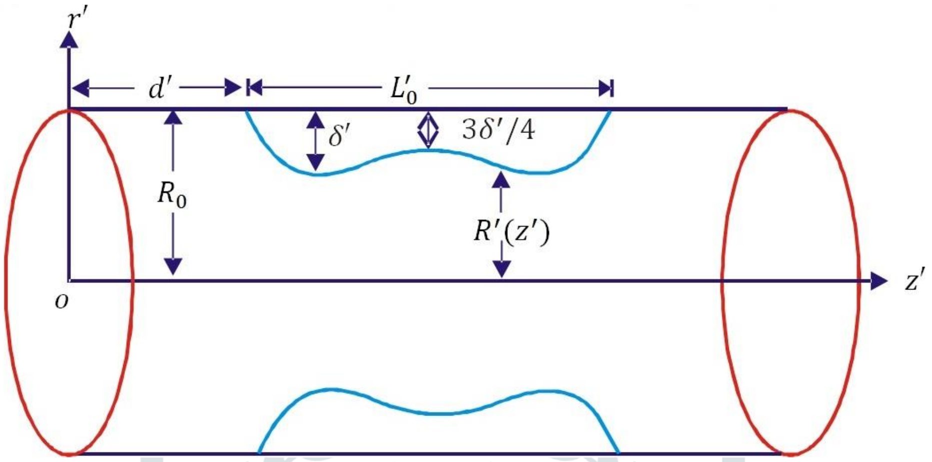

Figure 1: Overlapping stenoses in the arterial segment

Where $R'(z')$ is the radius of artery in the constricted region, R_0 is the radius outside the stenotic region, $L_0'(=3l_0'/2)$ is the length of the constriction, d' specifies the position of stenosis, stenosis has greatest height, δ', at the two throats of stenosis $z'=d'+l_0'/2$ and $z'=d'+l_0'$. At the middle of stenosis i.e. $z'=d'+3l_0'/4$, the height of stenosis is $3\delta'/4$ which is called critical height. The periodic body acceleration applied on the pulsatile flow of blood may be determined as:

$$A'(t')=A_0\cos\left(\omega_1 t'+\varphi\right) \quad (2)$$

Where $\omega_1=2\pi f_1$, f_1 is the frequency of body acceleration in Hertz, A_0 represents the amplitude of body acceleration, lead angle is denoted by ϕ . To neglect the wave effects, the frequency f_1 is taken to be small.

The pressure gradient at any z' may be expressed as follows:

$$-\frac{\partial p'}{\partial z'}=B_0+B_1\cos\left(\omega_2 t'\right) \quad (3)$$

Where $\omega_2=2\pi f_2$, f_2 is the pulse frequency, B_0 denotes the steady component and B_1 denotes the fluctuating component of pressure gradient. The equation of motion governing flow of blood can be obtained as

$$\rho\frac{\partial u'}{\partial t'}=A'(t')-\frac{\partial p'}{\partial z'}-\frac{1}{r'}\frac{\partial}{\partial r'}\left(r'\tau'\right) \quad (4)$$

The non-Newtonian nature of blood is represented by Bingham-Plastic fluid model whose constitutive equation is given by:

$$-\frac{\partial u'}{\partial r'}=\begin{cases}\frac{\tau'-\tau_c'}{\mu}, & \tau'\ge\tau_c' \\ 0, & \tau'<\tau_c'\end{cases} \quad (5)$$

Where τ' is designated for shearing stress, τ_c' for yield stress and μ for viscosity of blood.

The boundary conditions subject to the above equations are as follows:

$$u'=u_s' \text{ at } r'=R'(z') \quad (6)$$

$$\tau' \text{ is finite at } r'=0 \quad (7)$$

Where u_s' signifies the slip velocity at the constricted wall.

Now, dimensionless variables are set up as:

$$u=\frac{u'}{B_0R_0^2/4\mu},\ z=\frac{z'}{R_0},\ t=\omega_2 t',\ \omega=\frac{\omega_1}{\omega_2},\ \tau=\frac{\tau'}{B_0R_0/2},\ h=\frac{A_0}{B_0},\ d=\frac{d'}{R_0},\ L_0=\frac{L_0'}{R_0},$$
$$u_s=\frac{u_s'}{B_0R_0^2/4\mu},\ R(z)=\frac{R'(z')}{R_0},\ r=\frac{r'}{R_0},\ a=\frac{B_1}{B_0},\ \tau_c=\frac{\tau_c'}{B_0R_0/2},\ \delta=\frac{\delta'}{R_0},\ l_0=\frac{l_0'}{R_0}, \tag{8}$$

Then equation (4) transforms into:

$$\beta^2\frac{\partial u}{\partial t}=4h\cos(\omega t+\varphi)+4(1+a\cos t)-\frac{2}{r}\frac{\partial}{\partial r}(r\tau) \tag{9}$$

Where β symbolize Womersley frequency parameter and is given by:

$$\beta^2=\frac{\rho\omega_2R_0^2}{\mu}$$

The constitutive equation of Bingham-Plastic fluid changes into dimensionless form as:

$$-\frac{\partial u}{\partial r}=\begin{cases}2(\tau-\tau_c), & \tau\ge\tau_c\\ 0, & \tau<\tau_c\end{cases} \tag{10}$$

Non-dimensional boundary conditions from (6a & 6b) are as follows:

$$u=u_s \text{ at } r=R(z) \tag{11}$$

$$\tau \text{ is finite at } r=0 \tag{12}$$

Dimensionless variables from (7), change (1) into

$$R(z)=\begin{cases}1-\frac{32\delta}{L_0^4}\left[\frac{11}{32}(z-d)l_0^3-\frac{47}{48}(z-d)^2l_0^2+(z-d)^3l_0-\frac{1}{3}(z-d)^4\right], & d\le z\le d+L_0\\ 1, & otherwise\end{cases} \tag{13}$$

SOLUTION OF THE PROBLEM

Perturbation technique is used to solve the partial differential equations (9) and (10) expanding the shear stress and the axial velocity of blood in powers of small number, Womersley frequency parameter, β, which is given below:

$$u(z,r,t)=u_0(z,r,t)+\beta^2u_1(z,r,t)+\ldots \tag{14}$$

$$\tau(z,r,t)=\tau_0(z,r,t)+\beta^2\tau_1(z,r,t)+\ldots \tag{15}$$

Substituting equations (14) and (15) in (9) and comparing the coefficients of β^0 and β^2, we obtain,

$$\frac{\partial}{\partial r}(r\tau_0)=2rg(t) \tag{16}$$

where $g(t)=h\cos(\omega t+\varphi)+(1+a\cos t)$

$$\frac{\partial u_0}{\partial t}=-\frac{2}{r}\frac{\partial}{\partial r}(r\tau_1) \tag{17}$$

Integrating equation (16) and using boundary condition (12), we have,

$$\tau_0=rg(t) \tag{18}$$

From equation (11) and (14), it can be seen that,

$$u_0=u_s \text{ and } u_1=0 \text{ at } r=R(z) \tag{19}$$

On putting the values of and from equations (14) and (15) in (10), we get

$$-\frac{\partial u_0}{\partial r}=2(\tau_0-\tau_c) \tag{20}$$

$$-\frac{\partial u_1}{\partial r}=2\tau_1 \tag{21}$$

Integrating equation (20) and using (18) and (19), u_0 is obtained as

$$u_0=u_s+g(t)R^2\left[\left(1-\frac{r^2}{R^2}\right)-\frac{2\tau_c}{g(t)R}\left(1-\frac{r}{R}\right)\right] \tag{22}$$

Putting the value of u_0 from equation (22) in (17), τ_1 is expressed as

$$\tau_1 = \frac{g'(t)R^3}{8}\left[\left(\frac{r}{R}\right)^3 - 2\left(\frac{r}{R}\right)\right] \tag{23}$$

Using equation (23) in (21), u_1 may be written as

$$u_1 = \frac{g'(t)R^4}{16}\left[4\left(\frac{r}{R}\right)^2 - \left(\frac{r}{R}\right)^4 - 3\right] \tag{24}$$

Thus from equations (14), (15), (18), (22), (23) and (24), the expressions of velocity profile and shear stress are obtained as

$$u = u_s + g(t)R^2\left[\left(1 - \frac{r^2}{R^2}\right) - \frac{2\tau_c}{g(t)R}\left(1 - \frac{r}{R}\right)\right] + \frac{\beta^2 g'(t)R^4}{16}\left[4\left(\frac{r}{R}\right)^2 - \left(\frac{r}{R}\right)^4 - 3\right] \tag{25}$$

$$\tau = rg(t) + \frac{\beta^2 g'(t)R^3}{8}\left[\left(\frac{r}{R}\right)^3 - 2\left(\frac{r}{R}\right)\right] \tag{26}$$

From equation (26), shearing stress at the wall i.e. $r = R(z)$ is given by

$$\tau_w = Rg(t)\left[1 - \frac{\beta^2 R^2 f^2}{8}\right] \tag{27}$$

where $f = g'(t)/g(t)$

The non-dimensional volumetric flow rate Q is formulated as

$$Q(z,t) = \frac{4}{\mu}\int_0^{R(z)} ru(z,r,t)dr \tag{28}$$

where $Q(z,t) = \dfrac{Q'(z',t')}{\pi R_0^4 B_0/8}$

$$Q(z,t) = \frac{2R^2 u_s}{\mu} + \frac{g(t)R^3}{24\mu}\left[\frac{32\tau_c}{g(t)} + 24R - 3\beta^2 R^3 f\right] \tag{29}$$

The apparent viscosity in dimensionless form may be specified as

$$\frac{\mu_0}{\mu} = \frac{48u_s + g(t)\left[32k + 24 - 3\beta^2 f\right]}{48R^2 u_s + g(t)R^3\left[32k + 24R - 3\beta^2 R^3 f\right]} \tag{30}$$

where $k = \dfrac{\tau_c}{g(t)}$

NUMERICAL COMPUTATION AND DISCUSSION OF RESULTS

The significant computational effort has been done to quantify the effects of various parameters involved in the study. The purpose of the present investigations is to bring out the influences of periodic body acceleration, slip velocity and overlapping stenoses on the pulsatile flow of blood where blood is taken to be a non-Newtonian fluid characterizing by Bingham-Plastic fluid model. The analytical expressions of velocity profile, flow rate, wall shear stress and apparent viscosity are derived applying perturbation technique. The graphs of above expressions are exhibited using MATLAB 7.8. The parametric values of slip velocity (u_s = 0, 0.2, 0.5), Womersley frequency parameter (β = 0.5), the amplitude of pressure gradient (a = 0.1 to 0.5), yield stress (τ_c = 0.05 to 0.3), dimensionless stenosis height (δ = 0 to 0.6), the body acceleration parameter (h = 0 to 1) and lead angle (φ = 0.2) have been taken from Sarojamma et al [11], Varshney et al [12] and Kakati et al [14].

Figures 2 and 3 reveal the variation of axial velocity along radial distance and time (in degrees). In these figures, it can be observed that velocity is maximum near the centre and minimum near the wall of artery. Slip velocity augments the velocity profile and eases the blood flow. The profile is reduced by stenosis but enhances with the application of periodic body acceleration. The velocity decreases in the first half cycle from 0˚ to 180˚ and increases from 180˚ to 360˚.

The effect of body acceleration on wall shear stress is exposed in Figure 4 where one can see that shear stress attains greater values for increasing values of body acceleration parameter from t = 0˚ to 90˚ and 270˚ to 360˚ i.e., in the first and fourth quadrant and in the remaining space i.e., in the second and third quadrant, the shear stress decreases for increasing body acceleration parameter. The figure also depicts the comparison between the cases where body acceleration is applied and where it is not applied. In figure 5, shear stress is varied along axial length for various values of stenosis height where it is observed that wall shear stress is highest at the two stenosis throats and least at the extremities of stenosis and for increasing stenosis height it also augments considerably.

Figures 6 and 7 portray that apparent viscosity increases with increasing body acceleration parameter and stenosis height but decreased by slip velocity. It is minimum at 180˚ and maximum at 0˚ and 360˚. It diminishes in the first half of cycle from t = 0˚ to 180˚ and increases in the second half of cycle from t = 180˚ to 360˚

The changes in volumetric flow rate along with stenosis height and time for different values of body acceleration parameter and slip velocity are shown in figures 8 and 9. It can be noticed that acceleration parameter and slip velocity enhances the flow rate while it is decreased when stenosis height increases. It declines from highest to lowest in the first two quadrants and starts augmenting again from minimum (t = 180˚) to maximum (t = 360˚).

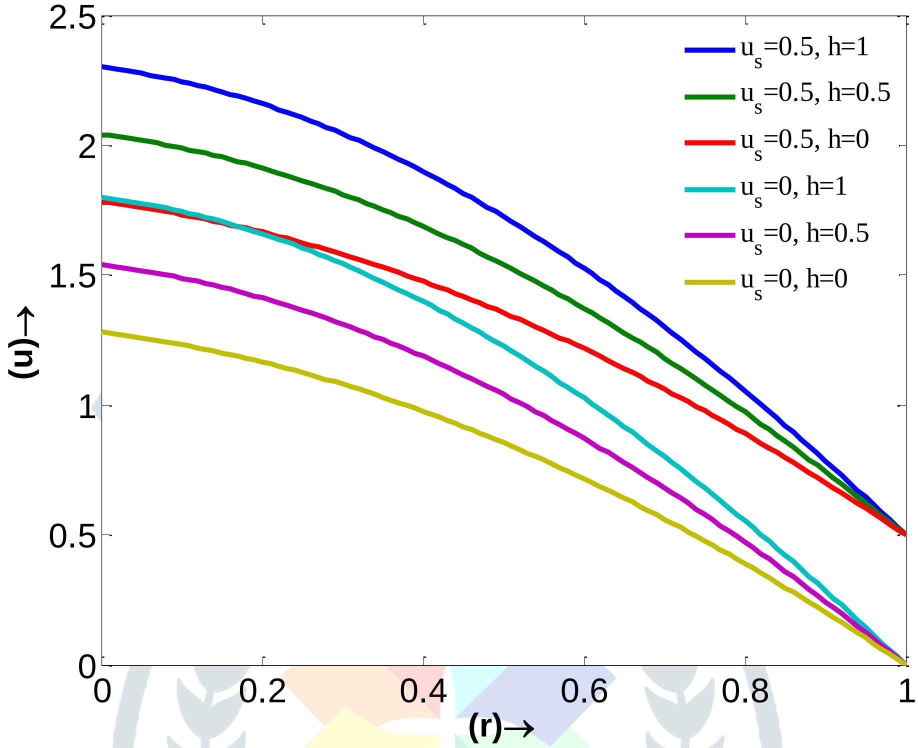

Figure 2: Axial velocity along radial distance for various values of slip velocity (u_s) and body acceleration parameter (h)

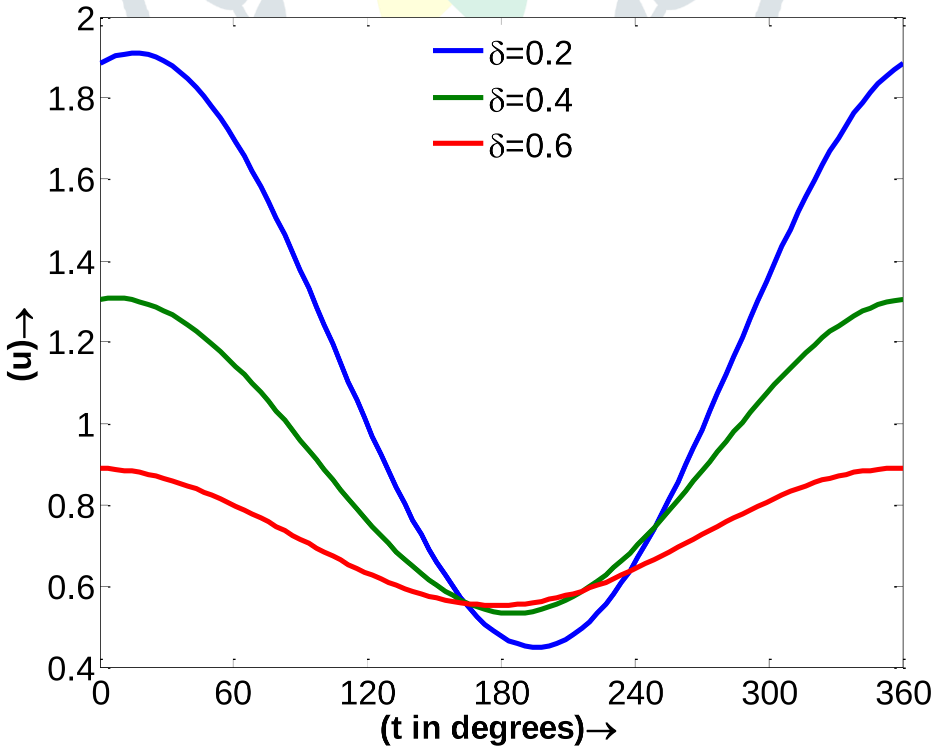

Figure 3: Axial velocity with time (in degrees) for various values of non dimensional stenosis height (δ)

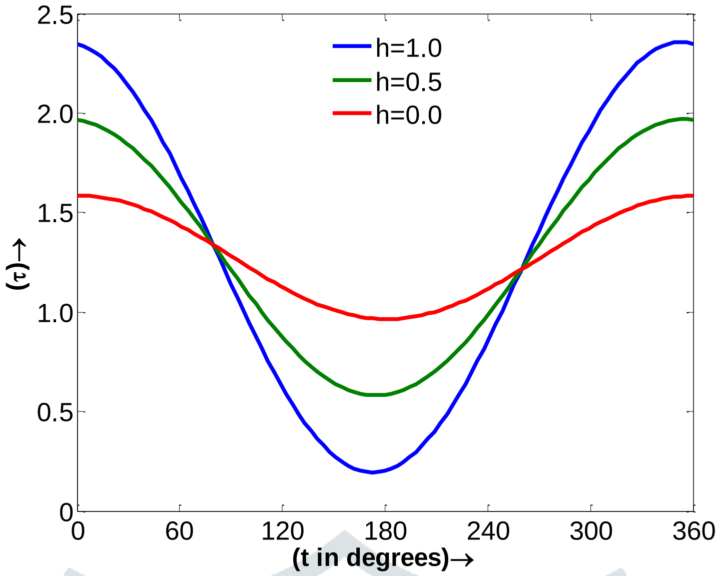

Figure 4: Wall shear stress with time for various values of body acceleration parameter (*h*)

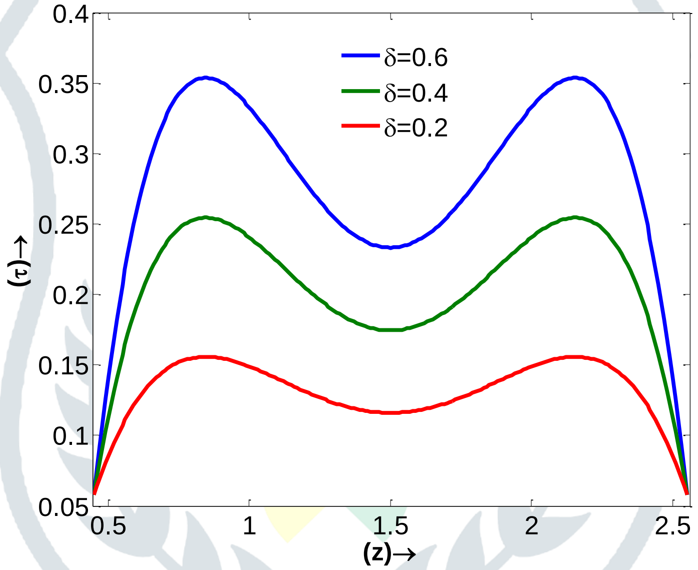

Figure 5: Wall shear stress along axial distance for various values of stenosis height (*δ*)

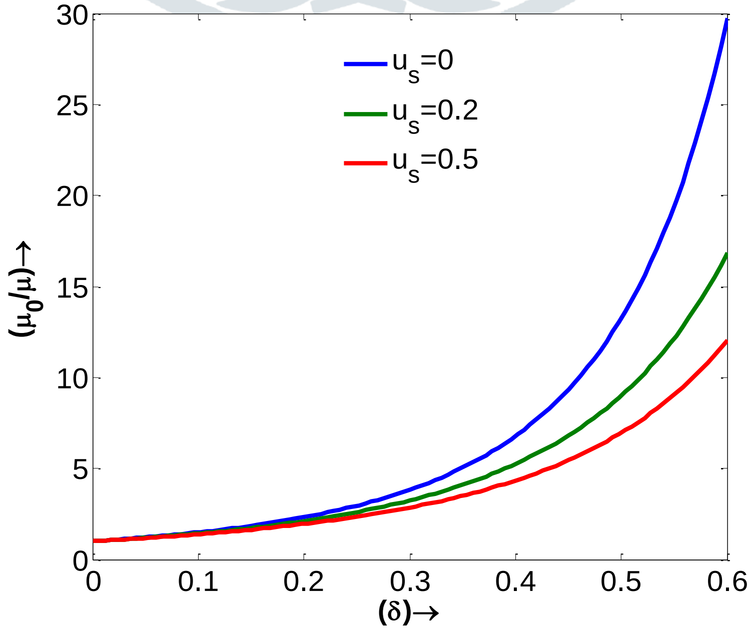

Figure 6: Apparent viscosity with stenosis height for various values of slip velocity (u_s)

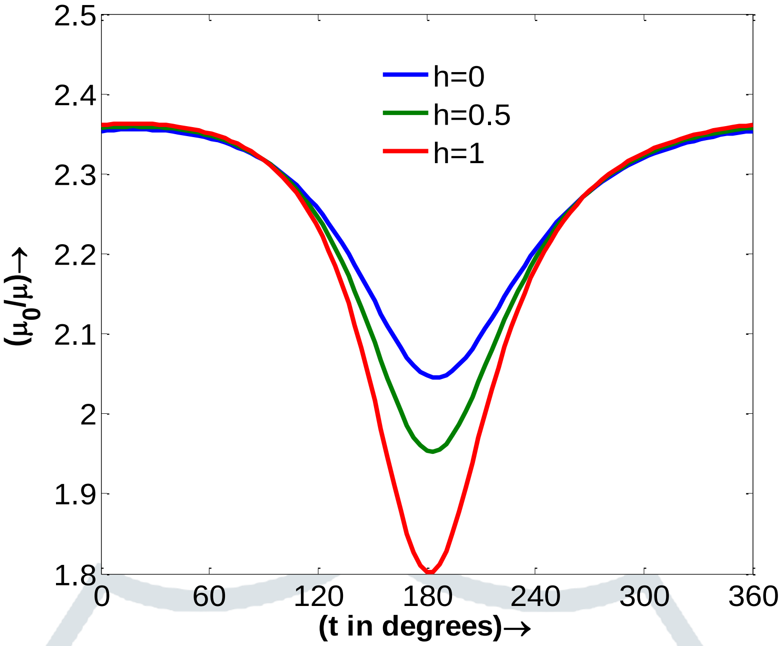

Figure 7: Apparent viscosity with time for various values of body acceleration parameter (*h*)

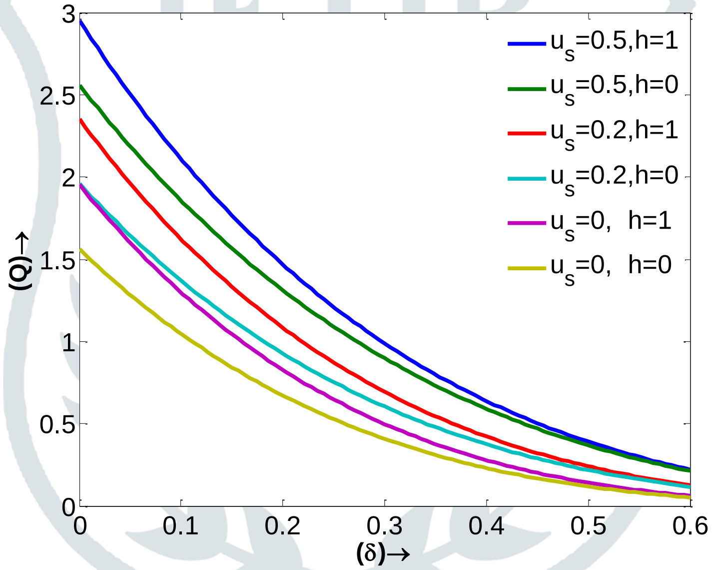

Figure 8: Volumetric flow rate with stenosis height for various values of slip velocity (u_s) and body acceleration parameter (*h*)

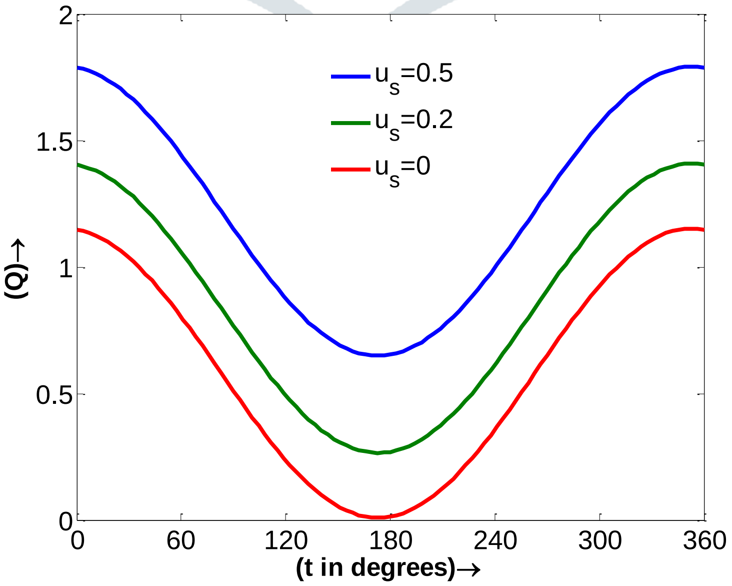

Figure 9: Volumetric flow rate with time for various values of slip velocity (u_s)

CONCLUSION

The present work investigates the pulsatile flow of blood through an arterial segment embedded with overlapping stenoses in the presence of periodic body acceleration. The velocity slip is applied at the vessel wall as a boundary condition. Blood is characterized as Bingham-Plastic fluid to represent the non-Newtonian behaviour of blood. Using the perturbation method, the governing equation of flow is deciphered and the analytical expressions for flow variables are derived. The axial velocity and flow rate are diminished with increasing stenosis height while apparent viscosity and wall shear stress are enhanced. Slip velocity helps in maintaining all these flow characteristics. Now body acceleration parameter increases velocity profile and flow rate but decreases apparent viscosity. Acceleration parameter also reduces wall shear stress from $t = 90^\circ$ to 270° in one cycle but increases for remaining part. This sort of variations in flow behaviour of blood may aid the medical practitioners, bio-engineers and physicians in treatment of diseased arterial circulation, making decision of application of such types of therapies, developing and improving artificial organs, better understanding of severity of stenosis and its consequences.

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