PARAMETRIC INTERACTION OF ACOUSTIC PHONONS IN MAGNETIZED SEMICONDUCTOR PLASMAS

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***ABSTRACT*-**Using the hydrodynamic model of semiconductor plasmas, a detailed analytical investigation is made to study parametric amplification in magnetized piezoelectric as well as non-piezoelectric semiconductors. The origin of nonlinear interaction is taken to be in the second-order optical susceptibility $\chi^{(2)}$ arising from nonlinear induced current density. The threshold value of pump intensity $I_{0,th}$ is obtained for crystals. Parametric gain constants are obtained for different situations of practical interest, i.e. (*i*) for piezoelectric coupling only g_p, (*ii*) for deformation potential coupling only g_d, (*iii*) for both the couplings g_b. Numerical estimates are made for *n*-InSb crystal duly irradiated by 10.6 μm CO_2 laser. It is found that the application of large d.c. magnetic field significantly reduces the threshold pump intensity $I_{0,th}$ for the onset of parametric amplification. At sufficiently high magnetic field (i.e. $\omega_c \sim \omega_0$), $I_{0,th}$ is reduced by approx. 3.56 times. The parametric gain constants (g_p, g_d and g_b) are found to increase with increasing pump intensity I_0 (> $I_{0,th}$), wave number k, magnetic field B_s (ω_c), and scattering angle θ. Moreover, it is found that $g_b > g_p > g_d$.

***Keywords*-Parametric interaction, parametric amplification, parametric gain coefficient, semiconductor plasmas, deformation potential, piezoelectric coefficient, dc magnetic field.**

INTRODUCTION

In the modern era plasma physics, the problem of the interaction of high-power laser radiation with plasmas is of outstanding interest [1 - 4]. In order to obtain fusion energy, high-power Q- switched lasers and strong radio-frequency sources are being developed or planned. The possibility of obtaining fusion energy depends to a large extent on the success of the technological developments of such high-power devices, as well as on a full understanding of the basic problem of how the electromagnetic energy of these intense radiation fields may couple, or may be forced to couple most efficiently into the plasma, at the high power levels that are now not only attracting the attention of the fusion plasma researchers [5 - 7] but also that of those dealing with nonlinear optics [8, 9].

In a nonlinear active medium, the breakdown of the superposition principle may lead to interaction among waves differing in frequencies. There exist a numerous nonlinear interactions which can be classified as parametric interaction (PI) of coupled mode. In the phenomena of PI of coupled modes, the external pump wave transfers its energy to the generated waves by a resonant mechanism that takes place when the pump field intensity is large enough to cause the vibration (with the external field frequency) of certain physical parameters of the system. Parametric amplifiers, parametric oscillators, optical phase conjugators, etc. are the devices based on PI in a nonlinear medium. Besides these technological uses, there are several other applications of PIs in which researchers are interested [10 - 12].

It is a totally accepted fact that the origin of PIs lies in the second-order nonlinear optical susceptibility $\chi^{(2)}$ of the nonlinear medium. $\chi^{(2)}$, the lowest order nonlinear optical susceptibility, is a third rank tensor and non-zero in a medium which lacks inversion symmetry. The nonlinear optical polarization which is quadratic in nature in terms of field amplitude leads to the nonlinear optical phenomena of second harmonic generation (SHG), sum frequency generation (SFG), difference frequency generation (DFG) etc. In general, the terms in the expression of $\chi^{(2)}$ provide a coupling among the set of three electromagnetic waves; each of which is characterized by frequency ω_i, wave number k_i, state of polarization ε_i, as well as complex amplitude $E_i = A_i \exp(i\omega_i t)$. $\chi^{(2)}$ has been studied in different frequency regimes and the sum and difference rules for the nonlinear susceptibilities in solids and other diluted media [13, 14]. Up till now a number of theoretical attempts have made to explain the behaviour of $\chi^{(2)}$ on the basis of multiple valence and conduction band theory [15, 16]; but nevertheless, the agreement between theoretically quoted values and experimental results can be said to be poor. All these theoretical studies were mainly developed with the induced polarizations arising from bound-electron nonlinearities.

It has been observed that in comparison to liquid and gaseous media, the crystalline nonlinear media are more suitable for fabrication of optoelectronic devices. The reason for this is two-fold. Firstly, $\chi^{(2)}$ is non-vanishing for non-centrosymmetric (NCS) crystalline nonlinear media. Secondly, in crystalline nonlinear media, by compensating the material dispersion, the birefringence could be used to phase match velocities of fundamental and harmonic waves. However, for nonlinear optical applications, nonlinear optical crystals should satisfy four basic conditions, viz. adequate nonlinearity, transparency in optical regime, proper birefringence, sufficient resistance to optical damage by intense laser radiation.

The doped semiconductor crystals offer considerable flexibility for fabrication of optoelectronic devices over other nonlinear optical crystals because their properties can be easily influenced by compositions, micro-structuring and externally applied fields [17]. For the construction of optical storage devices and switching elements, the optical properties of semiconductor materials are found to change strongly when charge carriers (electrons/holes) are excited optically. Hence, the supremacy of semiconductors as active media in fabrication of modern optoelectronic devices, optical communication, optical computing and all optical signal processing is unquestionable and hence the understanding of the phenomenon of PI in these crystals appears to be of fundamental significance.

The properties of nonlinear optical materials can be better understood when discussed with reference to nonlinear devices and the theory of nonlinear interactions. PI of waves has been studied deeply in the last five decades, there are tremendous possibilities for further exploration and exploitation due to the poor agreement between theories [13] and experiments [18]. The current trends in the field indicate that this old but fascinating phenomenon is still hotly pursued by both theoreticians as well as experimentalists, and an increasing number of interesting applications exploiting parametric interaction are being discovered or are yet to be discovered.

PI of acoustic waves with microwave electric fields in piezoelectric semiconductors was studied by Economou and Spector [19]. The role of d.c. magnetic field on parametric amplification was studied by Cohen [20]. The parametric excitation of hybrid mode has been studied by Ghosh and Aggarwal [21]. The parametric dispersive as well as absorptive characteristics while calculating $\chi^{(2)}$ originating from the finite induced current density produced in a semiconducting medium have been reported by Aghamkar et.al. [22]. Recently, parametric amplification in electrostrictively doped piezoelectric semiconductors has been studied by Pal et.al. [23]. Motivated by the intense interest in the field of study of PI based on $\chi^{(2)}$, in the present chapter the author has attempted to study the important process of parametric amplification originating from $\chi^{(2)}$ in n-InSb crystal of NCS structure immersed in an external d.c. magnetic field perpendicular to the direction of the pump wave propagation.

THEORETICAL FORMULATIONS

For studying the PI of acoustic phonons in magnetized semiconductor plasmas, arising due to effective noinlinear optical susceptibility ($\chi^{(2)}$), the hydrodynamic model of a homogeneous semiconductor plasmas (having both piezoelectric as well as deformation potential couplings) is considered. This model restricts the validity of the formulation to the limit $k_a l << 1$, where k_a is the wave number and l is the mean free path of electrons. The semiconductor medium is subjected to the d.c. magnetic field B_s (along the z-axis) perpendicular to the propagation direction (x-axis) of spatially uniform high frequency pump electric field $E_0 \exp(-i\omega_0 t)$. The scattered electromagnetic waves are propagating along a direction making an arbitrary angle θ with the direction of propagation of pump wave, i.e. in x-z plane making an angle θ with x- axis. Thus θ may be defined as the scattering angle, i.e. the angle between $\boldsymbol{k}_0$ and $\boldsymbol{k}_1$.

The basic equations describing parametric interaction of the pump with the medium are as follows:

$$\frac{\partial^2 u}{\partial t^2} + 2\Gamma_a \frac{\partial u}{\partial t} + \frac{\beta}{\rho}\frac{\partial E_a}{\partial x} + \frac{C_d \varepsilon}{\rho e}\frac{\partial^2 E_a}{\partial x^2} = \frac{C}{\rho}\frac{\partial^2 u}{\partial x^2} \quad (1)$$

$$\frac{\partial n_1}{\partial t} + n_0 \frac{\partial v_1}{\partial x} + n_1 \frac{\partial v_0}{\partial x} + v_0 \frac{\partial n_1}{\partial x} = 0 \quad (2)$$

$$\frac{\partial \boldsymbol{v}_0}{\partial t} + \nu \boldsymbol{v}_0 = -\frac{e}{m}(\boldsymbol{E}_0 + \boldsymbol{v}_0 \times \boldsymbol{B}_s) \quad (3)$$

$$\frac{\partial \boldsymbol{v}_1}{\partial t} + \nu \boldsymbol{v}_1 + \left(\boldsymbol{v}_0 . \frac{\partial}{\partial x}\right)\boldsymbol{v}_1 = -\frac{e}{m}(\boldsymbol{E}_1 + \boldsymbol{v}_1 \times \boldsymbol{B}_s) \quad (4)$$

$$\frac{\partial E_s}{\partial x} + \frac{\beta}{\varepsilon}\frac{\partial^2 u}{\partial x^2} - \frac{C_d}{e}\frac{\partial^3 u}{\partial x^3} = -\frac{n_1 e}{\varepsilon} \quad (5)$$

Eq. (1) is the equation of motion of the lattice in a crystal having piezoelectric and deformation potential couplings both. In this equation ρ, u, Γ_a, C, ε, β and C_d being the mass density of the crystal, displacement of the lattice, phenomenological damping parameter of acoustic mode, crystal elastic constant, the scalar dielectric, piezoelectric constant and deformation potential constant of the semiconductor, respectively. Conservation of charge is represented by continuity eq. (2) in which n_0 and n_1 are the un-perturbed and perturbed electron densities, respectively. Eqs. (3) and (4) are the linearised zeroth- and first-order momentum transfer equations of the oscillatory electron fluid, respectively; in which v_0 and v_1 are the zeroth- and first-order oscillatory fluid velocities having effective mass m and charge $-e$ and ν is the phenomenological electron collision frequency. The strong space charge electric field E_s is determined from Poisson's eq. (5) in which the second and third terms on the left and side give the piezoelectric and deformation potential contribution to polarization, respectively.

In a semiconductor plasma, the low frequency generated acoustic wave (ω_a) while interacting with high frequency pump electromagnetic wave (ω_0) produce density perturbations (n_1) at frequencies $\omega_0 \pm p\omega_a$, p being an integer. The low frequency perturbations (n_a) are proportional to $\exp[i(k_a x - \omega_a t)]$. These density perturbations can be obtained by using the standard approach [23]. Let us restrict ourselves only in the lowest order with $p = 1$ representing the first-order Stokes component. The perturbations at off-resonant frequencies $p \geq 2$ are neglected. Differentiating eq. (2) and then substituting the first-order differential coefficient of the equilibrium and perturbed fluid velocities through eqs. (3) and (4) and perturbed field through eq. (5), one obtains

$$\frac{\partial^2 n_1}{\partial t^2} + \nu \frac{\partial n_1}{\partial t} + \varpi_p^2 n_1 + \frac{n_0 e\beta}{m\varepsilon}\frac{\partial^2 u}{\partial x^2} - \frac{n_0 C_d}{m}\frac{\partial^3 u}{\partial x^3} = -\bar{E}\frac{\partial n_1}{\partial x} \tag{6}$$

where $\bar{E} = -\left[\frac{e}{m}E_0 + \omega_c v_{0y}\right]$, and $\varpi_p^2 = \left[\omega_p^2\left(\frac{\nu^2}{\nu^2+\omega_c^2}\right)\right]$, in which $\omega_c = eB_s/m$ is the electron cyclotron frequency, and $\omega_p = (n_0 e^2/m\varepsilon)^{1/2}$ is the plasma frequency of carriers in the medium.

In deriving eq. (6), Doppler shift has been neglected under the assumption that $\omega_0 >> \nu > kv_0$.

The density perturbations associated with the acoustic phonon mode (viz., n_s) and the scattered electromagnetic waves (n_f) arising due to the three wave parametric interaction will propagate at the generated frequencies ω_a and $\omega_0 \pm \omega_a$ respectively. The phase matching conditions which are to be satisfied for these modes are: $\hbar\omega_0 = \hbar\omega_1 + \hbar\omega_a$, and $\hbar\boldsymbol{k}_0 = \hbar\boldsymbol{k}_1 + \hbar\boldsymbol{k}_a$, i.e. the energy and momentum conservation relations should be satisfied.

Now since θ is the scattering angle, i.e. angle between $\boldsymbol{k}_1$ and $\boldsymbol{k}_0$, thus in writing the energy and momentum conservation relations one may assumed $k_{1y} = 0$, i.e. the scattered wave to propagate in the *x-z* plane. It worth pointed out here that these conservation equations could be satisfied over a wide range of scattering angle. Now for a spatially uniform laser irradiation $|k_0| \approx 0$ and one obtains $|k_1| = |k_a| = |k|$ (say). On resolving eq. (6) into two components (fast and slow) by denoting $n_1 = n_f + n_s$ under rotating wave approximation (RWA), one obtains:

$$\frac{\partial^2 n_f}{\partial t^2} + \nu\frac{\partial n_f}{\partial t} + \varpi_p^2 n_f = -\bar{E}\frac{\partial n_s^*}{\partial x} \tag{7a}$$

and

$$\frac{\partial^2 n_s}{\partial t^2} + \nu\frac{\partial n_s}{\partial t} + \varpi_p^2 n_s + \frac{n_0 e\beta}{m\varepsilon}\frac{\partial^2 u}{\partial x^2} - \frac{n_0 C_d}{m}\frac{\partial^3 u}{\partial x^3} = -\bar{E}\frac{\partial n_f^*}{\partial x}. \tag{7b}$$

In the above formulation, the author has restricted only to the Stokes component ($\omega_0 - \omega_a$) of the scattered electromagnetic waves. From eqs. (7a) and (7b), it is clearly understood that the slow and fast components of the density perturbations are coupled to each other via the pump electric field. Thus for PI to occur, the presence of the pump electric field is the fundamental necessity. The coupled wave equations are solved and are simplified for n_s, which is given by

$$n_s = \frac{ien_0 k_x^3 E_a\left(\beta^2 + \frac{C_d^2\varepsilon^2 k_x^2}{e^2}\right)[\Omega^2]^{-1}}{m\rho\varepsilon(\omega_a^2 - k_x^2 v_a^2 + 2i\Gamma_a\omega_a)} \tag{8}$$

where $\Omega^2 = \delta'^2 + i\nu\omega_a - \frac{k_x^2|\bar{E}|^2}{(\delta^2 - i\nu\omega_1)}$, in which $\delta'^2 = \varpi_p^2 - \omega_0^2$, $\delta^2 = \varpi_p^2 - \omega_1^2$, $\omega_1 = \omega_0 - \omega_a$, $k_x = k\cos\theta$, and $v_a^2 = C/\rho$; v_a being the velocity of acoustic wave.

It is clearly understood from eq. (8) that n_s depends upon the various powers of pump intensity, $I = (1/2)\eta\varepsilon_0 c_0 |E_0|^2$; η and c_0 being the background refractive index of the crystal and the velocity of light in vacuum, respectively. The density perturbations thus produced affects the propagation characteristics of the scattered waves, and can be studied by employing the electromagnetic wave equation:

$$\nabla^2\vec{E}_1 = \frac{1}{c_1^2}\frac{\partial^2\vec{E}_1}{\partial t^2} - \mu_0\frac{\partial\vec{J}_1}{\partial t} \tag{9}$$

where $c_1 = 1/(\mu_0\varepsilon_0\varepsilon_1)^{1/2}$ is the velocity of light in the medium, in which $\varepsilon_1 = \varepsilon/\varepsilon_0$ and $\mathbf{J}_1$ is the perturbed current density. The Stokes component of the induced current density may be obtained from the relation

$$J_1 = -n_s^* e v_0. \tag{10a}$$

Using eq. (8) and (10a), one obtains:

$$J_1 = \frac{-i\varepsilon e k_x^3 v_a^2 \omega_p^2 E_a^* E_0 (\kappa^2 + \zeta^2 k_x^2)[\Omega^2]^{-1}}{2m\Gamma_a \omega_a (\omega_c^2 - \omega_0^2)} \quad (10b)$$

where $\kappa^2 = \frac{\beta^2}{\varepsilon C}$ and $\zeta^2 = \frac{C_d^2 \varepsilon}{e^2 C}$.

In deriving eqs. (10a) and (10b), the author has used the expressions for v_{0x} and v_{0y} (i.e. the components of v_0 along *x*- and *y*-directions, respectively), which is the oscillatory electron fluid velocity in the presence of the pump wave and d.c. magnetic field. Using eq. (3), these expressions for v_{0x} and v_{0y} are obtained as:

$$v_{0x} = \frac{\bar{E}}{(\nu - i\omega_0)} \text{ and } v_{0y} = -\frac{e}{m}\frac{\omega_c E_0}{(\omega_c^2 - \omega_0^2)}. \quad (11)$$

In the coupled-mode approach, the time integral of nonlinear current density J_1 yields the nonlinear-induced polarization P_1 as:

$$P_1 = \int J_1 dt. \quad (12)$$

Neglecting the induced polarization due to transition dipoles, the second-order susceptibility obtained by using eqs. (10) – (12) as:

$$\chi^{(2)} = \frac{\varepsilon_1 e k_x^3 v_a^2 \omega_p^2 \omega_0 (\kappa^2 + \zeta^2 k_x^2)[\Omega^2]^{-1}}{2m\Gamma_a \omega_a (\omega_c^2 - \omega_0^2)\omega_1}. \quad (13)$$

As susceptibility being a complex quantity, can be expressed as:

$$\chi^{(2)} = \chi_r^{(2)} + \chi_i^{(2)}. \quad (14)$$

From eqs. (13) and (14), it is clearly understood that the second-order optical susceptibility $\chi^{(2)}$ is influenced by the free carrier concentration n_0 (through ω_p) and by d.c. magnetic field B_s (through ω_c). The dispersion characteristics of the scattered wave in a parametric process from $\chi_r^{(2)}$ and the parametric gain through $\chi_i^{(2)}$ can be seen from eq. (13).

Hence in the present chapter let us confine ourselves to the study of parametric gain in the presence of an external d.c. magnetic field and deformation potential coupling.

In a doped semiconductor, the parametric amplification is given by effective nonlinear absorption coefficient as:

$$\alpha_{para} = \left(\frac{\omega_1}{\eta c_0}\right)\chi_i^{(2)}, \quad (15)$$

The nonlinear growth of the signal ($\omega_1 = \omega_0 - \omega_a$) as well as idler ($\omega_a$) requires that α_{para} obtained from eq. (15) is negative.

Let us restrict ourselves to the analytical investigations followed by numerical estimations of the parametric growth in isotropic and magnetized semiconductors in heavily doped regime, i.e. $\varpi_p \approx \omega_0 (\approx \omega_1)$ with $\varpi_p >> \nu (\approx \omega_a)$. For this, solving eq. (13) to get real and imaginary parts of complex $\chi^{(2)}$ as:

$$\chi_r^{(2)} = \frac{\varepsilon_1 e K k_x \omega_p^2 \omega_0 \left(1 + \frac{\zeta^2 k_x^2}{\kappa^2}\right)(\varpi_p^2 \omega_1^2 \nu^2 - \delta^2 k_x^2 |\bar{E}|^2)}{2m\Gamma_a \omega_1 \omega_a (\omega_c^2 - \omega_0^2)[(\delta^2 \varpi_p^2 - k_x^2 |\bar{E}|^2)^2 + (\omega_1 \varpi_p^2 \nu)^2]} \quad (16)$$

and

$$\chi_i^{(2)} = \frac{-\varepsilon_1 e K k_x \omega_p^2 \omega_0 \left(1 + \frac{\zeta^2 k_x^2}{\kappa^2}\right)(\omega_a \omega_1 \nu^2 - k_x^2 |\bar{E}|^2)}{2m\Gamma_a \omega_1 \omega_a (\omega_c^2 - \omega_0^2)[(\delta^2 \varpi_p^2 - k_x^2 |\bar{E}|^2)^2 + (\omega_1 \varpi_p^2 \nu)^2]} \quad (17)$$

where $K = \kappa^2 k_x^2 v_x^2$.

The threshold value of pump electric field $E_{0,th}$ for the onset of parametric process, which is necessary condition for obtaining parametric amplification to occur, can be obtained by setting $\chi_i^{(2)} = 0$. The expression for $E_{0,th}$ is obtained for both magnetized as well as isotropic semiconductor crystals.

(*i*) In the presence of d.c. magnetic field, i.e. $\omega_c \neq 0$

$$(E_{0,th})_{\omega_c \neq 0} = \frac{m\nu}{ek_x}(\omega_1\omega_a)^{1/2}\left(1-\frac{\omega_c^2}{\omega_0^2}\right) \text{ and } (I_{0,th})_{\omega_c \neq 0} = \frac{\eta\varepsilon_0 c_0 m^2\nu^2\omega_1\omega_a}{2e^2k_x^2}\left(1-\frac{\omega_c^2}{\omega_0^2}\right)^2. \quad (18a)$$

(*ii*) In the absence of d.c. magnetic field, i.e. $\omega_c = 0$

$$(E_{0,th})_{\omega_c = 0} = \frac{m\nu}{ek_x}(\omega_1\omega_a)^{1/2} \text{ and } (I_{0,th})_{\omega_c = 0} = \frac{\eta\varepsilon_0 c_0 m^2\nu^2\omega_1\omega_a}{2e^2k_x^2}. \quad (18b)$$

In order to compare the threshold fields in the presence ($\omega_c \neq 0$) and absence ($\omega_c = 0$) of the magnetic field, the following ratio is obtained:

$$\frac{(I_{0,th})_{\omega_c \neq 0}}{(I_{0,th})_{\omega_c = 0}} = \frac{(E_{0,th})_{\omega_c \neq 0}}{(E_{0,th})_{\omega_c = 0}} = 1-\frac{\omega_c^2}{\omega_0^2}. \quad (18c)$$

From eq. (18c), it is clearly understood that the application of an external d.c. magnetic field considerably reduces the threshold pump field required for the onset of parametric amplification process to occur. It may also be observed that the expressions for the threshold fields are independent of the material parameters viz. piezoelectric and/or deformation potential coupling coefficients. Thus the threshold field will remain the same for any material belonging to NCS group.

The parametric growth (i.e. $\chi_i^{(2)}$ negative) can be achieved in the two different regions under the following conditions:

(*i*) if $k_x^2|\bar{E}|^2 > \omega_1\omega_a\nu^2$, $\omega_c^2 < \omega_0^2$; (19a)

(*ii*) if $k_x^2|\bar{E}|^2 < \omega_1\omega_a\nu^2$, $\omega_c^2 > \omega_0^2$. (19b)

It can be understood that under condition (19b) the expression for the parametric growth becomes nearly independent of the pump field amplitude and thus this case is of no practical interest.

Let us now discuss the different aspects of $\chi^{(2)}$ for different situations of practical interest.

(*i*) Piezoelectric coupling ($\beta \neq 0$, $C_d = 0$):

For this case, using eqs. (17) and (19b), one obtains the growth rate as:

$$g_p = \frac{\varepsilon_1 e^3 K k_x^3 \omega_p^2 \omega_0^5 \omega_1 \nu |E_0|^2}{2m^3\Gamma_a\omega_a\eta c_0(\omega_c^2-\omega_0^2)^3[(\delta^2\varpi_p^2 - k_x^2|\bar{E}|^2)^2 + (\omega_1\varpi_p^2\nu)^2]}. \quad (20a)$$

The suffix *p* is used for piezoelectric coupling.

(*ii*) Deformation potential coupling ($\beta = 0$, $C_d \neq 0$):

For this case, using eqs. (17) and (19a), one obtains the growth rate as:

$$g_d = \frac{\varepsilon_1 e^3 v_a^2 k_x^7 \omega_p^2 \omega_0^5 \omega_1 \nu \zeta^2 |E_0|^2}{2m^3\Gamma_a\omega_a\eta c_0(\omega_c^2-\omega_0^2)^3[(\delta^2\varpi_p^2 - k_x^2|\bar{E}|^2)^2 + (\omega_1\varpi_p^2\nu)^2]}. \quad (20b)$$

The suffix *d* is used for deformation potential coupling.

(*iii*) Both piezoelectric and deformation potential couplings ($\beta \neq 0$, $C_d \neq 0$):

In this case using the same pair of equations as above, the gain constant becomes:

$$g_b = \frac{\varepsilon_1 e^3 K k_x^3 \omega_p^2 \omega_0^5 \omega_1 \nu |E_0|^2 \left(1+\frac{\zeta^2 k_x^2}{\kappa^2}\right)}{2m^3\Gamma_a\omega_a\eta c_0(\omega_c^2-\omega_0^2)^3[(\delta^2\varpi_p^2 - k_x^2|\bar{E}|^2)^2 + (\omega_1\varpi_p^2\nu)^2]}. \quad (20c)$$

The suffix *b* stands for both the couplings.

RESULTS AND DISCUSSION

Let us now address a detailed numerical analysis of the parametric gain in a NCS semiconducting crystal, viz. n-InSb at 77 K duly irradiated by a nanosecond pulsed 10.6 μm CO_2 laser. The physical constants for the n-type InSb crystal have been considered as follows [23]:

$m = 0.0145 m_e$ (m_e the free mass of electron), $\varepsilon_1 = 15.8$, $v_a = 4\times10^3$ ms^{-1}, $\beta = 0.054$ Cm^{-2}, $C_d = 4.5$ eV, $\Gamma_a = 2\times10^{10}$ s^{-1}, $\eta = 3.9$, $\rho = 5.8\times10^3$ kg/m^3, $n_0 = 2\times10^{24}$ m^{-3}, $\omega_a = 2\times10^{11}$ s^{-1}, $\nu = 4\times10^{11}$ s^{-1} and $\omega_0 = 1.78\times10^{14}$ s^{-1}.

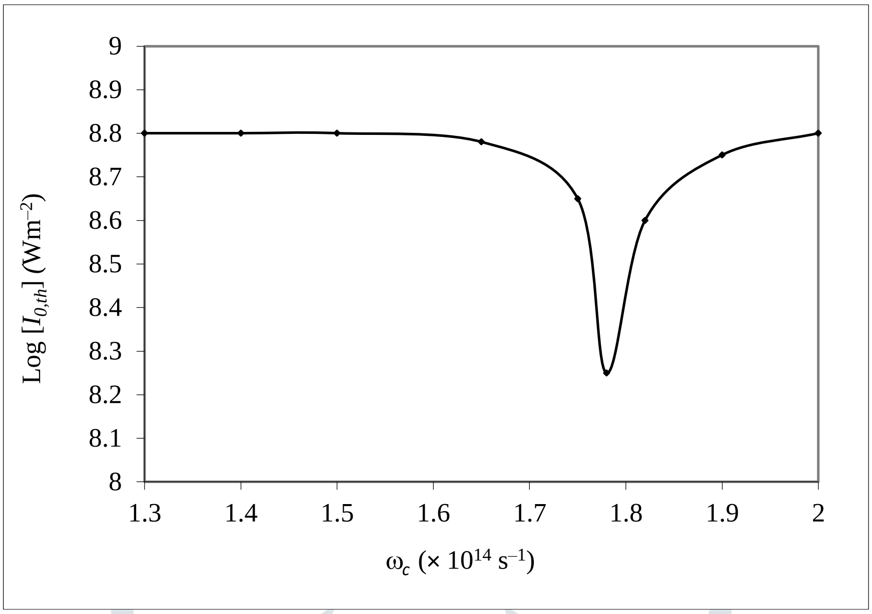

Fig. 1. Variation of $I_{0,th}$ with B_s (in terms of ω_c) with θ = 45° and $k = 2\times10^6$ m^{-1}.

The nature of dependence of $I_{0,th}$ with B_s (in terms of ω_c) is shown in Fig. 1. It can be seen that $I_{0,th}$ remains constant ($\text{anti}\log(8.8) = 6.3\times10^8$ Wm^{-2}) up to $\omega_c = 1.65\times10^{14}$ s^{-1}; if one applies magnetic field for which $\omega_c > 1.65\times10^{14}$ s^{-1}, $I_{0,th}$ starts decreasing sharply, attains a minimum value ($\text{anti}\log(8.25) = 1.77\times10^8$ Wm^{-2}) at $\omega_c = 1.78\times10^{14}$ s^{-1} ($B_s = 14.2$ T). With further increase in value of ω_c, $I_{0,th}$ increases sharply and saturates at high values of ω_c. This typical behaviour of $I_{0,th}$ arises due to the resonance condition $\omega_c^2 \approx \omega_0^2$, which can be seen in eq. (18b). Thus the applied d.c. magnetic field at resonance ($\omega_c \approx \omega_0$) reduces $I_{0,th}$ by approximately 3.56 times. In a doped n-InSb crystal, such lowering of $I_{0,th}$ by applying a magnetic field $B_s = 14.2$ T makes the crystal a potential candidate material for parametric amplification studies.

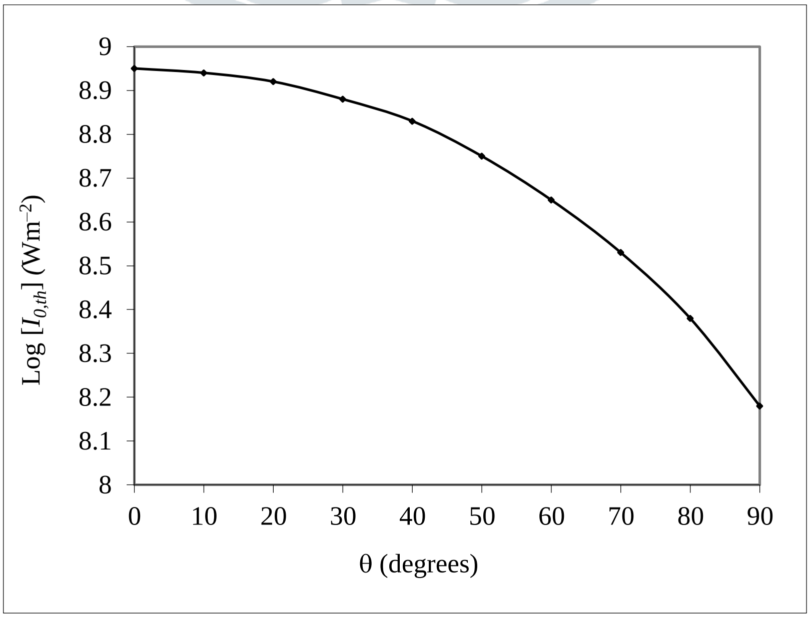

Fig. 2. Variation of $I_{0,th}$ with θ with $k = 2\times10^6$ m^{-1} and $\omega_c \sim \omega_0$.

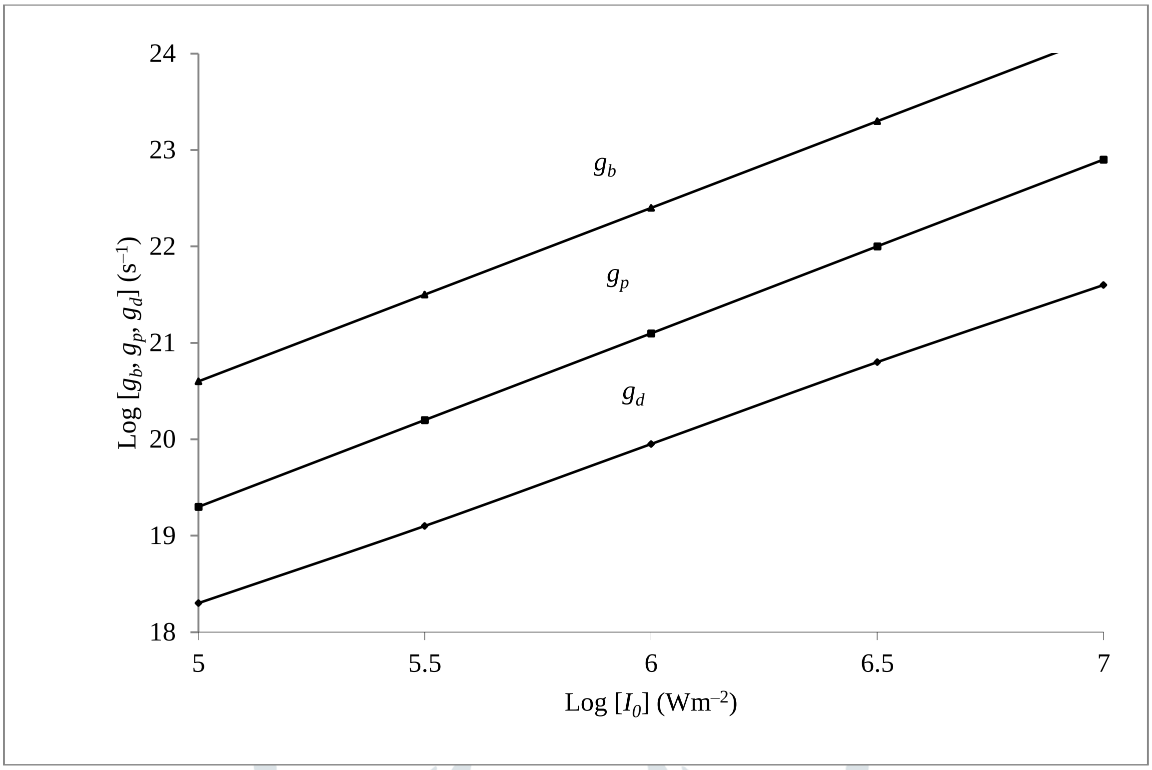

Fig. 3. Variation of g_d, g_p and g_b with I_0 with $k = 2\times10^6$ m^{-1}, $\omega_c \sim \omega_0$ and $\theta = 45^o$.

The nature of dependence of $I_{0,th}$ with θ is shown in Fig. 2. It can be observed that for a constant magnetic field, $I_{0,th}$ (starts with a value $\text{anti}\log(8.95) = 8.9\times10^8\ \text{Wm}^{-2}$ at $\theta = 0^o$), decreases parabolically with increasing value of θ ($I_{0,th} = \text{anti}\log(8.18) = 1.5\times10^8\ \text{Wm}^{-2}$ at $\theta = 90^o$).

Considering the pump intensities well above the threshold ($I_0 > I_{0,th}$), the gain constants for all the three types of couplings (i.e. piezoelectric g_p, deformation g_d and both g_b) can be estimated using eqs. (20a), (20b) and (20c) respectively. The nature of dependence of gain constants g_p, g_d, and g_b on different parameters such as pump intensity I_0, wave number k, magnetic field B_s (in terms of ω_c), and scattering angle θ are plotted in Figs. 3 – 6, respectively.

The nature of dependence of g_d, g_p and g_b with I_0 is shown in Fig. 3. It can be observed that all the three gain constants increase linearly with the pump intensity $I_0(> I_{0,th})$. It is found that the parametric gain constants are in the ratio $g_d : g_p : g_b :: 1:10:400$ at $k = 2\times10^6\ \text{m}^{-1}$, $\omega_c \approx \omega_0$ and $\theta = 45^o$.

The nature of dependence of g_d, g_p and g_b with k which varies from 10^6 to 10^7 m^{-1} is shown in Fig. 4. In this wavelength region, i.e. $10^6 < k < 10^7\ \text{m}^{-1}$, it can be observed that all the three gain constants increase parabolically with k. It can also be observed that for $k = 10^6\ \text{m}^{-1}$, all the three curves coincide, i.e. g_d, g_p and g_b are all equal ($\text{anti}\log(24.2) = 1.58\times10^{24}\ \text{s}^{-1}$). With increasing $k\,(> 10^6\ \text{m}^{-1})$, all the three gain constants increase separating each other with $g_b > g_p > g_d$. It is found that the parametric gain constants are in the ratio $g_d : g_p : g_b :: 1:18:158$ at $k = \text{anti}\log(6.8) = 6.3\times10^6\ \text{m}^{-1}$, $I_0 = 8\times10^8\ \text{Wm}^{-2}$, $\omega_c \approx \omega_0$ and $\theta = 45^o$.

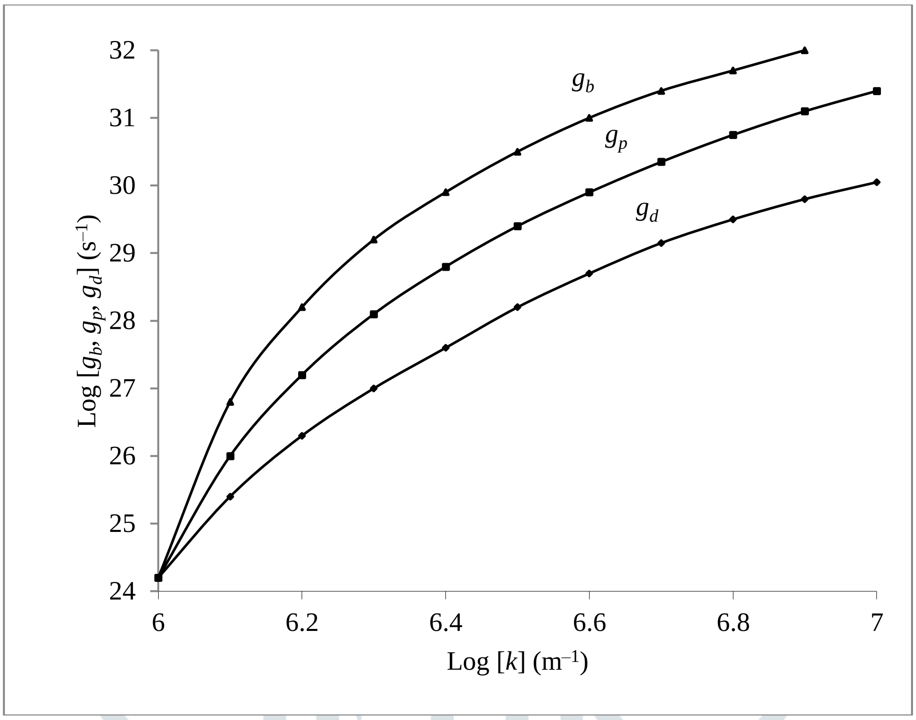

Fig. 4. Variation of g_d, g_p and g_b with k with $I_0 = 8\times10^8$ Wm^{-2}, $\omega_c \sim \omega_0$ and $\theta = 45^o$.

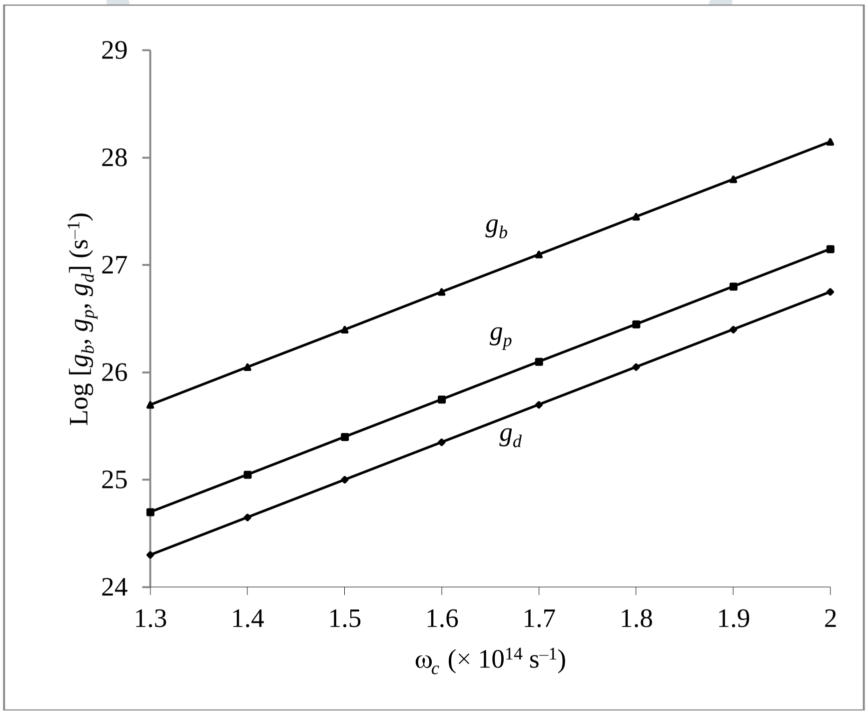

Fig. 5. Variation of g_d, g_p and g_b with B_s (in terms of ω_c) with $\theta = 45^o$ and $I_0 = 8\times10^8$ Wm^{-2}.

The nature of dependence of g_d, g_p and g_b with B_s (in terms of ω_c) is shown in Fig. 5. It can be observed that all the three gain constants increase linearly with ω_c (B_s). It is found that the parametric gain constants are in the ratio $g_d : g_p : g_b :: 1 : 2.5 : 25$ at $k = 2\times10^6\,\text{m}^{-1}$, $I_0 = 8\times10^8\,\text{Wm}^{-2}$ and $\theta = 45^o$.

The nature of dependence of g_d, g_p and g_b with θ is shown in Fig. 6. It can be seen that for $\theta = 0^o$, the gain constants are small. For $0^o < \theta < 40^o$, the gain constants remain constant. For $\theta > 40^o$, the gain constants increases rapidly, come closer and becomes independent for backscattered mode ($\theta = 90^o$). For backscattered mode, i.e. $\theta = 90^o$, one may obtain $g_p = g_b = g_d$ ($= \text{anti}\log(22.8) = 6.3\times10^{22}\,\text{s}^{-1}$).

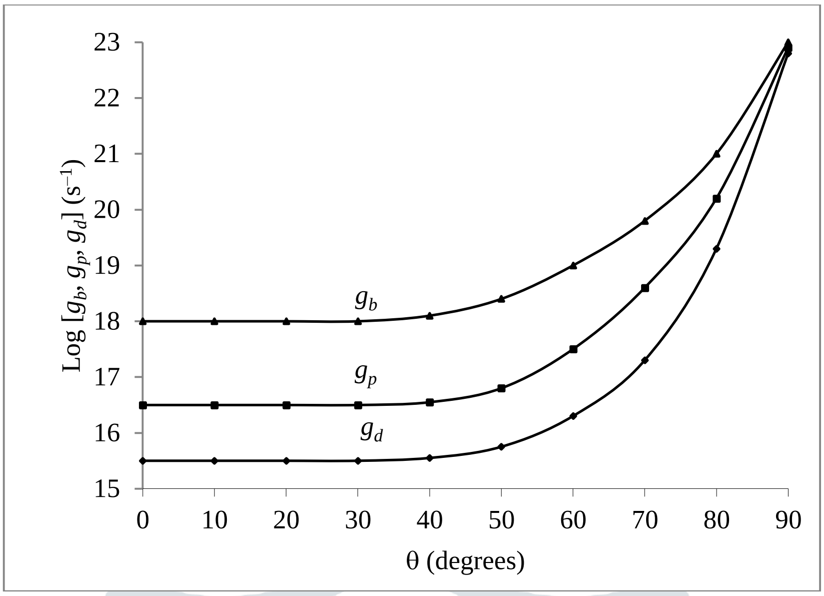

Fig. 6. Variation of g_d, g_p and g_b with θ with $k = 2\times10^6$ m^{-1} and $\omega_c \sim \omega_0$ and $I_0 = 8\times10^8$ Wm^{-2}.

From the above discussion, we conclude that (*i*) the application of large d.c. magnetic field significantly reduces the threshold pump intensity $I_{0,th}$ for the onset of parametric amplification. At sufficiently high magnetic field (i.e. $\omega_c \approx \omega_0$) $I_{0,th}$ is reduced by approx. 3.56 times; (ii) the parametric gain constants (g_d, g_p and g_b) are found to increase with increasing pump intensity $I_0 (> I_{0,th})$, wave number k, magnetic field B_s (ω_c), and scattering angle θ. Moreover, it is found that $g_b > g_p > g_d$. The present theory thereby provides an insight into developing potentially useful parametric backward wave amplifiers by incorporating the material characteristics of the medium.

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