MATHEMATICAL ANALYSIS OF SORET EFFECT ON NON-NEWTONIAN FLUID FLOW THROUGH POROUS MEDIUM IN PRESENCE OF MAGNETIC FIELD.

[1]Mr. Rahul P. Mehta, [2]Dr. Hari R. Kataria
[1]Assistant Professor, [2]Professor and Head

[1]Applied Science & Humanities Department,
Sardar Vallabhbhai Patel Institute of Technology, Vasad, India
[1]rahul1078@gmail.com
[1]Corresponding author (+91-9898066500)

[2]Department of Mathematics, Faculty of Science,
The M. S. University of Baroda, Vadodara, India
[2]hrkrmaths@yahoo.com

Abstract: Soret and magnetic field effects on non-Newtonian fluid flow with ramped boundary conditions are studied. The governing equations are transformed using appropriate change of variables to obtain the system of Linear ordinary differential equations. Laplace transform is employed to solve this problem. Analytic solutions for MHD flow of Non-Newtonian fluid through porous medium with ramped fluid velocity and constant velocity are derived. Effects of various parameters on flow are observed through various graphs.

KEYWORDS: MHD; Non-Newtonian fluid; Porous medium; Ramped Velocity

NOMENCLATURE:

u' Fluid velocity
t' Time
T' Fluid temperature
B_0 Uniform magnetic field
k' Permeability of porous medium
C' Concentration
k_1 thermal conductivity of the fluid
Gr Thermal Grashof number
D_T Thermal diffusion coefficient
k_2' Chemical reaction coefficient
Pr Prandtl number
ν Kinematic viscosity coefficient
β'_T Volumetric coefficient of thermal expansion
β'_c Volumetric coefficient of concentration expansion
$\emptyset$ Porosity of the porous medium

u Dimensionless fluid velocity
t Dimensionless time
θ Dimensionless fluid temperature
M Magnetic field parameter
k Permeability of porous medium parameter
C Dimensionless concentration
C_p Specific heat at constant pressure
Gm Mass Grashof number
D_M Mass diffusion coefficient
Kr Chemical reaction parameter
Sc Schmidt number
ρ Fluid density
g Acceleration due to gravity
σ Electrical conductivity

I. INTRODUCTION:

Study of MHD flow of Non-Newtonian fluid is instrumental, as there are many engineering applications. The flow pattern of such fluids have been studied for industrial oils, slurry flows, dilute polymer solutions etc. Kataria and Patel [1 – 4] have examined effects of heat and mass transfer on various fluids. Kataria and Mittal [5 – 6] have discussed MHD Nanofluid flow. Sheikholeslami at el. [7 – 8] also have studied free convective Nanofluid flow.

The objective of the present paper is to investigate impact of magnetic field on heat and mass transfer. Novelty of the article is the analytic study of the MHD non-Newtonian fluid flow with ramped boundary conditions. The derived ordinary differential equations are solved using the Laplace transform. The effects of the pertinent parameters governing the problem are discussed.

II. MATHEMATICAL FORMULATION OF THE PROBLEM:

Sketch of the physical problem is drawn in Fig. 1. Axes are chosen as follows. $x'-$ axis which is drawn vertically is the wall and y' – axis is drawn horizontally. As shown in that figure, magnetic field of strength B_0 is in the opposite direction to the fluid flow. When time $t' \leq 0$, the plate and the fluid are stationary having surface concentration C'_∞ and constant temperature of fluid and the plate is assumed to be T'_∞. When time is $0 < t' \leq t_0$, fluid velocity, temperature and concentration are $u'_\infty + (u'_w - u'_\infty)\, t'/t_0$, $T'_\infty + (T'_w - T'_\infty)\, t'/t_0$ and $C'_\infty + (C'_w - C'_\infty)\, t'/t_0$ respectively. Whereas for time $t' > t_0$, their values become constant u'_w, T'_w and C'_w respectively. Effect of viscous dissipation induced by magnetic and electrical field are neglected. Here flow is considered as one dimensional laminar flow, and the fluid is incompressible Non-newtonian fluid. The equations which are governed for all these assumptions, are derived using Boussinesq's approximation. They are as follows.

$$\frac{\partial u'}{\partial t'} = \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} u' - \frac{\mu\phi}{\rho k'} u' + g\beta'_T (T' - T'_\infty) + g\beta'_C (C' - C'_\infty) \qquad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k_1}{\rho c_p} \frac{\partial^2 T'}{\partial y'^2} \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D_M \frac{\partial^2 C'}{\partial y'^2} + D_T \frac{\partial^2 T'}{\partial y'^2} - k'_2 (C' - C'_\infty) \qquad (3)$$

With following initial and boundary conditions:

$$u' = 0,\ T' = T'_\infty,\ C' = C'_\infty;\ as\ \ y' \geq 0\ and\ \ t' \leq 0, \qquad (4)$$

$$u' = \begin{cases} u'_\infty + (u'_w - u'_\infty)\, t'/t_0 & if\ \ 0 < t' < t_0 \\ u'_w & if\ \ \ t' \geq t_0 \end{cases},\ as\ t' > 0\ and\ y' = 0\ , \qquad (5)$$

$$T' = \begin{cases} T'_\infty + (T'_w - T'_\infty)\, t'/t_0 & if\ \ 0 < t' < t_0 \\ T'_w & if\ \ \ t' \geq t_0 \end{cases},\ as\ t' > 0\ and\ y' = 0\ , \qquad (6)$$

$$C' = \begin{cases} C'_\infty + (C'_w - C'_\infty)\, t'/t_0 & if\ \ 0 < t' < t_0 \\ C'_w & if\ \ \ t' \geq t_0 \end{cases};\ as\ t' > 0\ and\ y' = 0\ , \qquad (7)$$

$$u' \to 0, T' \to T'_\infty,\ C' \to C'_\infty;\ as\ y' \to \infty\ and\ t' \geq 0$$

Introducing the following dimensionless quantities:

$$y = \frac{y'}{U_0 t_0},\ u = \frac{u'}{U_0},\ t = \frac{t'}{t_0},\ \theta = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)},\ C = \frac{(C' - C'_\infty)}{(C'_w - C'_\infty)},\ Gr = \frac{vg\beta'_T (T'_w - T'_\infty)}{U_0^3}$$

$$Gm = \frac{vg\beta'_C (C'_w - C'_\infty)}{U_0^3},\ M^2 = \frac{\sigma B_0^2 v}{\rho U_0^2},\ P_r = \frac{\rho v C_p}{k_1}, Sc = \frac{v}{D_M}$$

$$Kr = \frac{v k'_2}{U_0^2},\ Sr = \frac{D_T (T'_w - T'_\infty)}{v (C'_w - C'_\infty)} \qquad (8)$$

Equations (1), (2) and (3) become, (dropping out the " ′ " notation (for simplicity))

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \left(M^2 + \frac{1}{k}\right) u + G_r \theta + G_m C \qquad (9)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} \qquad (10)$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2} - krC \qquad (11)$$

With initial and boundary condition

$$u = \theta = C = 0\ , y \geq 0, t \leq 0\ ,$$

$$u = \begin{cases} t, & 0 < t \leq 1 \\ 1 & t > 1 \end{cases},\quad at\ \ y = 0,\quad t > 0$$

$$\theta = \begin{cases} t, & 0 < t \leq 1 \\ 1 & t > 1 \end{cases},\ at\ \ y = 0,\ t > 0$$

$$C = \begin{cases} t, & 0 < t \leq 1 \\ 1 & t > 1 \end{cases}\ at\ \ y = 0,\ t > 0\ ,$$

$$u \to 0, \theta \to 0, C \to 0\quad at\ \ y \to \infty,\ t > 0 \qquad (12)$$

Solution of the problem:

We find the analytic solutions for fluid velocity; Temperature and Concentration. They are obtained from equations (9) to (11) with initial and boundary conditions (12) using the Laplace transform technique.
Solution of the problem for ramped velocity, ramped wall temperature and ramped surface concentration:

$$\theta(y, t) = f_7(y, t) - f_7(y, t - 1) H(t - 1) \qquad (13)$$

$$C(y,t) = [g_4(y,t) - g_5(y,t)] - [g_4(y,t-1) - g_5(y,t-1)]H(t-1) \quad (14)$$
$$u(y,t) = f_2(y,t) - f_2(y,t-1)H(t-1) + h_1(y,t) - h_1(y,t-1)H(t-1) \quad (15)$$

Solution of the problem for constant velocity, ramped temperature and ramped surface concentration

In order to understand effects of ramped velocity of the fluid flow, we must compare our results with constant velocity. In this case, the initial and boundary conditions are same as Eq. (12) except $u = 1$ for $y = 0,\ t > 0$. The fluid velocity $u(y,t)$ found for this case using Laplace Transform will be

$$u(y,t) = f_1(y,t) + h_1(y,t) - h_1(y,t-1)H(t-1) \quad (16)$$

Where,

$$h_1(y,t) = g_1(y,t) + g_2(y,t) + g_3(y,t) \quad (17)$$
$$g_1(y,t) = a_{29}f_1(y,t) + a_{30}f_2(y,t) + a_{31}f_3(y,t) + a_{32}f_4(y,t) + a_{33}f_5(y,t) \quad (18)$$
$$g_2(y,t) = a_{34}f_6(y,t) - a_{17}f_7(y,t) - a_{31}f_8(y,t) + a_{28}f_9(y,t) \quad (19)$$
$$g_3(y,t) = a_{35}f_{10}(y,t) + a_{20}f_{11}(y,t) + a_{32}f_{12}(y,t) + a_{25}f_{13}(y,t) \quad (20)$$
$$g_4(y,t) = a_{37}f_{10}(y,t) - a_{36}f_{13}(y,t) \quad (21)$$
$$g_5(y,t) = a_{36}f_6(y,t) - a_{36}f_9(y,t) \quad (22)$$
$$f_1(y,t) = \frac{1}{2}\left[e^{-y\sqrt{a_1}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{a_1 t}\right) + e^{y\sqrt{a_1}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{a_1 t}\right)\right] \quad (23)$$
$$f_2(y,t) = \frac{1}{2}\left[\left(t - \frac{y}{2\sqrt{a_1}}\right)e^{-y\sqrt{a_1}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{a_1 t}\right) + \left(t + \frac{y}{2\sqrt{a_1}}\right)e^{y\sqrt{a_1}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{a_1 t}\right)\right] \quad (24)$$
$$f_3(y,t) = \frac{e^{a_8 t}}{2}\left[e^{-y\sqrt{\frac{1}{a_8}(a_1+a_8)}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(a_1+a_8)t}\right) + e^{y\sqrt{\frac{1}{a_8}(a_8-a_1)}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(a_1-a_8)t}\right)\right] \quad (25)$$
$$f_4(y,t) = \frac{e^{a_{13} t}}{2}\left[e^{-y\sqrt{\frac{1}{a_{13}}(a_1+a_{13})}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(a_1-a_{13})t}\right) + e^{y\sqrt{\frac{1}{a_{13}}(a_1-a_{13})}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(a_1-a_{13})t}\right)\right] \quad (26)$$
$$f_5(y,t) = \frac{e^{a_5 t}}{2}\left[e^{-y\sqrt{\frac{1}{a_5}(a_1+a_5)}}\, erfc\left(\frac{y}{2\sqrt{t}} - \sqrt{(a_1+a_5)t}\right) + e^{y\sqrt{\frac{1}{a_5}(a_5-a_1)}}\, erfc\left(\frac{y}{2\sqrt{t}} + \sqrt{(a_1-a_5)t}\right)\right] \quad (27)$$
$$f_6(y,t) = erfc\left(\frac{1}{2}\sqrt{\frac{1}{a_2 t}}\, y\right) \quad (28)$$
$$f_7(y,t) = \left(\frac{y^2}{2a} + t\right) erfc\left(\frac{y}{2\sqrt{a_2 t}}\right) - \frac{y}{2}\frac{\sqrt{t}}{\sqrt{a_2\pi}}e^{-\frac{y^2}{4a_2 t}} \quad (29)$$
$$f_8(y,t) = \frac{e^{bt}}{2}\left[e^{-y\sqrt{\frac{a_8}{a_2}}}\, erfc\left(\frac{y\sqrt{a_2}}{2\sqrt{t}} - \sqrt{a_8 t}\right) + e^{y\sqrt{\frac{a_8}{a_2}}}\, erfc\left(\frac{y\sqrt{a_2}}{2\sqrt{t}} + \sqrt{a_8 t}\right)\right] \quad (30)$$
$$f_9(y,t) = \frac{e^{bt}}{2}\left[e^{-y\sqrt{\frac{a_5}{a_2}}}\, erfc\left(\frac{y\sqrt{a_2}}{2\sqrt{t}} - \sqrt{a_5 t}\right) + e^{y\sqrt{\frac{a_5}{a_2}}}\, erfc\left(\frac{y\sqrt{a_2}}{2\sqrt{t}} + \sqrt{a_5 t}\right)\right] \quad (31)$$
$$f_{10}(y,t) = \frac{1}{2}\left[e^{-y\sqrt{kr\,Sc}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{kr\,t}\right) + e^{y\sqrt{kr\,Sc}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{kr\,t}\right)\right] \quad (32)$$
$$f_{11}(y,t) = \frac{1}{2}\left[\left(t - \frac{y\sqrt{Sc}}{2\sqrt{kr}}\right)e^{-y\sqrt{Sc\,kr}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{kr\,t}\right) + \left(t + \frac{y\sqrt{Sc}}{2\sqrt{kr}}\right)e^{y\sqrt{Sc\,kr}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{kr\,t}\right)\right] \quad (33)$$
$$f_{12}(y,t) = \frac{e^{-a_{13} t}}{2}\left[e^{-y\sqrt{Sc(kr-3)}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(kr-a_{13})t}\right) + e^{y\sqrt{Sc(kr-a_{13})}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(kr-a_{13})t}\right)\right] \quad (34)$$
$$f_{13}(y,t) = \frac{e^{a_5 t}}{2}\left[e^{-y\sqrt{Sc(kr+a_5)}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} - \sqrt{(kr+a_5)t}\right) + e^{y\sqrt{Sc(kr+a_5)}}\, erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}} + \sqrt{(kr+a_5)t}\right)\right] \quad (35)$$

III. RESULT AND DISCUSSION:

We have inspected the fluid velocity, temperature and concentration for various parameters like Magnetic field parameter M, Grashof number (Gr), mass Grashof number (Gm), thermal radiation parameter Nr, chemical reaction parameter Kr, Soret effect Sr and Prandtl number Pr. For plotting graphs as shown in figures 2 – 13, change is made in values of one parameter at a time and all remaining parameters remain fixed.

The fig. 2 shows effect of Magnetic field parameter M on velocity profiles for both velocity conditions. It is seen that velocity decreases with increment in M values. Lorentz force induced on boundary is the reason for such decrement. Permeability of porous medium K has positive impact on both velocity conditions which is reflected in Fig.3. Fig.4 indicates that different values of Schmidt number Sc have reciprocal effect on concentration. It is seen from Fig. 5 and Fig. 6 that when we increase Sr, velocity and concentration also increased. Effects of Grashof number Gr and mass Grashof number Gm has positive impact on velocity as described in fig. 7 and fig. 8. Fig. 9 shows that increment in Pr will reduce the temperature.

IV. CONCLUSION:

The concept of this research is to get analytical solution for MHD flow of Non-Newtonian fluid passing through a vertical plate with ramped fluid velocity and observe various effects like effects of radiation, chemical reaction and Soret effect. Results are derived for constant and variable velocity.

Key remarks for the conclusions can be summarized as:

- Magnetic field parameter M has impeding effects with constant or ramped velocity.
- Permeability of porous medium k, thermal Grashof number Gr and mass Grashof number Gm have affirmative correlation with velocity.

- Temperature of the fluid also has negative impact on Pr.
- Soret number Sr has positive correlation with velocity or concentration.
- Schmidt number Sc has reverse impact on concentration.

V. APPENDIX:

$$a_1 = M^2 + \frac{1}{k} \qquad a_2 = \frac{1+Nr}{Pr} \qquad a_3 = \frac{1}{Sc\, a_3} - \frac{1}{Sc}$$

$$a_4 = \frac{Kr}{Sc} \qquad a_5 = \frac{a_4}{a_3} \qquad a_6 = \frac{Sc}{a_3 a_2}$$

$$a_7 = \frac{1}{a_2} - 1 \qquad a_8 = \frac{a_1}{a_7} \qquad a_9 = \frac{Gr}{a_7}$$

$$a_{10} = Sc - 1 \qquad a_{11} = Sc + 1 \qquad a_{12} = Sc\, Kr - a_1$$

$$a_{13} = \frac{a_{12}}{a_{10}} \qquad a_{14} = \frac{Gm}{a_{10}} \qquad a_{15} = \frac{Gm\, a_6}{a_{10}}$$

$$a_{16} = \frac{Gm\, a_6}{a_{10}} \qquad a_{17} = \frac{-a_9}{a_8} \qquad a_{18} = \frac{a_9}{{a_8}^2}$$

$$a_{19} = \frac{a_9}{1-a_8} - a_{17} - \frac{a_{18}}{1-a_8} \qquad a_{20} = \frac{a_{14}}{a_{13}} \qquad a_{21} = \frac{a_{14}}{{a_{13}}^2}$$

$$a_{22} = \frac{a_{14}}{1+a_{13}} - a_{20} - \frac{a_{21}}{1+a_{13}} \qquad a_{23} = \frac{-a_{15}}{a_{13} a_5} \qquad a_{24} = \frac{a_{15}}{a_{13}(a_{13}+a_5)}$$

$$a_{25} = \frac{a_{15}}{a_5(a_{13}+a_5)} \qquad a_{26} = \frac{a_{16}}{a_8 a_5} \qquad a_{27} = \frac{a_{16}}{a_8(a_8-a_5)}$$

$$a_{28} = \frac{-a_{16}}{a_5(a_8-a_5)} \qquad a_{29} = a_{19} + a_{22} + a_{23} - a_{26} \qquad a_{30} = a_{17} + a_{20}$$

$$a_{31} = a_{18} - a_{27} \qquad a_{32} = a_{21} + a_{24} \qquad a_{33} = a_{25} - a_{28}$$

$$a_{34} = -a_{19} + a_{26} \qquad a_{35} = a_{22} + a_{23} \qquad a_{36} = \frac{a_6}{a_5}$$

$$a_{37} = a_{36} + 1$$

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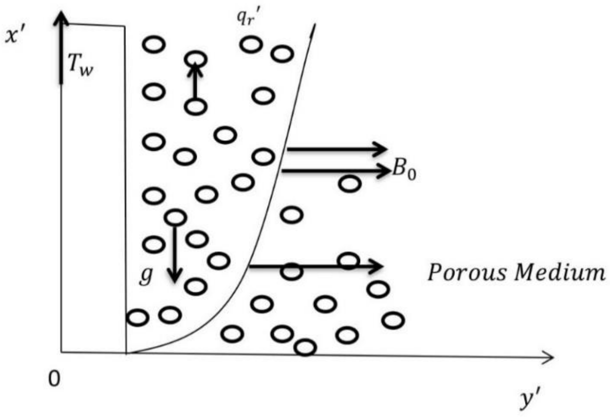

Figure 1: Physical Sketch of the problem

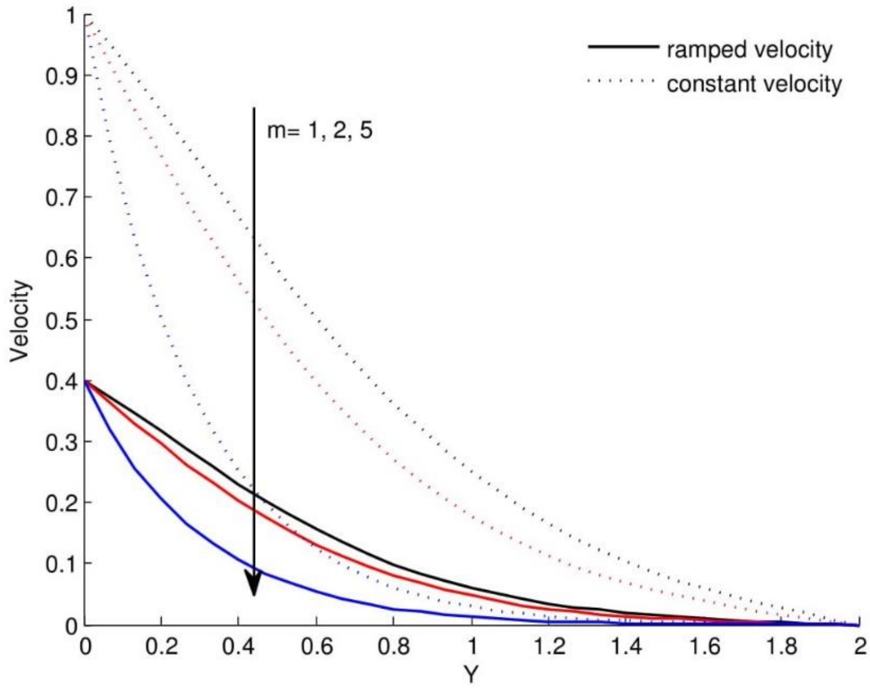

Figure 2: Velocity profile u for different values of y and M for k=0.8, Pr=10, Sc=6.2, Gr=3, Gm=2, kr=10, Sr=5

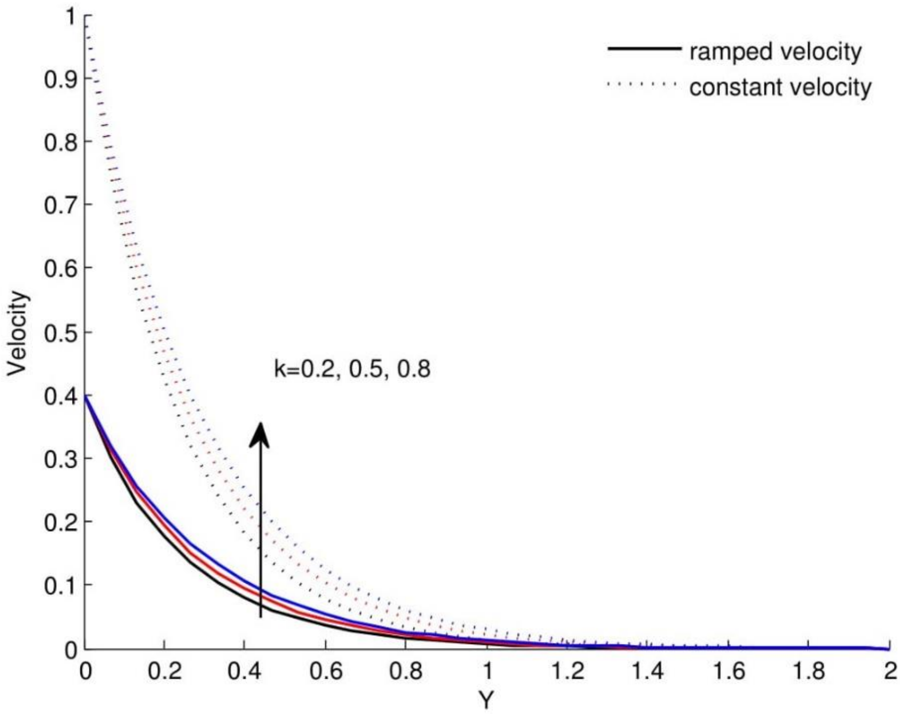

Figure 3: Velocity profile u for different values of y and k for M=5, Pr=10, Sc=6.2, Gr=3, Gm=2, kr=10, Sr=5

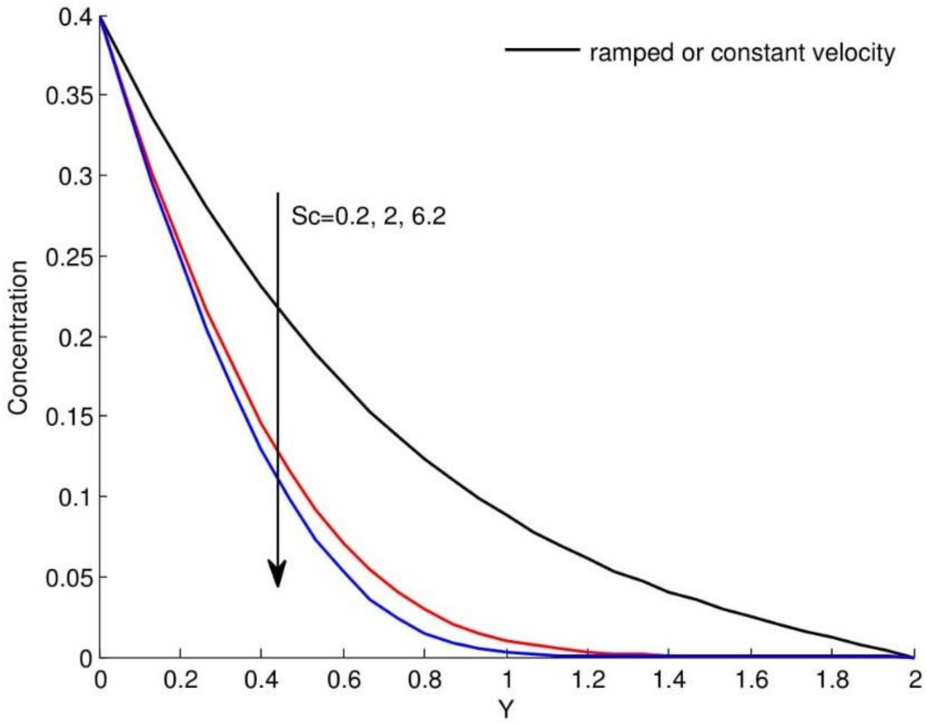

Figure 4: Concentration profile for different values of y and Sc for M=5, k=0.8, Pr=10, Gr=3, Gm=2, kr=10, Sr=5

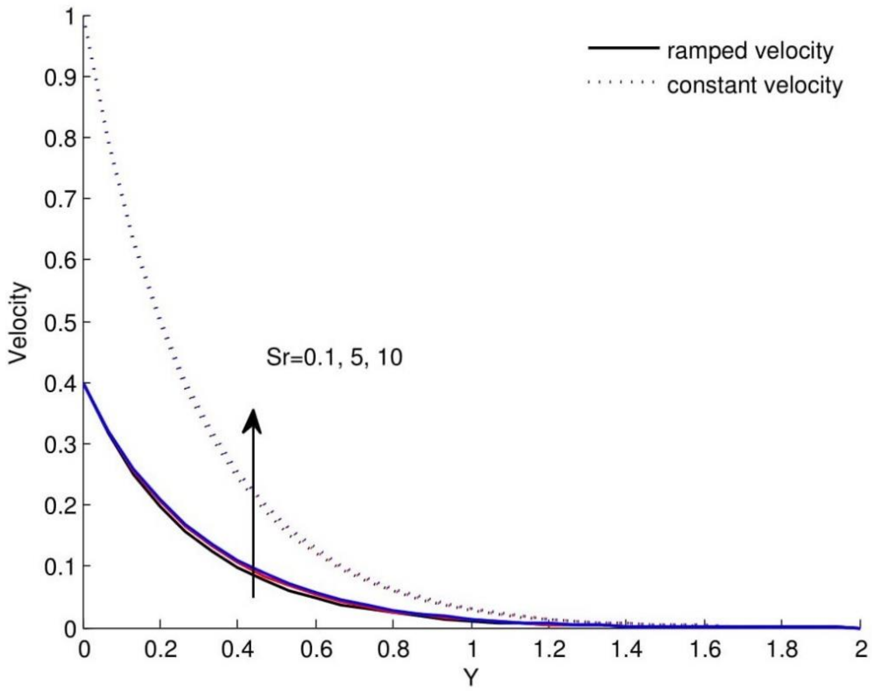

Figure 5: Velocity profile u for different values of y and Sr for M=5, k=0.8, Pr=10, Sc=6.2, Gr=3, Gm=2, kr=10

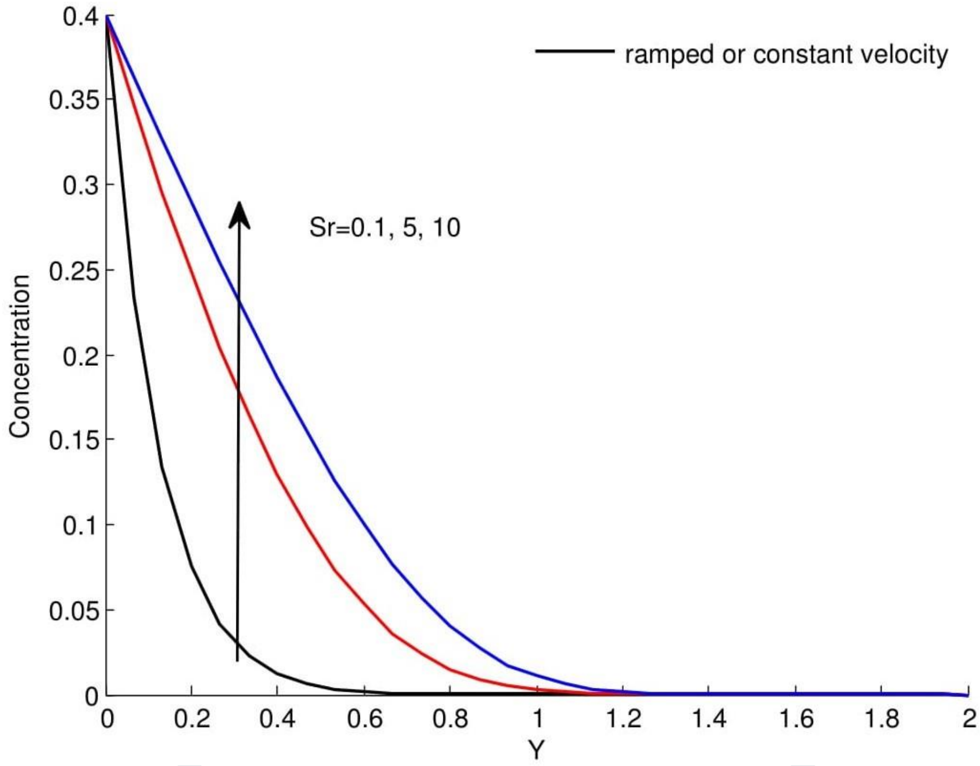

Figure 6: Concentration profile for different values of C and Sr for M=5, k=0.8, Pr=10, Sc=6.2, Gr=3, Gm=2, kr=10

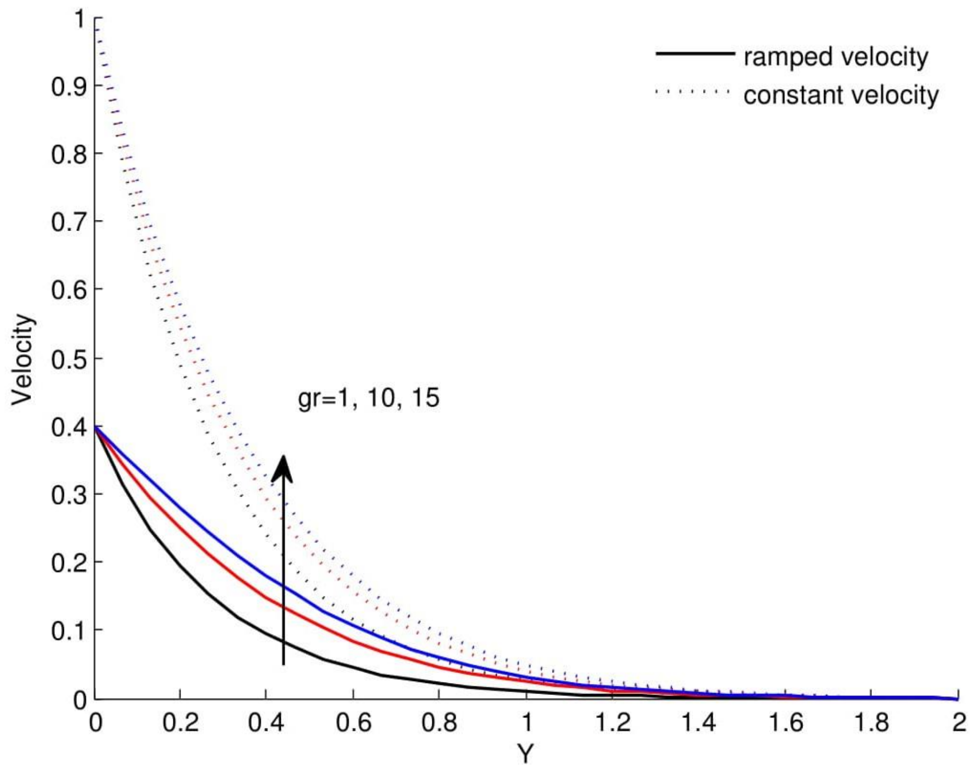

Figure 7: Velocity profile u for different values of y and Gr for M=5, k=0.8, Pr=10, Sc=6.2, Gm=2, kr=10, Sr=5

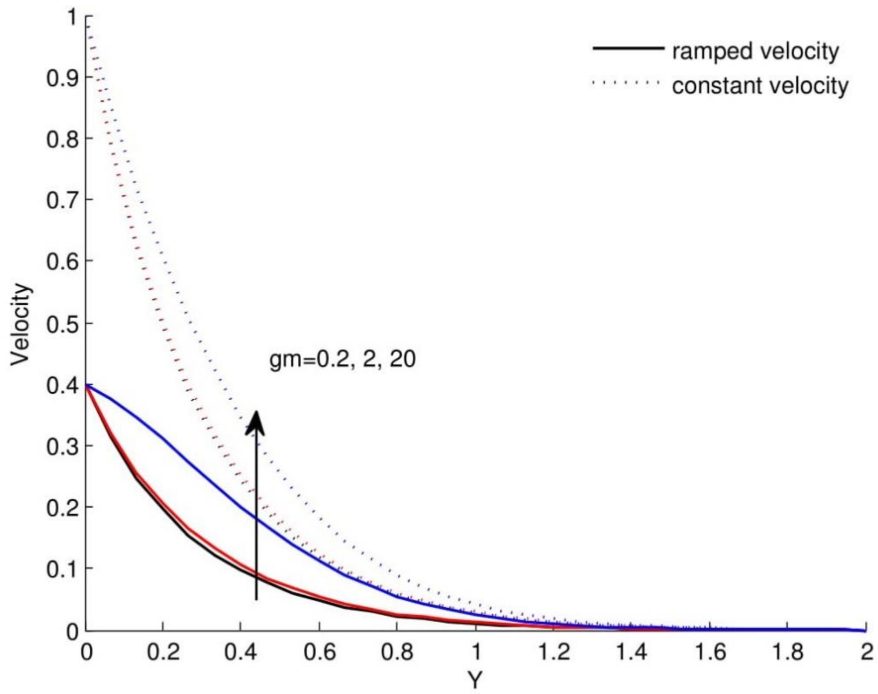

Figure 8: Velocity profile u for different values of y and Gm for M=5, k=0.8, Pr=10, Sc=6.2, Gr=3, kr=10, Sr=5

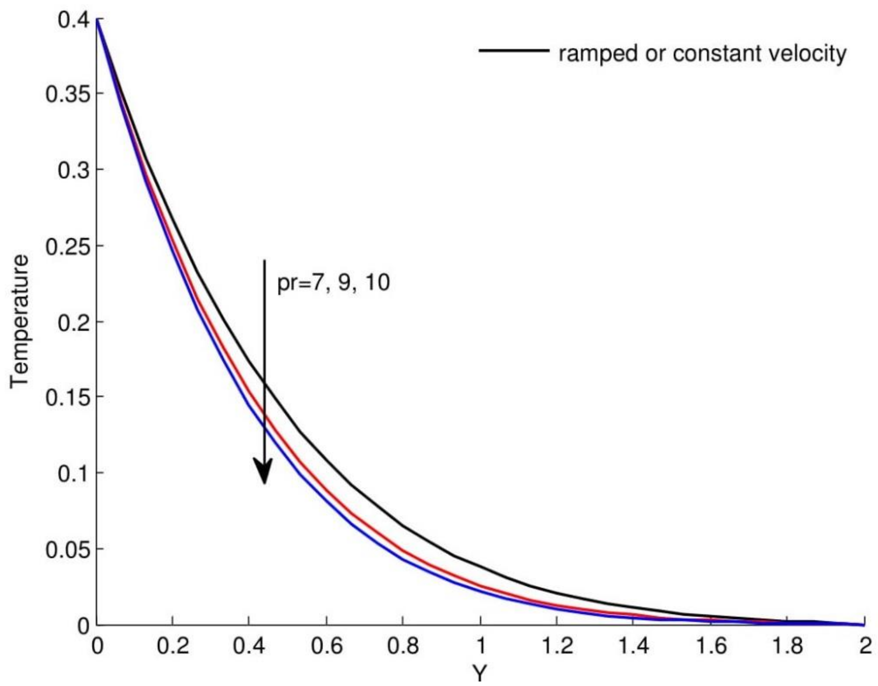

Figure 9: Temperature profile for different values of θ and Pr for M=5, k=0.8, Sc=6.2, Gr=3, Gm=2, kr=10, Sr=5