SYSTEM RELIABILITY WITH IMPERFECT FAULT COVERAGE

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Abstract

In this Paper, we deal with the Reliability Analysis of a System which comprises two subsystems connected in series. Each subsystem has an active and a spare unit with constant failure rates. Reliability of the subsystems with covered and uncovered faults has been calculated with known coverage parameter and failure rates for a particular time period. The mean time to failure and hazard rate function of the System has been calculated. Numerical illustration is also provided by graphical representation of Reliability analysis.

Keywords

Imperfect Fault Coverage, Standby Redundancy, MTTF and hazard rate function.

1. INTRODUCTION

Reliability of a System is defined as the probability that it performs its purpose within tolerance for the period of time planned under the given operating conditions. The concept of Reliability stresses four elements namely probability, tolerance, time and operating conditions. It tells how much reliable a system is. [3] It is a key concern of the System engineer in any repairable system. Complex systems like computer and telecommunication networks, electric appliances and many more are real life examples. Designing the systems and determining their reliability are very important task for the system managers and analyst.

In many practical situations, the production may be disturbed due to the sudden failure of the machines which brings unwanted loss of income as well as goodwill in the market. Such situation can be controlled up to some extent using the support of standby units and repairs. With the help of standbys, the system remains active and continues to perform its job in case of failure. The standby concept has attracted many researchers working in the field of Reliability theory. Reliability analysis has also been done for Machine repair problem with standbys. Reliability modeling with imperfect coverage has been examined by many authors time to time. Due to imperfect coverage, the Reliability of Repairable systems cannot be improved with the increase of redundancy. [2]

In this paper, we discuss the analysis of Reliability in Repairable system with imperfect fault coverage. The perfect detection and recovery of an active unit is done with probability **c** which is known as coverage parameter and imperfect detection and recovery has been done with the probability **1-c**. We formulate the model involved in our study and analyze its System Reliability using fault coverage parameter and failure rates of active and spare units in the subsystems.

In section 1, we give the introduction, in section 2, basic concepts have been given and in section 3 we have dealt with Reliability of a System with Imperfect fault coverage. In section 4, we analyze the System Reliability of the model involved in our study. In section 5, we present numerical illustration with graphical representation. Finally in section 6, we draw the conclusion.

2. BASIC CONCEPTS

Reliability

Let the random variable X be the lifetime or the time to failure of a component. The probability that the component survives until sometime *t* is called the reliability R(t) of the component.

$$R(t)=P(X>t)=1-F(t).$$

Mean Time To Failure

Let X denote the lifetime of a component so that its Reliability R(t)=P(X>t), then the expected life or mean time to failure (MTTF) of the component is given by

$$E[X]=\int_0^\infty R(t)\,dt$$

Covered and Not-covered fault

Reliability models of systems with standby redundancy are not very realistic. Reliability of such items strongly depends on effectiveness of recovery mechanism. In particular, some spare parts are impossible to recover from the failure. Such faults are said to be *not covered* and probability of such faults are denoted by $1-\varsigma$, where ς is the probability of occurrence of *covered* faults and is known as the **coverage factor** or **coverage parameter.**

Standby Redundancy

The Redundancy wherein all alternative means of performing the function is inoperative until needed, and is switched on upon failure of the primary means of performing the function**.**

Laplace- Stieltjes Transform

If X is a nonnegative continuous random variable, then we define the Laplace-Stieltjes transform (LST) of X,

$$L_X(S)=\int_0^\infty e^{-st} f(t)\,dt$$

3. RELIABILITY ANALYSIS OF A SYSTEM

Reliability models of Systems with dynamic redundancy developed earlier are not very realistic. It has been demonstrated that the Reliability of such Systems depends strongly on the effectiveness of recovery mechanisms.

Consider a System which comprises two subsystems connected in series. Each subsystem has an active and a standby spare with constant failure rates λ and μ(presumably $0 \le \mu \le \lambda$), thus making it a warm spare. Let c_1 be the probability of the successful recovery on the failure of an active and let c_2 is the probability of the successful recovery following the failure of a spare unit.

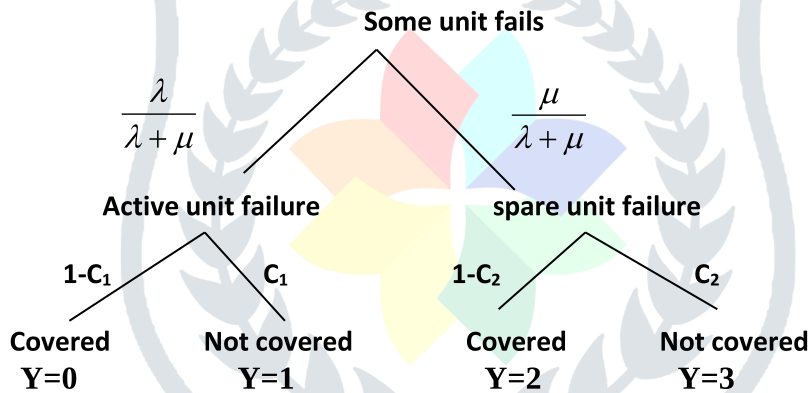
We compute the Laplace-Stieltjes transform of the system lifetime X as follows.

Let X_1 and X_2 be the random variable that denotes the lifetime of an active and spare units respectively, we assume X_1~EXP(λ) and X_2~EXP(μ). The Residual lifetime of the System is W~EXP(λ). Denote the random variable Y taking values as

$$Y = \begin{cases} 0, & \text{not-covered failure in the active unit,} \\ 1, & \text{covered failure in the active unit,} \\ 2, & \text{not-covered failure in the standby unit,} \\ 3, & \text{covered failure in the standby unit.} \end{cases}$$

the probability mass function of Y by noting that the probability of the active unit failing first is $\frac{\lambda}{\lambda+\mu}$, while the probability of the spare unit failing first is $\frac{\mu}{\lambda+\mu}$.

Consider the tree diagram,



and for covered and uncovered faults, we consider the probabilities to be,

$$p_Y(1) = \frac{\lambda C_1}{\lambda+\mu}, \qquad p_Y(0) = \frac{\lambda(1-C_1)}{\lambda+\mu}$$

$$p_Y(3) = \frac{\mu C_2}{\lambda+\mu}, \qquad p_Y(4) = \frac{\mu(1-C_2)}{\lambda+\mu}$$

If not covered fault has occurred (i.e., Y=0 or Y=2) the life time of the system is simply min(X_1,X_2), while a covered fault (i.e., Y=1 or Y=3) implies a system life time being min(X_1,X_2)+W
where, min(X_1,X_2)~EXP(λ+μ) and W~EXP(λ)

$\therefore$ The Conditional Laplace-Stieltjes transform of X for each type of not-covered fault is given by,

$$L_{X/Y}(S/Y=0) = L_{X/Y}(S/Y=2) = \frac{\lambda+\mu}{S+(\lambda+\mu)}, \qquad \text{.........(1)}$$

And for the covered fault, X is the sum of two independent exponentially distributed random variables and hence,

$$L_{X/Y}(S/Y=1)=L_{X/Y}(S/Y=3)=\frac{\lambda+\mu}{S+(\lambda+\mu)}\left[\frac{\lambda}{S+\lambda}\right] \qquad ....(2)$$

Using the theorem of total transform,

$$L_X(S)=\frac{\lambda+\mu}{S+\lambda+\mu}\left[\frac{\lambda(1-c_1)+\mu(1-c_2)}{\lambda+\mu}\right]+\frac{(\lambda+\mu)\lambda}{(S+\lambda+\mu)(S+\lambda)}\left[\frac{\lambda c_1+\mu c_2}{\lambda+\mu}\right] \qquad ....(3)$$

Thus,

$$L_X(S)=\frac{\lambda+\mu}{S+\lambda+\mu}\left[\frac{\lambda c_1+\mu c_2}{\lambda+\mu}\frac{\lambda}{S+\lambda}+\frac{\lambda(1-c_1)+\mu(1-c_2)}{\lambda+\mu}\right]$$

$$=L_{Y_1}(S)L_{Y_2}(S)$$

Where,

$$L_{Y_1}(S)=\frac{\lambda+\mu}{S+\lambda+\mu}$$

And

$$L_{Y_2}(S)=\frac{\lambda c_1+\mu c_2}{\lambda+\mu}\frac{\lambda}{S+\lambda}+\frac{\lambda(1-c_1)+\mu(1-c_2)}{\lambda+\mu}$$

$$=c\frac{\lambda}{S+\lambda}+[1-c]$$

Considering $c=\frac{\lambda c_1+\mu c_2}{\lambda+\mu}$ ....(4)

Equation(3) is a mixture of 100(1-c)% of an exponential, EXP(λ+μ) and c% of hypoexponential, HYPO(λ+μ, λ).

$$\therefore f_X(t)=(1-c)(\lambda+\mu)e^{-(\lambda+\mu)t}+\frac{c\lambda(\lambda+\mu)}{\mu}\left[e^{-\lambda t}-e^{-(\lambda+\mu)t}\right],\ t>0 \qquad ....(5)$$

We evaluate the Reliability of the given System,

$$R(t) = \int_t^{\infty} f(t)\,dt$$

$$= (1-c)e^{-(\lambda+\mu)t} + \frac{c}{\mu}(\lambda+\mu).e^{-\lambda t} - \frac{c}{\mu}\lambda.e^{-(\lambda+\mu)t},\ t \geq 0 \qquad \text{................(6)}$$

Where c is given by (4)

The Mean time to failure of the System is given by,

$$\text{MTTF} = \int_0^{\infty} R(t)\ dt = \frac{1-c}{\lambda+\mu}\left[c\left(\frac{\lambda+\mu}{\mu}\right)\left(\frac{1}{\lambda}\right)+1\right]$$

The Hazard rate of the system is given by,

$$h(t) = \frac{f(t)}{R(t)} = \frac{(1-c)(\lambda+\mu)e^{-(\lambda+\mu)t} + \frac{c\lambda(\lambda+\mu)}{\mu}\left[e^{-\lambda t} - e^{-(\lambda+\mu)t}\right]}{(1-c)e^{-(\lambda+\mu)t} + \frac{c}{\mu}(\lambda+\mu).e^{-\lambda t} - \frac{c}{\mu}\lambda.e^{-(\lambda+\mu)t}} \qquad \text{.........(7)}$$

Consider a system with two subsystems with failure rates of Active and Spare of the two systems being λ, μ and β, δ respectively, the coverage factor of two systems being 'C' and 'd', so that the Reliability of the two subsystems,

The **Reliability** of the system is given by

$$R_S(t) = R_X(t)R_Z(t)$$

$$= \begin{cases} (1-c)(1-d)e^{-(\lambda+\beta+\mu+\delta)t} + \frac{d}{\delta}(1-c)(\beta+\delta)e^{-(\lambda+\beta+\mu)t} - \frac{d}{\delta}(1-c)\beta e^{-(\lambda+\mu+\beta+\delta)t} \\ +(1-d)\frac{c}{\mu}(\lambda+\mu)e^{-(\beta+\lambda+\delta)t} - \frac{c\lambda}{\mu}(1-d)e^{-(\lambda+\mu+\beta+\delta)t} \\ +\frac{Cd}{\delta\mu}\begin{cases}(\lambda+\mu)(\beta+\delta)e^{-(\lambda+\beta)t} - (\lambda+\mu)\beta e^{-(\lambda+\beta+\delta)t} \\ -\lambda(\beta+\delta)e^{-(\lambda+\mu+\beta)t} + \lambda\beta e^{-(\lambda+\mu+\beta+\delta)t}\end{cases} \end{cases} \qquad \text{......(8)}$$

MTTF of the system is given by

$$\text{MTTF} = \begin{cases} \dfrac{(1-c)(1-d)}{\lambda+\beta+\mu+\delta} + \dfrac{d}{\delta}\dfrac{(1-c)(\beta+\delta)}{\lambda+\beta+\mu} - \dfrac{d}{\delta}\dfrac{(1-c)\beta}{\lambda+\mu+\beta+\delta} \\ +\dfrac{c}{\mu}\dfrac{(\lambda+\mu)(1-d)}{\beta+\lambda+\delta} - \dfrac{c\lambda}{\mu}\dfrac{(1-d)}{\lambda+\mu+\beta+\delta} \\ +\dfrac{cd}{\delta\mu}\begin{Bmatrix} \dfrac{(\lambda+\mu)(\beta+\delta)}{\lambda+\beta} - \dfrac{(\lambda+\mu)\beta}{\lambda+\beta+\delta} \\ -\dfrac{\lambda(\beta+\delta)}{\lambda+\mu+\beta} + \dfrac{\lambda\beta}{\lambda+\mu+\beta+\delta} \end{Bmatrix} \end{cases} \quad \text{........(9)}$$

The

Hazard rate of the system is given by,

$$h(t) = \frac{f_X(t) f_Z(t)}{R_S(t)} \quad (\because \text{X and Z are independent random variables})$$

$$= \frac{\left[(1-c)(\lambda+\mu)e^{-(\lambda+\mu)t} + \dfrac{c\lambda(\lambda+\mu)}{\mu}\left[e^{-\lambda t} - e^{-(\lambda+\mu)t}\right]\right]\left[(1-d)(\beta+\delta)e^{-(\beta+\delta)t} + \dfrac{d\beta(\beta+\delta)}{\delta}\left[e^{-\beta t} - e^{-(\beta+\delta)t}\right]\right]}{\begin{Bmatrix} (1-c)(1-d)e^{-(\lambda+\beta+\mu+\delta)t} + \dfrac{d}{\delta}(1-c)(\beta+\delta)e^{-(\lambda+\beta+\mu)t} - \dfrac{d}{\delta}(1-c)\beta e^{-(\lambda+\mu+\beta+\delta)t} \\ +(1-d)\dfrac{c}{\mu}(\lambda+\mu)e^{-(\beta+\lambda+\delta)t} - \dfrac{c\lambda}{\mu}(1-d)e^{-(\lambda+\mu+\beta+\delta)t} \\ +\dfrac{cd}{\delta\mu}\begin{Bmatrix} (\lambda+\mu)(\beta+\delta)e^{-(\lambda+\beta)t} - (\lambda+\mu)\beta e^{-(\lambda+\beta+\delta)t} \\ -\lambda(\beta+\delta)e^{-(\lambda+\mu+\beta)t} + \lambda\beta e^{-(\lambda+\mu+\beta+\delta)t} \end{Bmatrix} \end{Bmatrix}} \quad \text{......(10)}$$

For **c**=0.6, **μ**=0.005, **λ**=0.008, **d**=0.8, **β**=0.009, **δ**=0.006**,** we tabulate the results for various values of t in months, the graphical representations are given below. From (9) **MTTF** of the system is 157.199 = 13 Years

The Reliability R(t) and Hazard rate H(t) are tabulated below in different values of t

t (time in months)	R(t)	H(t)
2	0.299	0.0000810
3	0.302	0.0000807
4	0.315	0.0000786
5	0.321	0.0000779
6	0.329	0.0000771
7	0.335	0.0000763
8	0.341	0.0000756
9	0.349	0.0000746
10	0.353	0.0000743

The Graphs of R(t) and H(t) are given below

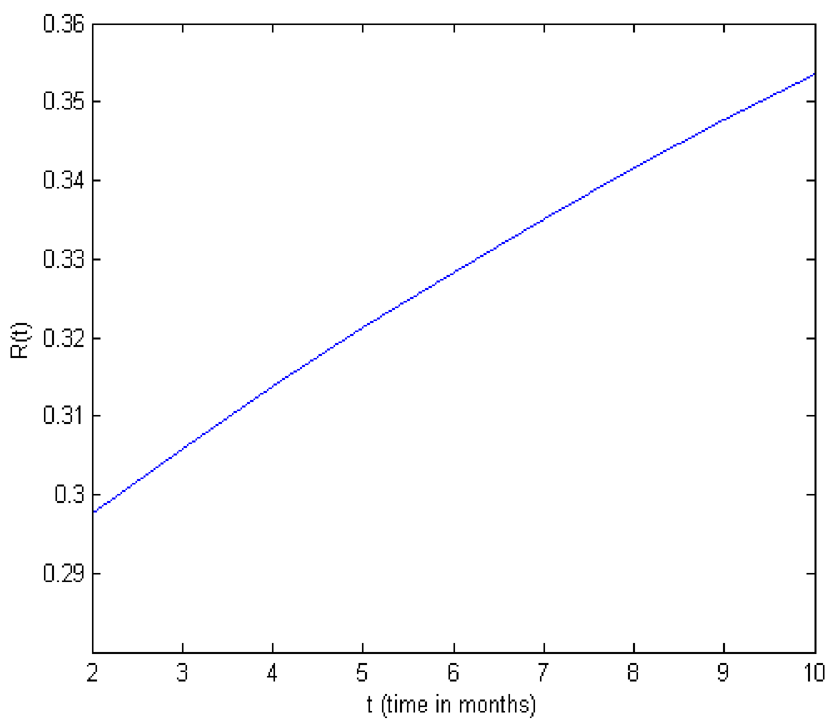


Figure 1

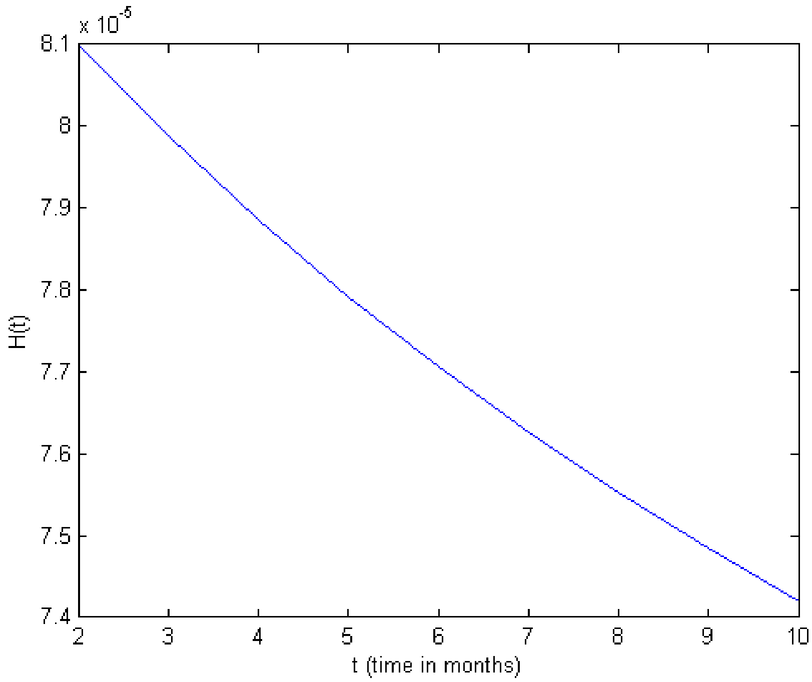


Figure 2

4. CONCLUSION

In this paper, the System Reliability is analyzed using imperfect fault coverage method. We have considered a Series system and calculated its Reliability, MTTF, and Hazard rates and have been dealt with numerical examples. This method is applicable to all the Systems involving active and spare units with imperfect fault coverage.

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