A Study on Fuzzy Queuiing Model Using Dsw Algorithm with Pentagonal Fuzzy number

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Abstract : In this paper we study fuzzy single server queuing model in pentagon fuzzy numbers using α-cut method.The arrival rate and service rate are fuzzy natures and also analyzed the performance measures of this model in pentagon fuzzy numbers. Numerical example shows the efficiency of the algorithm.

IndexTerms - Membership function,Pentagon fuzzy number, α-cuts, standard interval analysis,DSW algorithm.

1. INTRODUCTION

Queueing models have great extent applications in service organizations as well as manufacturing firms. Various customers getservice by different types of serve's according to specific queue discipline within the context of queuing theory, the inter arrival time and service times are required to follow certain distributions. Fuzzy Logic was initiated in 1965 by Zadeh[4]. Fuzzy queuing model was first introduced by R.J.Lie and E.S.Lee[5] in 1989, further developed this model by many authors namely J.J.Buckely[1] in 1990, R.S.Negi and E.S.Lee[6] in 1992, S.P.Chen in 2005 and R. Srinivasan[7]in 2014.

Also fuzzy queuing models have been described by such researchers like Chen[2,3] analyzed bulk arrival queuing model with fuzzy parameters and varying batch sizes using Zadeh's extension principle and recently he developed (FM/FM/1): (α/FCFS) and (FM/FM(k)/1): (α/FCFS) queueing models. Here the parameters, fuzzy arrival rate and fuzzy service rate are best described by linguistic terms very high, high, low, very low and moderate.

2. FUZZY SET THEORY:

2.1 Fuzzy Set: Let X be a classical set of objects, called the universe, whose generic elements are denoted by x .Membership in a classical subject A of X is often denoted as characteristic function μ_A: X→[0,1] such that $\mu_A(x)=\begin{cases} 1, & if\ x \in A \\ 0, & if\ x \notin A \end{cases}$ [0,1] is called a valuation set. If the valuation set is allowed to be the real interval [0,1] .A is called a fuzzy set.

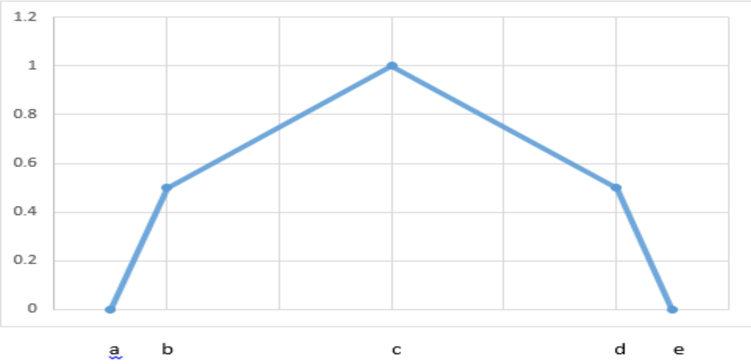
2.2Membership Function of Fuzzy Set: In fuzzy sets , each elements is mapped to [0,1] by membership function μ_A:X→[0,1] where [0,1] means real numbers between 0 and 1(including 0 and 1).

2.3 α- cut: An α cut of a fuzzy set $\tilde{A}$ is a crisp set A_α that contains all the elements of the universal set X that have a membership grade in A greater than or equal to the specified value of α. Thus

$A_\alpha=\{x \in X: \mu_A(x) \geq \alpha$,$0\leq \alpha\leq 1$

2.4 Pentagon fuzzy number with membership function: A fuzzy number $\tilde{A}$=(a,b,c,d,e) where a ≥ b ≥ c ≥ d ≥ e is said to be a pentagon fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & for\ x < a \\ \frac{x-a}{b-a}, & for\ a \leq x \leq b \\ \frac{x-b}{c-b}, & for\ b \leq x \leq c \\ 1, & for\ x = c \\ \frac{d-x}{d-c}, & for\ c \leq x \leq d \\ \frac{e-x}{e-d}, & for\ d \leq x \leq e \\ 0, & if\ x > e \end{cases}$$



3. FORMULATION OF THE PROBLEM: Let us consider a single server FM/FM/1 queueing system first come first served disicipline. The inter arrival time A and service time S are described by the following fuzzy sets.

A=$\{a, \mu_{\tilde{A}(a)} | a \in X\}$

S=$\{s, \mu_{\tilde{S}(s)} | s \in Y\}$

Here X is the set of the inter arrival time and Y is the set of the service time.

$\mu_{\tilde{A}}$(a)is membership function of the inter arrival time.

$\mu_{\tilde{S}(s)}$is membership function of the service time.

The α cuts of inter arrival time, service time are represented as

A(α) ={a∈X/,$\mu_{\tilde{A}(a)}$≥α}

S(α)={s∈Y/,$\mu_{\tilde{S}(s)}$≥α}

Using these α cuts we have to define the membership function P(A,S) as follows

$$\mu_{P(A,S)(x)}= \begin{cases} 0\ , & for\ x < a \\ \frac{x-a}{b-a}\ , & for\ a \le x \le b \\ \frac{x-b}{c-b}, & for\ b \le x \le c \\ 1, & for\ x = c \\ \frac{d-x}{d-c}\ , & for\ c \le x \le d \\ \frac{e-x}{e-d}, & for\ d \le x \le e \\ 0\ , & if\ x > e \end{cases}$$

4. THE (FM\FM\1) :(∞\FCFS) QUEUE MODEL: In this model we consider an infinite source population with first Come first served discipline where both the interarrival time$\tilde{\lambda}$ and the service time $\tilde{\mu}$ follow an exponential Distribution.

The performance measures of the single server queueing system are

The expected number of customer in the system ,$L_S=\frac{x}{y-x}$

The expected number of customers in the queue ,Lq =$\frac{x.x}{y(y-x)}$

The average waiting time of a customer in the system ,$W_S=\frac{1}{y-x}$

The average waiting time of a customer in the queue ,$W_q=\frac{x}{y(y-x)}$

5. STANDARD INTERVAL ANALYSIS FOR ARITHMETIC:

Let I_1 and I_2 be two interval numbers defined by ordered pairs of real numbers with lower and upper bounds.

I_1=[a,b],a≤b;I_2=[c,d],c≤d

Definea general arithmetic property with the symbol * ,where*=[+,-,×,÷] symbolically the operation

I_1*I_2= [a,b]*[c,d]

Represents another interval. The interval calculation depends on the magnitudes and signs of the element a,b,c,d.

[a,b]+[c,d]=[a+c,b+d]

[a,b]-[c,d]=[a-d,b-c]

[a,b].[c,d]=[min(ac,ad,bc,bd),max(ac,ad,bc,bd)]

[a,b]÷ $[c,d]$=[a,b].[$\frac{1}{d}$ ‘$\frac{1}{c}$] provided that 0∉[c,d]

$\propto[a,b]=\begin{cases}[\propto a, \propto b]\ for\ \propto > 0\\ [\propto b, \propto a]\ for \propto < 0\end{cases}$

Where ac,ad,bc,bd are arithmetic products and $\frac{1}{d},\frac{1}{c}$ are quotients.

6. DSW ALGORITHM: DSW (Dong,Shah,Wong) is one of the approximate methods makes use of intervals at various ∝-cut levels in defining membership functions. It was the full ∝-cut intervals in a standard interval analysis.

The DSW Algorithm Consists of The Following Steps:

1.Selet a ∝-cut value where 0≤∝≤ 1.

2.Find the intervals in the input membership functions that correspond to this ∝.

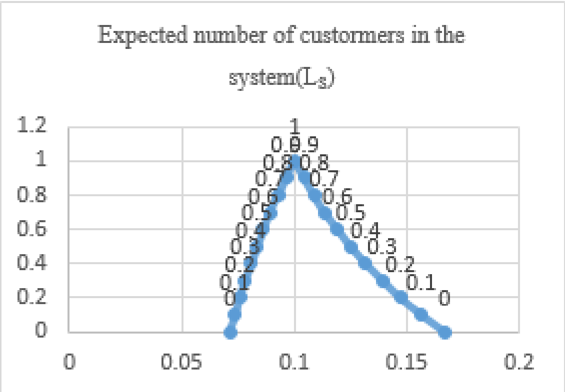
3.Using standard binary interval operations compute the interval for the output membership function for the selected ∝-cut level.

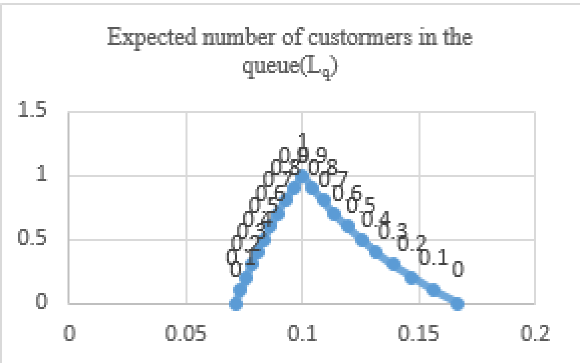
4.Repeat steps 1-3 for different values of ∝to complete ∝-cut representation of the soiution

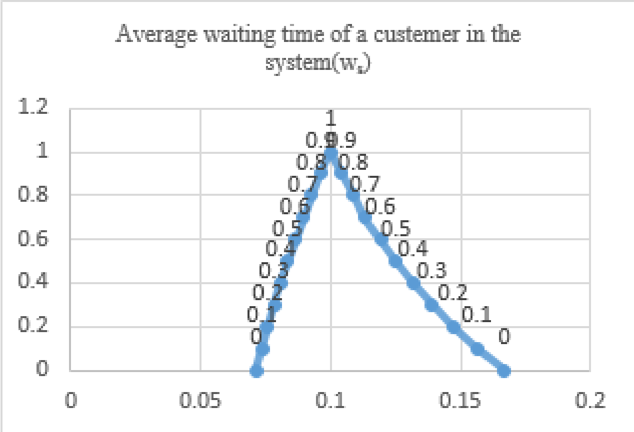
7. NUMERICAL EXAMPLE: Consider a FM/FM/1 queue where the both arrival rate and service rate are pentagonal fuzzy numbers represented by $\tilde{\lambda}$=[1,2,3,4,5] and $\tilde{\mu}$=[11,12,13,14,15]. The interval of confidence level∝ as [1+2∝ ,5-2∝] and [11+2∝ ,15-2∝] Where x=[1+2∝ ,5-2∝] and y=[11+2∝ ,15-2∝] By taking ∝ values from 0,0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9,1 the following results are obtaind as shown in Table.

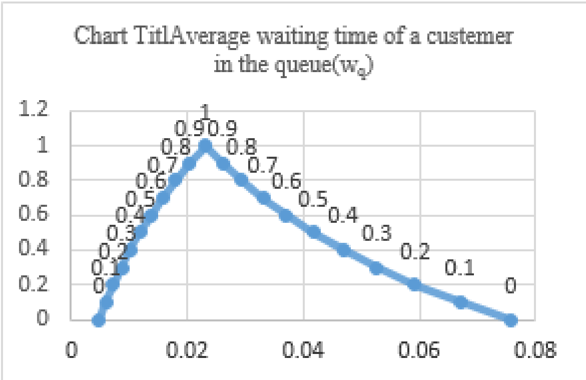
Table:1 The ∝ cuts of Ls,Lq,Ws,Wq at ∝ *valus*

∝	Ls	Lq	Ws	Wq
0	[0.07143,0.83335]	[0.00476,0.37875]	[0.07143,0.16667]	[0.00476,0.07575]
0.1	[0.08824,0.75020]	[0.00716,0.32141]	[0.07353,0.15625]	[0.00596,0.06696]
0.2	[0.10605,0.67648]	[0.01016,0.27275]	[0.07575,0.14706]	[0.00726,0.05929]
0.3	[0.12501,0.61111]	[0.01388,0.23174]	[0.07813,0.13889]	[0.00868,0.05267]
0.4	[0.14517,0.55264]	[0.01840,0.19668]	[0.08065,0.13158]	[0.01022,0.04683]
0.5	[0.16667,0.5]	[0.02380,0.16672]	[0.08333,0.125]	[0.01190,0.04168]
0.6	[0.18966,0.45239]	[0.03025,0.14093]	[0.08621,0.11905]	[0.01375,0.03708]
0.7	[0.21429,0.40910]	[0.03778,0.11871]	[0.08929,0.11364]	[0.01574,0.03298]
0.8	[0.24073,0.36955]	[0.04671,0.09976]	[0.09259,0.10869]	[0.01797,0.02934]
0.9	[0.26922,0.33331]	[0.05708,0.08335]	[0.09615,0.10416]	[0.02038,0.02605]
1	[0.3,0.3]	[0.06921,0.06921]	[0.1,0.1]	[0.02307,0.02307]









8. RESULT AND DISCUSSION: Using Microsoft Excel we perform α cuts of arrival rate and service rate and fuzzy expected number of jobs in queue at eleven distinct α levels:0,0.1,0.2,0.3,0.40.5,0.6,0.7,0.8,0.9,1.Crisp intervals for fuzzy expected number of jobs in queue at different possibility α levels are presented in table The performance measures such as expected number of customers in the system(Ls),expected length of queue number of customers in the queue(Lq), the average waiting time of a customers in the system(Ws) and the average waiting time of a customers in the queue(Wq) also derived in table.The α cut represent the possibility that these four performance measure will lie in the associate range. Specially α =0 the range,the performance measures could appear and for α =1 the range.

1. The Expected number of customers in the system Ls is 0.3 and impossible falls outside [0.07143,0.83335].
2. The Expected number of customers in the queue Lq is 0.06921 and impossible falls outside [0.00476,0.37875].
3. The Average waiting time of a customers in the system Ws is 0.1 and impossible falls outside [0.07143,0.16667].
4. The Average waiting time of a customers in the queue Wq is 0.02307 and impossible falls outside [0.00476,0.07575].

The above results will be very useful for designing a queueing system.

9.CONCLUSION: In this paper the performance of FM/FM/1 pentagon fuzzy numbers is studied. We infer that fuzzy set theory has been applied to queueing theory also. The inter arrival time and fuzzy nature. The performance measure such as system length,queuelength,systemtime,queue time are also in fuzzy nature. Numerical example shows that the efficiency of DSW algorithm.

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