

# INTERDISCIPLINARY APPROACH OF TOPOLOGY IN APPLIED SCIENCES

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**Abstract:** Imagination of an idea in its abstractness has the potential to emerge in order to spread its seeds in reality. Topology is an investigation of invariant properties of space in presence of a continuous deformation. With the passage of time, this branch has shown its interdisciplinary nature in disciplines of applied sciences leading to the new advancement in science of materials. Here, aim is to present analysis on recent trends arising out of the art of topology. This includes topological materials [1] and their existing properties founded in the concept of spin-orbit coupling. Along with this, comments on topological transition in materials, electrical properties and topological insulators [2] with examples have been made. In addition, this in-depth review is having details of related upcoming technical applications [3] in computing at quantum levels, energy storage, catalysis and low energy electronic machinery along with their concerned challenges.

**IndexTerms– Topology, topological insulators, spin-orbit coupling.**

## I. INTRODUCTION

Physical models of evolutionary processes are typically framed in terms of parameter, spaces and time. Understanding the existence, structure and bifurcation of invariant properties often forms the focal point for the qualitative study of these systems. Modern data sciences uses topological methods to find the structural features of data sets before further supervised or unsupervised analysis. Geometry and topology are very natural tools for analyzing massive amounts of data since geometry can be regarded as the study of distance functions. The explosive growth in web-based storage, management, processing and accessibility of social, medical, scientific and engineering data has been driven by our need of fundamental understanding of the processes which produce the data. The challenges of managing massive amounts of data include extracting, analyzing, visualizing, sharing, storing, transferring and searching such data. Modern data sciences uses so called topological methods to find the structural features of data sets before the supervised or unsupervised analysis.

## II. TOPOLOGICAL MATERIALS

The idea of topological materials grew out of work in early 1970s, when Physicists J. Michael and David J. Thouless used the concept of topology to explain why superconductivity happens in certain materials at extremely low temperatures but disappears at higher ones. In the 1980s, F. Duncan used topology to explain some properties of magnets. Topological materials have been working their way from theoretical physics into the world of experimental chemistry over the past decade, and the pace is quickening. Topological materials promise useful applications, such as more energy- efficient microelectronic components, better catalysts, improved thermoelectric convertors, or new magnetic storage media. The field of research may enable the design of quantum computers more powerful than any classical computer ever could be or the realization of higher-temperature superconductors.

Topological materials boast certain electronic states that persist in the face of disruptions of their physical structures. In fact, when applying the idea of topological states to materials, what's important is not the shape of the material itself but the structure of its electronic bands. The electronic band structure is the digital print of a solid or crystalline structure. The energy region at which electrons can flow without resistance is called conduction band-typically within metallic materials, whereas if the material was in absolute zero, the electrons would be confined in the lower energetic valence band, where the energy needs to be injected in order for electrons to jump into conduction band and flow. So, the conduction band if the lowest closest range of energies to the valence bands, more difficult it is to jump the gap, and materials in the situation are called insulators. In metals these energy ranges overlap, so electrons move easily into the conduction band, allowing current to flow. Insulators have a wide band gap, so electrons cannot jump from the valence band to conduction band. Semiconductors have a small band gap, so current can flow if the electrons absorb the right amount of energy. In topological materials, there can be the case where the bulk of the material is insulator, whereas the surface of the material is conductive. Although a lot of topological materials are heavy-metal compounds, some simpler, more familiar materials also have topological properties, Gold is a topological metal, the Bell labs physicist who won a Nobel prize for developing transistors, recognized that gold has a special surface state- it is what gives the metal its famous luster- but at the time there was no theory of topological matter to explain it. People have long time known that the conduction band in gold crosses the valence band in such a way that it absorbs more blue light and gives off its yellow color. That crossing is a topological effect. The surface states of topological metals are not as robust as they are in insulators, but they also exist in materials such as platinum and tin. Some transition metals compounds- including niobium phosphide, tantalum phosphate, niobium arsenate and tantalum arsenide have been identified as topological materials and they are excellent catalysts also.

## III. TOPOLOGY AND TOPOLOGICAL INSULATORS

Topological insulators are new quantum states theoretically proposed in [1, 2, 3, 4 ] and they have been experimentally observed by various ways. They are nonmagnetic insulators in bulk, but have gapless edge/surface states. They can be realized in two dimensions(2D)(Fig. 1(a)) and in three dimensions(3D)(Fig. 1(b)). In the topological insulators, the time-reversal symmetry is assumed, i.e. nonmagnetic. The edge/surface states consist of pairs of states which have opposite spins and propagate in the opposite directions. Any nonmagnetic insulators are classified into either topological insulators or ordinary insulators by using  $Z_2$  topological numbers. In the 2D topological insulators, the  $Z_2$  topological number  $\nu$  can take two values:  $\nu = 1$  for topological

insulator, and  $v = 0$  for the ordinary insulator. Nonmagnetic insulators without the spin-orbit coupling are ordinary insulators, and when the spin-orbit coupling becomes stronger, insulators may become topological insulator. Hence, the edge/surface states of topological insulators arise from the spin-orbit coupling. The spin-orbit coupling act as “spin-dependent magnetic field”, and it gives rise to spin-dependent quantum hall effect [ 3, 5 ]. The edge states from this spin dependent quantum hall effect consist of counterpropagating states with opposite spins. These edge/surface states are called “helical”. The important point for these helical edge/surface states is that any time-reversal-symmetric perturbation, such as nonmagnetic impurities or electron-electron interaction, cannot open a gap[ 6,7 ]. This can be understood in the following way: the  $Z_2$  topological number cannot change continuously when nonmagnetic perturbation is added, and the system remains the topological insulator, even in the presence of perturbation. This robustness of topological insulator against perturbations is called the topological protection.

Not all materials with strong spin-orbit coupling are topological insulators; in order to distinguish between topological and ordinary insulators, one has to calculate the  $Z_2$  topological numbers. The topological invariants can be determined from the parity of occupied Bloch function at the time-reversal Brillouin zone. If number of states crossing Fermi level is odd then material is topological insulator, otherwise it is ordinary insulator.

Schematics for the edge states of the ordinary insulators and topological insulators are shown in the fig. 2(a) and 2(b), respectively. We can see the difference how the edge bands are connected to bulk valence or conduction bands. In ordinary insulators the both sides of the edge-state dispersion are connected to the same bulk bands, whereas in topological insulators the edge-state dispersion connects between the bulk conduction and bulk valence bands. We note that because of the spin-orbit coupling, the edge states are spin-split in fig. 2(a)(b), which is called the Rashba splitting. From the time-reversal symmetry, the edge states which are symmetric with respect to  $k = 0$  are Kramers degenerate, and having opposite spins. It is noted that figs. 2(a) and (b) cannot be deformed into each other continuously, without closing the bulk gap. This is guaranteed when the time-reversal symmetry is preserved, and it is a manifestation that these two insulators belong to different topological phases.

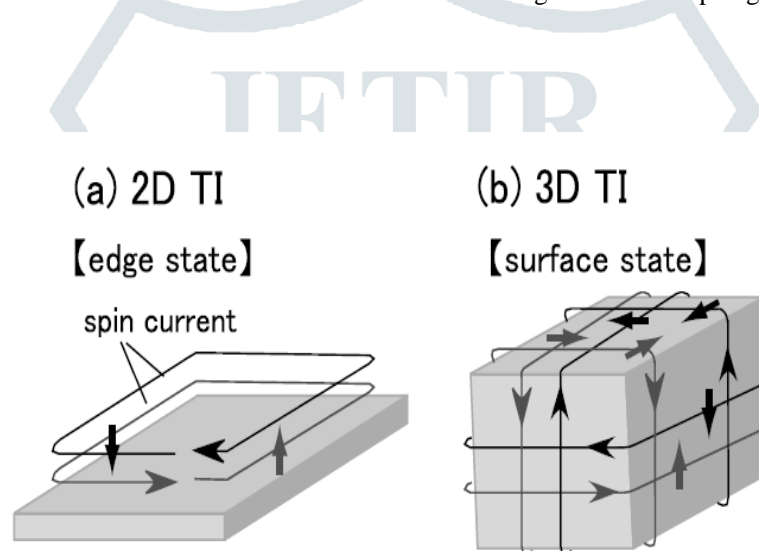


Fig. 1: Schematic figures for (a) a 2D topological insulator and (b) a 3D topological insulator.

Figures 2(a) and (b) are the simplest cases, and we can consider other types of edge states. Any cases of edge states in nonmagnetic insulators can be classified either into ordinary or topological insulators, by counting the number of Kramers pairs of edge states on the Fermi energy. Even and odd numbers of kramers pairs correspond then to the ordinary insulators ( $v = 0$ ) and topological insulators ( $v = 1$ ), respectively. In fig. 2(a), the number of pairs is either two or zero, depending on the value of the Fermi energy within the gap, and in (b) it is one. Thus (a) belongs to the ordinary insulator and (b) does to the topological insulators, in accordance with the previous explanations. This topological number  $v$  will not change under continuous change of parameters, unless the bulk gap closes. When the bulk gap closes at some point, the topological number may change. One can classify how the bulk gap closes by changing parameters, and can associate this classification with the change of the topological number [ 8 ]. As a result, universal phase diagrams between the topological and ordinary insulators have been found [ 9 ].

Here we briefly mention the 3D topological insulators. One of the typical form of surface states of the 3D topological insulators is the single Dirac cone. On the (111) surface of  $\text{Bi}_2\text{Se}_3$  or  $\text{Bi}_2\text{Te}_3$ , the surface Fermi surface is a single Fermi surface encircling the gamma point. The surface state is linear in the wavenumber, schematically shown in fig. 2(d). This is called a Dirac cone. In  $\text{Bi}_2\text{Se}_3$  and  $\text{Bi}_2\text{Te}_3$  [ 10,11 ], the surface states observed in experiments form a single Fermi surface around the gamma point, and these surface states form a single Dirac cone (fig. 2(d)).

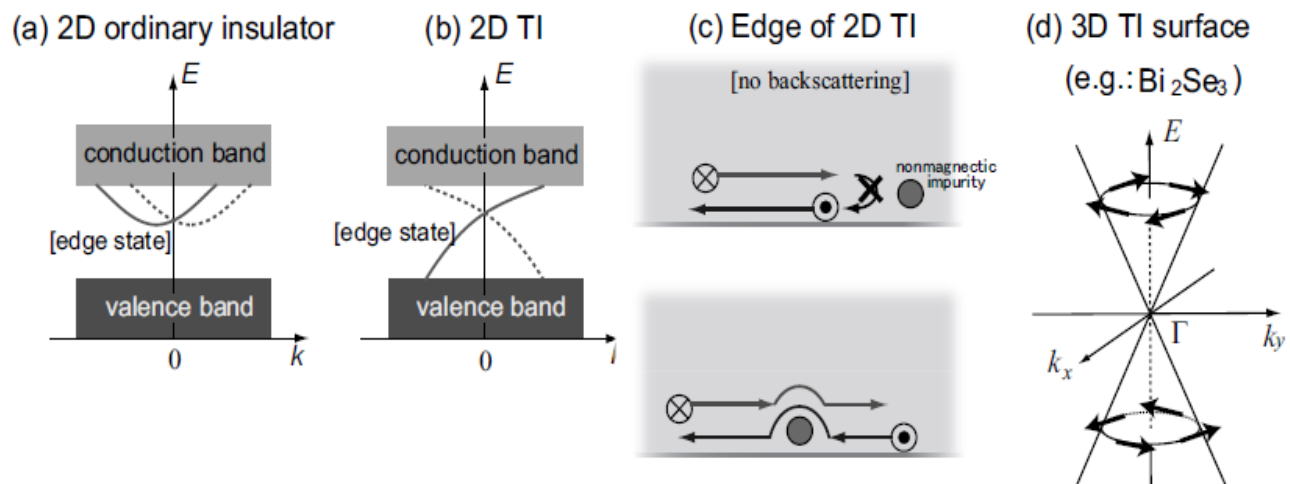


Fig. 2: Schematic figure of edge states for (a) 2D ordinary insulator and (b) topological insulator. (c) Backscattering is prohibited for edge states of 2D topological insulators. (D) Dirac cone of the surface states of 3D topological insulators.

#### IV. APPLICATIONS OF TOPOLOGICAL INSULATORS

Topological insulators are a hot topic in condensed-matter physics on account of their special properties. Electrical insulator in the bulk, these materials have surface states that conducts electrons as well. These materials don't open a gap for time-reversal-perturbation, such as non-magnetic perturbation. In these materials current is not disrupted by small defects in crystal, the current simply flows around it. So these materials have the potential to form high temperature superconductors, low energy electronic machinery. As these states are topologically protected, researchers are looking for their applications in computing at quantum levels.

#### V. SUMMARY

Topological materials have been working their way from theoretical physics into the world of experimental chemistry over a decade. It is surprising that such topological phases can be realized into zero magnetic field, by the spin-orbit coupling only. In the coming years, we can expect more discoveries and surprises in the field of topological insulators.

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