Oscillatory convection in a rotating fluid layer under gravity modulation

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Abstract: A study of thermal instability driven by buoyancy force is carried out in an initially quiescent infinitely extended horizontal rotating fluid layer. The time periodic gravity field is considered and its effect on the system has been investigated. A weakly nonlinear stability analysis is performed for the oscillatory mode of convection, and heat transport in terms of the Nusselt number, which is governed by the complex non-autonomous Ginzburg-Landau equation, is calculated. The influence of external controlling parameters like amplitude and frequency of modulation on heat transfer has been investigated. For lower values of Pr < 1 oscillatory mode of instabilities are possible. The duel effect of rotation on the system for oscillatory mode of convection either is to stabilize or destabilize the system has been found. Further, the study establishes that the heat transport can be controlled effectively by a mechanism that is external to the system. Finally oscillatory mode of convection strengthens the heat transfer rather than stationary mode.

***Index Terms* - Oscillatory convection; Gravity modulation; Weak nonlinear stability; Rotating fluid layer.**

Nomenclature

Latin Symbols

$\mathbb{A}$	Amplitude of convection
δ	Amplitude of thermal modulation
$\vec{g}$	Acceleration due to gravity
R_{aT}	Thermal Rayleigh number, $\mathrm{Ra} = \dfrac{\beta_T g \Delta T d K}{\nu k_T}$
R_0	Critical Rayleigh number
a	Critical wave number
d	Depth of the fluid layer
(x, z)	Horizontal and vertical co-ordinates
Nu	Nusselt number
p	Reduced pressure
λ	Stress relaxation time
$\bar{\varepsilon}$	Strain retardation time
T	Temperature
ΔT	Temperature difference across the porous layer
Ta	Taylor number, $\mathrm{Ta} = \left(\dfrac{2\Omega d^2}{\nu}\right)^2$
t	Fast Time scale

Greek Symbols

α_T	Coefficient of thermal expansion
ω	Dimensionless oscillatory frequency
μ	Dynamic viscosity of the fluid
κ_T	Effective thermal diffusivity
ρ	Fluid density
Ω	Frequency of modulation
υ	Kinematic viscosity, $\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}\left(\dfrac{\mu}{\rho_0}\right)$
χ	Perturbation parameter
θ	Phase angle
γ	Heat capacity ratio

s	Slow time scale, s=χ^2t
ψ	Stream function
$\hat{k}$	Vertical unit vector
Other symbols	
∇_1^2	$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$
∇^2	$\nabla_1^2+\frac{\partial^2}{\partial z^2}$
Subscripts	
b	Basic state
c	Critical
0	Reference value
Superscripts	
*	Dimensionless quantity
'	Perturbed quantity

1. INTRODUCTION

Gravity modulation, which can be realized by vertically oscillating a fluid layer, is a mecha-nism which can be used to alter the convective flow. The problem of convection in a fluid layer in the presence of complex body forces has gained considerable attention in recent decades due to its promising applications in engineering and technology. The time dependent gravity field one of the complex forces, is of interest in space laboratory experiments, in areas of crystal growth etc. It is also of importance in the large scale convection in atmospheres, oceans, planetary mantles, and it provides the mechanism of heat transfer for a large fraction of the outermost interiors of our sun and all stars. The random fluctuations of gravity field both in magnitude and direction can be experienced in space laboratories significantly influence natural convection. Gresho and Sani [1] have studied the linear stability of a fluid layer with rigid boundaries and found the in uence of the gravity modulation on the convection threshold of the system. Wadih and Roux [2] have studied the case of a fluid occupying a cylindrical cavity of infinite length and submitted to a negative temperature gradient maintained in the upward direction. Clever et al. [3] considered the problem of two dimensional oscillatory convection in a gravitationally modulated fluid layer. They have also analyzed three dimensional oscillatory convection under gravity modulation in [4]. Gravity modulation in a fluid layer has been studied Bhadauria et al. [5]. Malashetty and Swamy [6] reported the effect of gravity modulation on the onset of thermal convection in a fluid and porous layer using linear stability analysis. One can readily analyze from the results that the effect of gravity modulation on RayleighBenard convection is similar to that of asymmetric thermal modulation. Siddheshwar et al. [7] performed a local nonlinear stability analysis of Rayleigh Benard magneto-convection using GinzburgLandau equation. They showed that modulation can be used to enhance/diminish the heat transport in a stationary magnetoconvection. Bhadauria et al. [8-19] studied the effect of internal heating on Rayleigh-Benard convection under gravity modulation, they found that internal heat generation, amplitude of modulation is to enhance the heat transfer and frequency of modulation is to reduce the heat transfer.

Rotating the fluid layer around an axis perpendicular to the layer, the classic pattern of convective flow above a critical value of the rotation speed can be unstable even the smallest temperature differences the patterns change with time. These changes do not happen locally constant transition from one pattern to another. Rather, the change is done globally and suddenly. This phenomenon in the literature is known as Kuppers-Lortz instability. This effect plays a role in brain research in the explanation of spontaneous perceptual change in optical illusions, for example the Necker cube. Buoyancy driven convection in a horizontal rotating fluid layer rotating about the vertical provides one of the classical examples of hydrodynamic instability (Veronis [9], Chandrasekhar [10]). The system is of interest because convection can set in either stationary mode or via oscillatory mode depending on the values of Prandtl and Taylor numbers. In the weakly nonlinear regime classical perturbation theory has revealed branches of steady convection near the steady state bifurcation and branches of different types of standing and travelling waves near the Hopf bifurcation. When these bifurcations are near one another one can study the interaction between steady and oscillatory convection at small amplitude. This study has revealed different ways whereby oscillations, be they in the form of standing or travelling waves; give way to steady convection as the Rayleigh number increases. Kuppers-Lortz [11] the stability of convective flow is investigated in a rotating fluid layer for rigid boundary conditions. The critical Taylor number above which the only stable two dimensional flows become unstable is calculated as a function of the Prandtl number.

Donnelly [12] he found that sinusoidally modulated rotation of the inner cylinder can be inhabited instability of Couette flow. The origin of the inhibition has been shown experimentally to be due to the viscous wave propagated across the annulus by the modulated cylinder. Rauscher and Kelly [13] analyzed the combined effect of thermal modulation and rotation on the onset of convection in a rotating fluid layer for the case of the lower wall temperature modulation. They reported that high Taylor number has destabilizing effects over a range of frequencies, contradictory to the classical stabilizing effect of Taylor number at small values. For small Prandtl numbers, convection in a rotating fluid layer can begin in an oscillatory manner and the modulation might be expected to have more of a resonant effect. Knobloch [14] analyzed Kuppers-Lortz instability. He found that in this instability a pattern of parallel rolls becomes unstable to another set of rolls oriented at an angle with respect to the first, once the rotation rate exceeds a critical value. This new set is itself unstable in the same fashion etc., resulting in complex dynamics right at onset. He also found instabilities (standing or traveling rolls) are triggered by the formation of heteroclinic orbits connecting two sets of standing or traveling rolls with different orientation.

Kloosterziel and Carnevale [15] investigated onset of convection in a closed form of Rayleigh-Benard convection considering rigid stress free upper and lower boundaries. They determined analytically critical points on the marginal stability boundary above which an increase of either viscosity or diffusivity is destabilizing. Finally, they show that if the fluid has zero viscosity the system is always unstable, in contradiction to [10] conclusion. Malashetty and Swamy [16] investigated temperature modulation in a rotating fluid layer using linear theory. It is observed that the instability can be enhanced by the rotation at low frequency symmetric modulation and with moderate to high frequency lower wall modulation, whereas the stability can be enhanced by the rotation in case of asymmetric modulation. They also found that by proper tuning of modulation frequency, Taylor number and Prandtl number it is possible to advance or delay the onset of convection. Bhadauria et al.[17] considering stationary mode of convection in a rotating fluid layer they have investigated effects of temperature and gravity modulation while assuming stress-free isothermal boundaries. They found that rotation reduces heat transfer for both modulations while both modulations can be used to regulate heat transfer by suitably tanning amplitude and frequency of modulations. Bhadauria et al. [18] investigated rotation speed modulation of Rayleigh Benard convection while accounting the effects of internal heating and temperature dependant viscosity. They found that, rotation and frequency of modulation reduces heat transfer while internal Rayleigh number enhances heat transfer in the system.

Thus, it is clear that, although a few studies are available in the literature on weakly nonlinear stationary convection under modulation, not even a single study is reported on weak nonlinear oscillatory mode of convection under modulation. Recently Bhadauria and Kiran [19, 20] investigated weak nonlinear analysis for supercritical flow in a viscoelastic fluid and porous layer under gravity modulation. Where they derived an amplitude of convection using non autonomous complex Ginzburg-Landau equation ([21]- [23], [35]-[40]). They revealed that the heat transfers more in oscillatory mode rather than in stationary mode. Further, it is also said the modulation can be used effectively to regulate the system. This motivated us to study a weak nonlinear convection problem in a rotating fluid layer under gravity modulation for oscillatory mode, using a complex Ginzburg-Landau equation, and in the process quantify the heat transfer across the fluid layer in terms of the Nusselt number.

2. GOVERNING EQUATIONS

We consider a layer of an incompressible viscous fluid, confined between two infinitely extended parallel horizontal planes, z = 0 and z = d, at a distance 'd' apart. The fluid layer is heated from below and cooled from above.

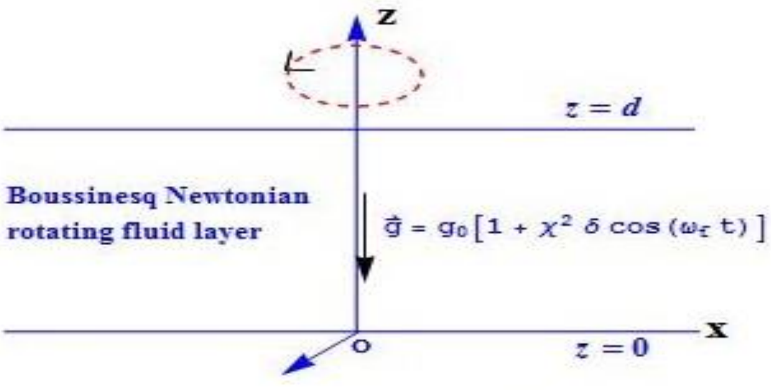

Fig.1:Physical representation of the problem

A Cartesian frame of reference is chosen in such a way that the origin lies on the lower plane and the z-axis is vertical upward. The layer is rotating about the z-axis with a constant angular velocity (Fig. 1). The fluid layer is considered to be Boussinesq and under these conditions the governing equations for a rotating viscous fluid layer are given by ([10, 17]):

$$\Delta\,.\,\vec{q} = 0, \qquad (1)$$

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} + (2\Omega \times \vec{q}) = -\frac{1}{\rho_0}\nabla_p + \frac{\rho}{\rho_0}\vec{g} + \upsilon \nabla^2 \vec{q}, \qquad (2)$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\Delta)T=\mathrm{K}_T\Delta^2 T, \tag{3}$$

$$\rho=\rho_0\left[1-\alpha_T(T-T_0)\right], \tag{4}$$

where $\vec{q}=(u,v,w)$ is the fluid velocity, p is pressure, Ω is rotational velocity about z-axis, υ kinematic viscosity, ρ is the density of the fluid, ρ_0 reference viscosity, κ_T is thermal diffusivity of the fluid layer, $\vec{g}$ is gravity field, T is temperature, α_T thermal expansion coefficient. The externally imposed thermal boundary conditions and gravity field are given by

$$\begin{aligned} T&=T_0+\Delta T, && at \quad z=0 \\ T&=T_0, && at \quad z=d \end{aligned} \tag{5}$$

$$\vec{g}=g_0\left(1+\chi^2\delta\cos(\omega_f t)\right)\hat{k}, \tag{6}$$

Where , the temperature difference is across the fluid layer, are the reference values of temperature and density of the fluid, small amplitude of modulation, is modulation frequency. A hydrostatic solution exists for the governing equations with boundary conditions described above. The solution is given by $\vec{q}_b=0,\ p=p_b(z,t), T=T_b(z,t), \rho=\rho_b(z,t).$

$$\kappa_T\frac{\partial^2 T_b}{\partial z^2}=0, \tag{7}$$

The solution of Eq. (7) subject to the thermal boundary conditions Eq. (5), is given by:

$$T_b=T_0+\frac{\Delta T}{2}\left[1-\frac{2z}{d}\right]. \tag{8}$$

Now introducing perturbed quantities $\vec{q}=\vec{q}_b+\vec{q}',\ \vec{p}=\vec{p}_b+\vec{p}',\vec{T}=\vec{T}_b+\vec{T}',\vec{\rho}=\vec{\rho}_b+\vec{\rho}',$ for two-dimensional convection, using stream function ψ as $u'=\frac{\partial\psi}{\partial z}, w'=-\frac{\partial\psi}{\partial x}$, we non dimensionalize the above equations with the following scales; $(x,y,z)=d\left(x^*,y^*,z^*\right), t=\frac{d^2}{\kappa_T}t^*, \vec{q}=\frac{\kappa_T}{d}\vec{q}^*, \psi=\kappa_T\psi^*,\ \ T'=\Delta T\, T^*\ and\ \omega_f=\frac{d^2}{\kappa_T}\omega_f{}^*$. Finally by eliminating the pressure term and dropping asterisk, we obtain the non-dimensional governing system as

$$\left(\frac{1}{\Pr}\frac{\partial}{\partial t}\nabla^2-\nabla^4\right)\psi+Ra_T(1+\chi^2\delta\cos(\Omega t))\frac{\partial T}{\partial x}-\sqrt{Ta}\frac{\partial V}{\partial z}=\frac{1}{\Pr}\frac{\partial(\psi,\nabla^2\psi)}{\partial(x,z)}, \tag{9}$$

$$\frac{\partial\psi}{\partial x}+\left(\frac{\partial}{\partial t}-\nabla^2\right)T=\frac{\partial(\psi,T)}{\partial(x,z)}, \tag{10}$$

also from the momentum equation (9), we may write the following equation for υ

$$\sqrt{Ta}\frac{\partial\psi}{\partial z}+\left(\frac{1}{\Pr}\frac{\partial}{\partial t}-\nabla^2\right)V=\frac{1}{\Pr}\frac{\partial(\psi,V)}{\partial(x,z)}. \tag{11}$$

The above system of equations (9)-(11) will be solved by considering stress free and isothermal boundary conditions as given bellow

$$\psi=\frac{\partial^2\psi}{\partial z^2}=\frac{\partial V}{\partial z}=T=0 \qquad z=0, \quad z=1. \tag{12}$$

We now introduce a small perturbation parameter χ that shows deviation from the critical state of the onset of convection, then the variables for a weak nonlinear state may be expanded as power series of χ as:

$$
\begin{aligned}
Ra_T &= R_0 + \chi^2 R_2 + \chi^4 R_4 + \ldots\ldots., \\
\psi &= \chi\psi_1 + \chi^2\psi_2 + \chi^3\psi_3 + \ldots\ldots., \\
T &= \chi T_1 + \chi^2 T_2 + \chi^3 T_3 + \ldots\ldots., \\
V &= \chi V_1 + \chi^2 V_2 + \chi^3 V_3 + \ldots\ldots., \\
\delta &= \delta_0 + \chi\delta_1 + \chi^2\delta_2 + \chi^3\delta_3 + \ldots\ldots.,
\end{aligned} \tag{13}
$$

where R_0 is the critical value of the critical Rayleigh number at which the onset of convection takes place in the absence of gravity modulation. The expression δ of is consistent with the basic state solution provided that δ_0 vanishes at the lowest order. In addition, unless δ_1 vanishes, the equations obtained at order χ and χ^2 present a singularity in the solution. These observations indicate that the effects of gravity modulation should be introduced at $\delta = \chi^2\delta_2$ thereby enabling consistency. The reader may also note that, this type of expansion (13) was first used by Malkus & Veronis [30] (reader may also see Venezian [31]) in connection with convection problems to consider the effects of finite amplitude convection. It is required that expressions (13) satisfy the equations of motion (9-11) for all values of χ less than some maximum χ . The coefficients of each power of χ generated by substituting the Eq.(13) into the system of Eqs.(9-11) must vanish individually and the resulting series for each of the variables must converge if relations (13) are to represent a satisfactory solution to the problem.

3. BIFURCATION OF PERIODIC SOLUTION

In order to allow for anticipated frequency shift along the bifurcation solution, we introduce the fast time scale of time τ and the slow time scale of s. Therefore, the scaling of time variable is such that $\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau} + \chi^2 \frac{\partial}{\partial s}$ given in [35-40]. In the first order problem, the nonlinear term in energy equation will vanish. Therefore, the first order problem reduces to the linear stability problem for over stability.

3.1 AT THE LOWEST ORDER

At first order we consider the following system in which the nonlinearity skipping:

$$
\begin{pmatrix}
\left(\frac{1}{\Pr}\frac{\partial}{\partial r}\nabla^2 - \nabla^4\right) & R_0\frac{\partial}{\partial x} & -\sqrt{Ta}\frac{\partial}{\partial z} \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial r} - \nabla^2 & 0 \\
\sqrt{Ta}\frac{\partial}{\partial z} & 0 & \left(\frac{1}{\Pr}\frac{\partial}{\partial r} - \nabla^2\right)
\end{pmatrix}
\begin{bmatrix} \psi_3 \\ T_3 \\ V_3 \end{bmatrix}
=
\begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \end{bmatrix} \tag{14}
$$

The solution of the lowest order system subject to the boundary conditions, Eq. (12), is assumed to be of the form

$$\psi_1 = (\mathbb{A}_1(s)e^{i\omega T} + \mathbb{A}_1\overline{(s)}e^{-i\omega T})\sin ax \sin \pi z, \tag{15}$$

$$T_1 = (\mathbb{B}_1(s)e^{i\omega T} + \mathbb{B}_1\overline{(s)}e^{-i\omega T})\cos ax \sin \pi z, \tag{16}$$

$$V_1 = (\mathbb{C}_1(s)e^{i\omega T} + \overline{\mathbb{B}_1}(s)e^{-i\omega T})\sin ax \cos \pi z, \tag{17}$$

The above Eqs.(15-17) represents standing waves. Travelling waves are excluded, the reason is that, the symmetry around the rotation axis has to be imposed, the later requiring the stream function to be zero at x=0. The undetermined amplitudes are functions of slow time scale and are related by the following relation:

$$\mathbb{B}(s) = -\frac{a}{c + i\omega}\,\mathbb{A}(s), \tag{18}$$

$$\mathbb{C}(s) = -\frac{\pi\sqrt{Ta}}{\left(c + \frac{i\omega}{\Pr}\right)}\,\mathbb{A}(s), \tag{19}$$

where c = a2 + π2. The thermal Darcy-Rayleigh number for stationary mode of convection can be given as:

$$R_0 = \frac{c^3 - \frac{c\omega^2}{\Pr}}{a^2} + \frac{\pi^2 Ta}{a^2}\left(\frac{c^2 + \frac{\omega^2}{\Pr}}{c^2 + \frac{\omega^2}{\Pr^2}}\right), \tag{20}$$

$$\omega = \Pr\sqrt{\frac{\pi^2 Ta(1-\Pr)}{c(1+\Pr)} - c^2} \tag{21}$$

It is obvious from the above expression (Eq.21) that, becomes complex quantity when Pr exceeds one, since must be a positive real quantity (for the oscillatory convection to be possible), Pr values are considered less than one. Therefore, for pure fluids, over stability is possible only for values of Prandtl numbers less than 1 to obtain real frequencies. For stationary mode of convection the critical Rayleigh number we obtain as when (= 0),

$$R_0 = \frac{c^3 + \pi^2 Ta}{a^2} \tag{22}$$

which is the result obtained by Bhadauria et al. [17]. It can be observed that the mode of oscillatory convection exists when the Taylor number satisfies the following inequality:

$$Ta > \frac{(1+\Pr)}{\pi^2(1-\Pr)}c^3 \tag{23}$$

The wave number corresponding Rayleigh numbers can be obtained by minimizing Rayleigh

number with respect to critical wave number.

3.2 IN THE SCOND ORDER

In this state we consider the nonlinear terms in the equations:

$$\frac{\partial(\psi_1, T_1)}{\partial(x,z)} = \frac{\pi a}{2}, \quad \{\mathbb{A}1(s)\ \mathbb{B}1(s) + \mathbb{A}1\ (s)\ \mathbb{B}1(s) + \mathbb{A}1(s)\ \mathbb{B}1(s) + \mathbb{A}1(s)\ \mathbb{B}1(s)\}\sin 2\pi z. \tag{24}$$

From the above relation, we can deduce that the velocity and temperature fields have the terms having frequency 2ω and independent of past time scale. Thus, we write the second order temperature term as follows:

T2 = {T20 + T22e2iωτ + T 22e−2iωτ} sin 2πz (25)

where T_{22} and T_{20} are temperature fields having the terms having the frequency 2ω and independent of fast time scale, respectively. The solutions of the second order problem are:

$$\psi_0 = 0, \tag{26}$$

$$T_{20} = \frac{a}{8\pi}, \quad \{\mathbb{A}1(s)\ \mathbb{B}1(s) + \mathbb{A}1(s)\ \mathbb{B}1(s)\}, \tag{27}$$

$$T_{22} = \frac{\pi a}{8\pi^2 + 4i\omega}, \quad \mathbb{A}1(s)\ \mathbb{B}1(s), \tag{28}$$

Similarly we consider (for mathematical traceability)

$$V_{20} = \frac{a}{8\pi}, \quad V_2 = \{V_{20} + V_{22}e^{2i\omega\tau} + \bar{V}_{22}e^{-2i\omega\tau}\}\sin ax \tag{29}$$

For solving the system. The solutions are:

$$V_{20} = \frac{a}{8\pi}\ \{\mathbb{A}(s)\ \mathbb{C}(s) + \mathbb{A}(s)\ \mathbb{C}\ (s)\}, \tag{30}$$

$$V_{22} = \frac{\pi a}{8a^2 + \frac{2i\omega}{\Pr}}\ \mathbb{A}(s)\ \mathbb{C}(s) \tag{31}$$

The horizontally averaged Nusselt number Nu(s) for the oscillatory mode of convection is given

by:

$$Nu(s) = 1 - \chi^2 \left(\frac{\partial T_2}{\partial z}\right)_{z=0} \tag{32}$$

Substituting the expression of T2, given in Eqn. (25), one can simplify Eq. (32) as

$$Nu = 1 + a^2 \left(\frac{c}{2(c^2+\omega^2)} + \frac{\pi^2}{2\sqrt{(4\pi^4+\omega^2)}\sqrt{(c^2+\omega^2)}} \right) |A_1(s)|^2 \tag{33}$$

The above Nusselt number is defined at lower boundary z = 0 only, since we are heating from the bottom and analyze how does heat transport varying in the system. It is clear that the gravity modulation is effective at third order and affects Nu(s) through $\mathbb{A}$(s) which is evaluated at third order. We calculate the mean value of Nusselt number (Nu) for better understanding of the results on heat transport. However, a representative time interval that allows a clear comprehension of the effect needs to be chosen. The interval (0, 2π) seems to be an appropriate interval to calculate (Nu). The time-averaged Nusselt is defined as:

$$N\bar{u} = \frac{1}{2\pi} \int_0^{2\pi} Nu(\tau)\, d\tau. \tag{34}$$

3.3 AT THE THIRD ORDER

At this stage we have the following relations:

$$\begin{pmatrix} \left(\frac{1}{\Pr}\frac{\partial}{\partial r}\nabla^2 - \nabla^4\right) & R_0 \frac{\partial}{\partial x} & -\sqrt{Ta}\frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial r} - \nabla^2 & 0 \\ \sqrt{Ta}\frac{\partial}{\partial z} & 0 & \left(\frac{1}{\Pr}\frac{\partial}{\partial r} - \nabla^2\right) \end{pmatrix} \begin{bmatrix} \psi_3 \\ T_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} R_{31} \\ R_{32} \\ R_{33} \end{bmatrix} \tag{35}$$

The expressions for R_{31}, R_{32} and R_{33} given in appendix can be evaluated using the values of, T_1, V_1 and T_2. Now under the following salvability condition

$$\int_0^{\frac{2\pi}{a}} \int_0^1 \left(R_{31}\hat{\psi}_1 + R_{32}\hat{T}_1 + R_{33}\hat{V}_1 \right) dx\, dz = 0 \tag{36}$$

yield the following complex Ginzburg-Landau equation that describes the temporal variation of the amplitude A(s) of the convection cell

$$\frac{\partial A_1(s)}{\partial s} - \Upsilon_1^{-1} F(s) + \Upsilon_1^{-1} k_1 |A_1(s)|^2 A_1(s) = 0. \tag{37}$$

Writing A1(s) in the phase-amplitude form, we get

$$A_1(s) = |A_1(s)|\ e^{i\varphi} \tag{38}$$

Now substituting the expression Eq.(38)in Eq.(37), we get the following equation for the amplitude |A1(s)|:

$$\frac{d\left|A_1(s)\right|^2}{ds} = 2p_r\left|A_1(s)\right|^2 - 2l_r\left|A_1(s)\right|^4 \tag{39}$$

$$\frac{d(ph(A_1(s)))}{ds} = p_i - l_i\left|A_1(s)\right|^2 \tag{40}$$

where $\gamma_1^{-1}F(s) = p_r + ip_i$, $\gamma_1^{-1}k_1 = l_r + il_i$ and $ph(.)$ represents the phase shift. We solve equation (39) using the inbuilt solution NDsolve of Mathematica 8, subjected to the condition (0)=a_0 where a_0 is chosen initial amplitude of convection. In our calculations we may use R_2= R_0 to keep the parameters to the minimum.

4. RESULTS AND DISCUSSION

In the light of earlier work Bhadauria et al. [17] studied a weak nonlinear stability analysis for stationary mode of convection of related problem. It is quite important to study thermal instability in a rotating fluid layer under oscillatory mode of convection where instabilities set in before stationary convection. As of now a lot of research work has been reported for heat transport under various physical models while considering weak nonlinear theory for stationary mode of convection. A close observation on modulation of Rayleigh-B'enard convection is to find externally adjusting parameters which controls convective phenomenon. Such a candidate gravity modulation is considered in this paper to investigate supercritical flow in a rotating fluid layer. As pointed out by Bhadauria et al. [19, 20] the oscillatory mode of convection exist when the Taylor Ta and Prandtl Pr numbers satisfies the relation given in Eq.(23). With this relation it is quite important to consider Pr<1 for the present problem, hence the effect of local acceleration term $\frac{1}{\Pr}\frac{\partial}{\partial s}$ is effective and affect the momentum equation. Vadasz [26] pointed out that for only low values of Pr oscillatory mode of convection is possible for fluid layer whereas for porous medium is not limited to a particular domain of Prandtl number for over stable convection. We also consider low values of amplitude δ_2 and frequency of ω_f of modulation since for low values of amplitude δ_2, frequency of ω_f of modulation is the maximum heat transfer in the system. The results corresponding to the gravity modulation has been depicted in figures [2-11], where we have plotted Nu with respect to the slow time s. At lower values of time, there obviously is only conduction and we then have a value of Nu=1 at these times. Nusselt number increases with increase in time and exceeds the value of 1 indicating the convective contribution to the heat transport.

The gravity plots presented in figure 2-11, where the nature of the figures is oscillatory due to modulation effect. We know that Prandtl number is the ratio between momentum diffusivity and thermal diffusivity. The diffusion of mass due to density differences arising due to variation in density is the momentum diffusivity. Thermal diffusivity is the rate at which the diffusion of heat takes place in the fluid due to its conductive properties. Pr increases means momentum diffusivity is very high and thermal diffusivity may be low or high (i.e. if the fluid has high thermal conductivity, this is the case in which maximum heat transfer takes place). Due to high thermal conductivity the heat also diffuses faster and also the movement of fluid is faster due to quick response of mass flow due to density differences.

In Fig.2 it is found that increment in Pr is to enhance the heat transfer in fluid layer, these are the results compatible with the one obtained by Bhadauria et al. [19] while considering low viscous fluids for stationary mode of convection. In rotating fluid flows, the Taylor number (Ta) characterizes the importance of inertial forces due to rotation, relative to viscous forces. When Ta increases, which means either centrifugal forces may grow or kinematic viscosity may reduce as a consequences fluid will not move freely and then system may stabilizes. In Fig.3 the effect of Taylor number is to stabilize the system for certain finite range of Taylor number (up to 6000 nearly), suddenly an opposite effect takes place for large values of Ta>6000, confirming the results of Rauscher and Kelly [13] given in Fig 4. In general, obtaining Ta very large is not possible but for justifying what happens if one consider high rotation in the system. From the Fig.5 one can see that, the effect of an amplitude of modulation enhance the heat transfer as increases but opposite in the case of frequency given in figure 6. From the Fig.6 we find that the effect of gravity modulation decreases as the frequency of modulation increases, and finally when ω_f is very large the effect of modulation disappears altogether, thus conforming the results of ([24, 25], [27, 28]).

In Fig.7 we have shown the difference between modulated and unmodulated cases. The nature is quite different in unmodulated case, for lower values of s, Nu increases and for high values of s steady state achieves. The heat transfer is quantified for gravity modulation and compared with the present results in Fig.7. But, as for gravity modulation, the modulated flows transport less heat than their corresponding unmodulated flows conform the results of ([32]-[40]). In figure 8 we compare the results of oscillatory and stationary mode of convection. It is found that heat transfer is more in oscillatory mode of convection than in stationary mode. It can be observed that $Nu_{St} < Nu_{OSC}$ for the same wave number. This implies that oscillatory instabilities can set in before stationary mode. Similar results has also been obtained by Kim et al. [29] and Bhadauria and Kiran ([19,20]).

In Fig.9 we have presented our results in terms of averaged Nusselt number to see the effect of frequency of gravity modulation. From the Fig.9(a) it is evident that for lower values of frequency of modulation there is an enhancement in ($\overline{Nu}$), further increases ω_f achieves steady state. We also can observe effect of Pr in Fig.9 (a) is to enhance the heat transfer and effect of Ta in Fig.9(b) is to diminish the heat transfer.

In Figures 10 and 11, the stream lines and the corresponding isotherms are depicted for gravity modulation, respectively at s = 0:0; 1:0; 1:5; 2:0; 3:0; 6:0 for P r = 0:1; T a = 500; 2 = 0:1 = 2:0 and = 0:5. From the Figures we found that initially when the time is small the magnitude of streamlines is also small given in figures 10(a) and (b), and isotherms are straight that is the system is in conduction state figure 11(a) and (b). However, as time increases, the magnitude of streamlines increases and the isotherms loses their evenness. This shows that the convection is taking place in the system. Convection becomes faster on further increasing the value of time s. However, the system achieves the study state beyond s = 0:16 as there is no change in the streamlines and isotherms gures 10(d,e,f)-11(d,e,f).

5. CONCLUSIONS

The effect of gravity modulation on a rotating Rayleigh Benard convection for oscillatory mode has been investigated considering a weak nonlinear stability analysis resulting in the complex Ginzburg Landau amplitude equation. The following conclusions are drawn:

1. Effect of Prandtl number Pr is to advance the onset of convection and hence enhance the heat transport.

2. The Taylor number Ta has duel effect on Rayleigh-Benard convection either is to decrease or increase for (Ta > 6000) the heat transport for all three types of modulations.

3. Amplitude of modulation is advance the convection and hence heat transfer.

4. The frequency ω_f of modulation is decrease the heat transfer.

5. The maximum heat transfer (Nu) for lower range of ω_f , diminishes with higher values of ω_f .

6. APPENDIX

The governing equations of the problem are given by Chandrasekhar [10]:

$$\Delta.\vec{q}=0,$$

$$\left(\bar{\lambda}\frac{\partial}{\partial t}+1\right)(-\Delta P+\rho\vec{g})-\frac{\mu}{K}\left(\bar{\varepsilon}\frac{\partial}{\partial t}+1\right)\vec{q}=0,$$

$$\frac{\partial T}{\partial t}+(\vec{q}.\Delta)T=\mathrm{K}_T\Delta^2 T,$$

$$\rho=\rho_0\left[1-\alpha_T(T-T_0)\right],$$

The coefficients given in the Eq. (31) are

$$R_{31}=-\frac{1}{\Pr}\frac{\partial}{\partial s}\left(\nabla^2\psi_1\right)-R_0\left(\frac{\partial T_2}{\partial x}\right)-\left(R_2+R_0\delta_2\cos(\omega_f s)\right)\frac{\partial T_1}{\partial x},$$

$$R_{32}=\frac{\partial T_1}{\partial s}+\frac{\partial\psi_1}{\partial x}\left(\frac{\partial T_2}{\partial z}\right),\ R_{33}=-\frac{1}{\Pr}\frac{\partial V_1}{\partial s}-\frac{1}{\Pr}\frac{\partial\psi_1}{\partial z}\frac{\partial V_2}{\partial s}$$

The coefficients given in the Eq. (37) are defined by

$$\Upsilon=\left[\frac{c}{\Pr}-\frac{a^2R_0}{(c+i\omega)^2}-\frac{\pi^2 Ta}{\Pr\left(c+\frac{i\omega}{\Pr}\right)^2}\right],\ F(S)=\left[\frac{a^2}{(c+i\omega)}\left(R_2+R_0\delta_2\cos(\omega_f s)\right)\right],$$

$$k = \left[\frac{a^4 R_0 c}{4(c^2 + \omega^2)(c + i\omega)} + \frac{\pi^2 a^4 R_0}{(8\pi^2 + 4i\omega)(c + i\omega)^2} + \frac{\pi^4 a^4 c Ta}{4\left(c^2 + \frac{\omega^2}{\Pr^2}\right)\left(c + \frac{i\omega}{\Pr}\right)} + \frac{\pi^4 a^2 Ta}{2\left(4a^2 + \frac{i\omega}{\Pr}\right)\left(c + \frac{i\omega}{\Pr}\right)^2} \right],$$

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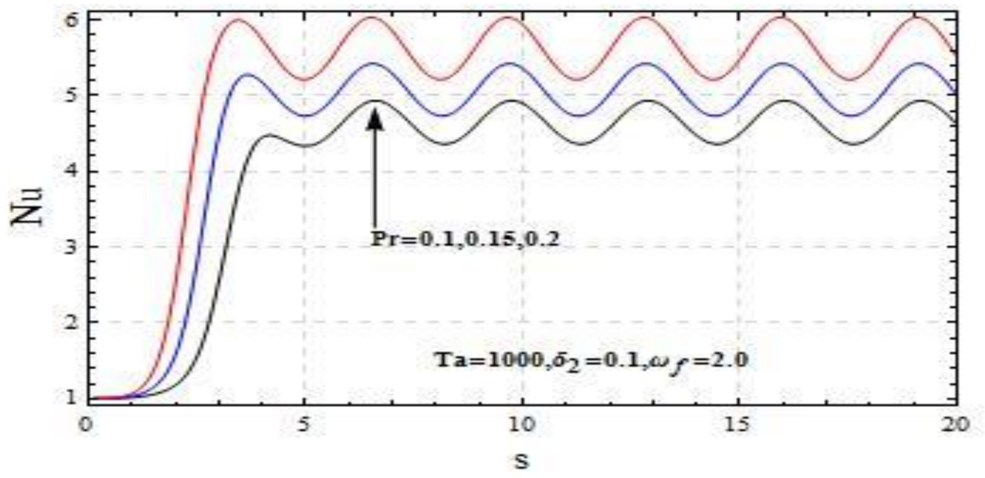
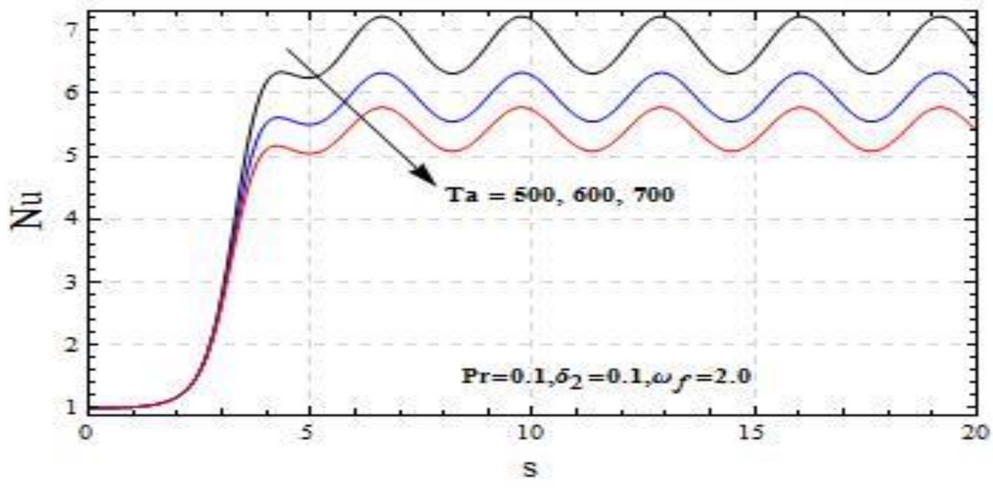


Fig .2 : Effect of system parameters on heat transfer



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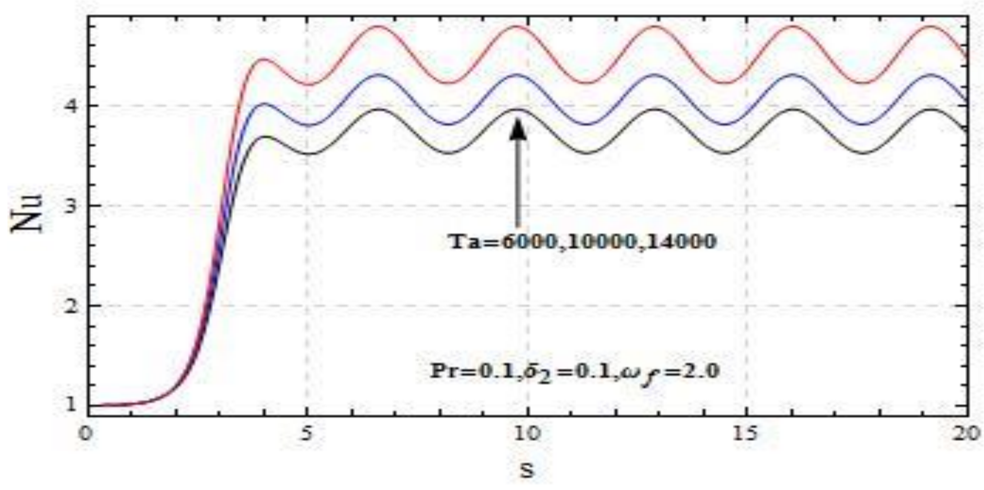

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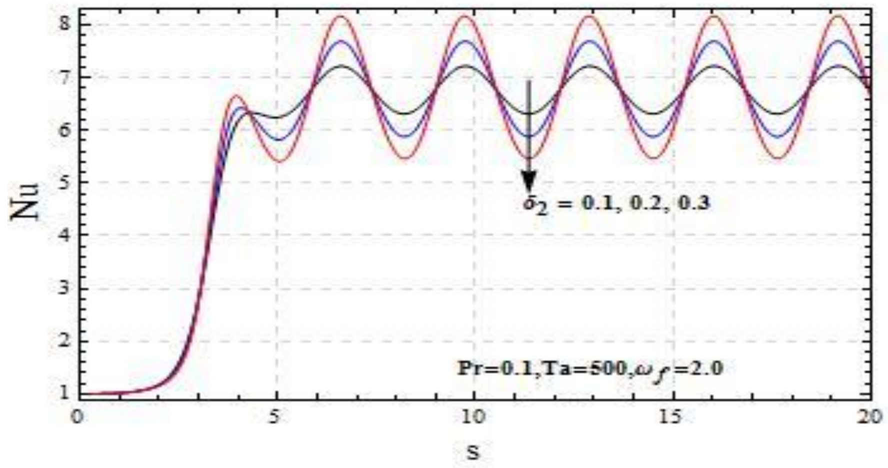

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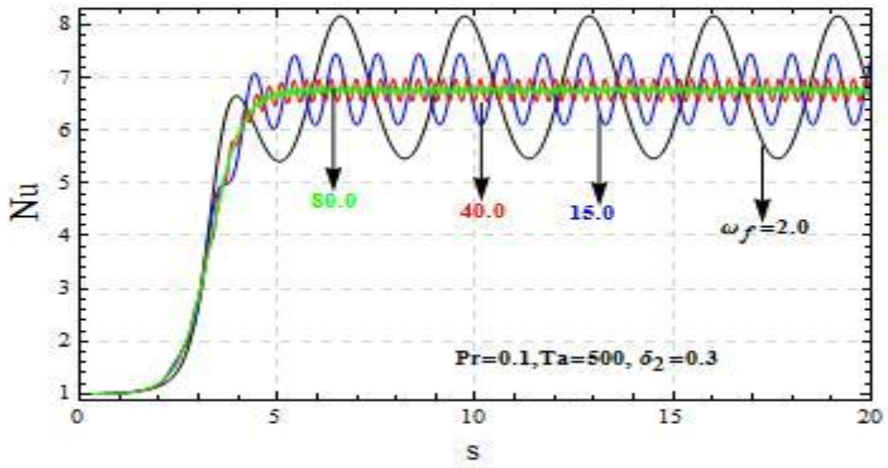

Fig .6 : Effect of system parameters on heat transfer

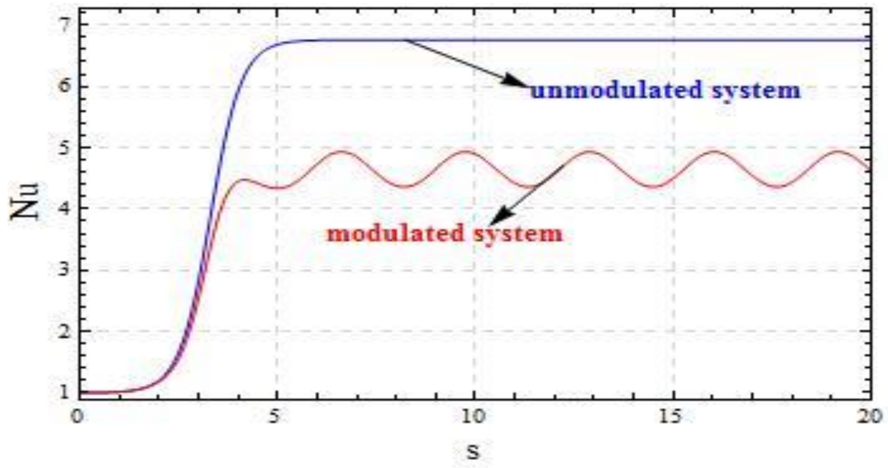

Fig .7 : Comparision between stationary and oscillatory convection

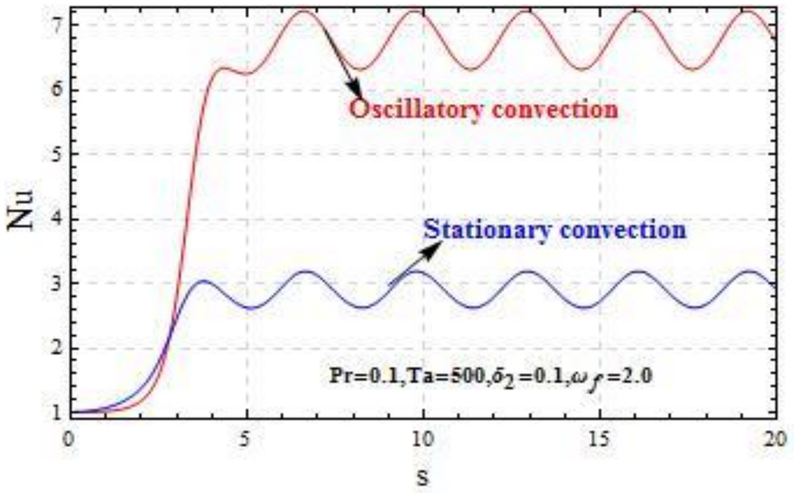

Fig .8 : Comparision between stationary and oscillatory convection

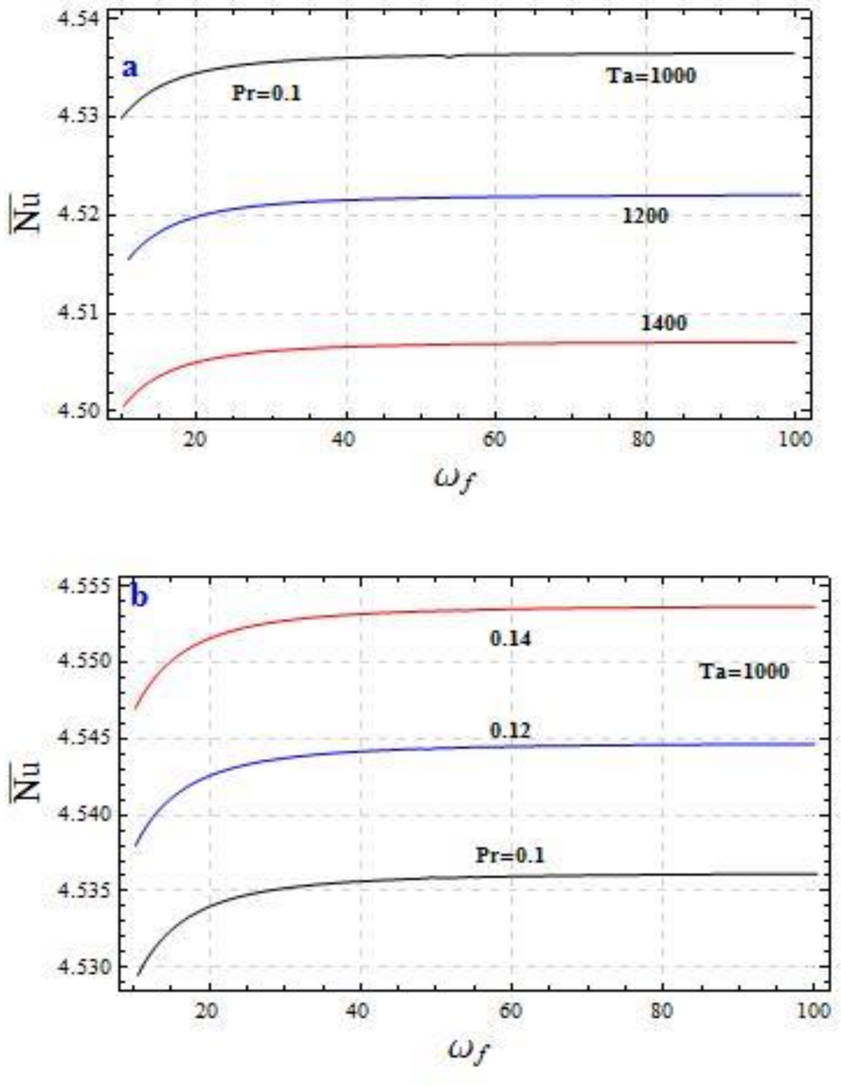

Fig .9 : Effect ofω_f on $\overline{Nu}$ for various fvalues of Ta and Pr

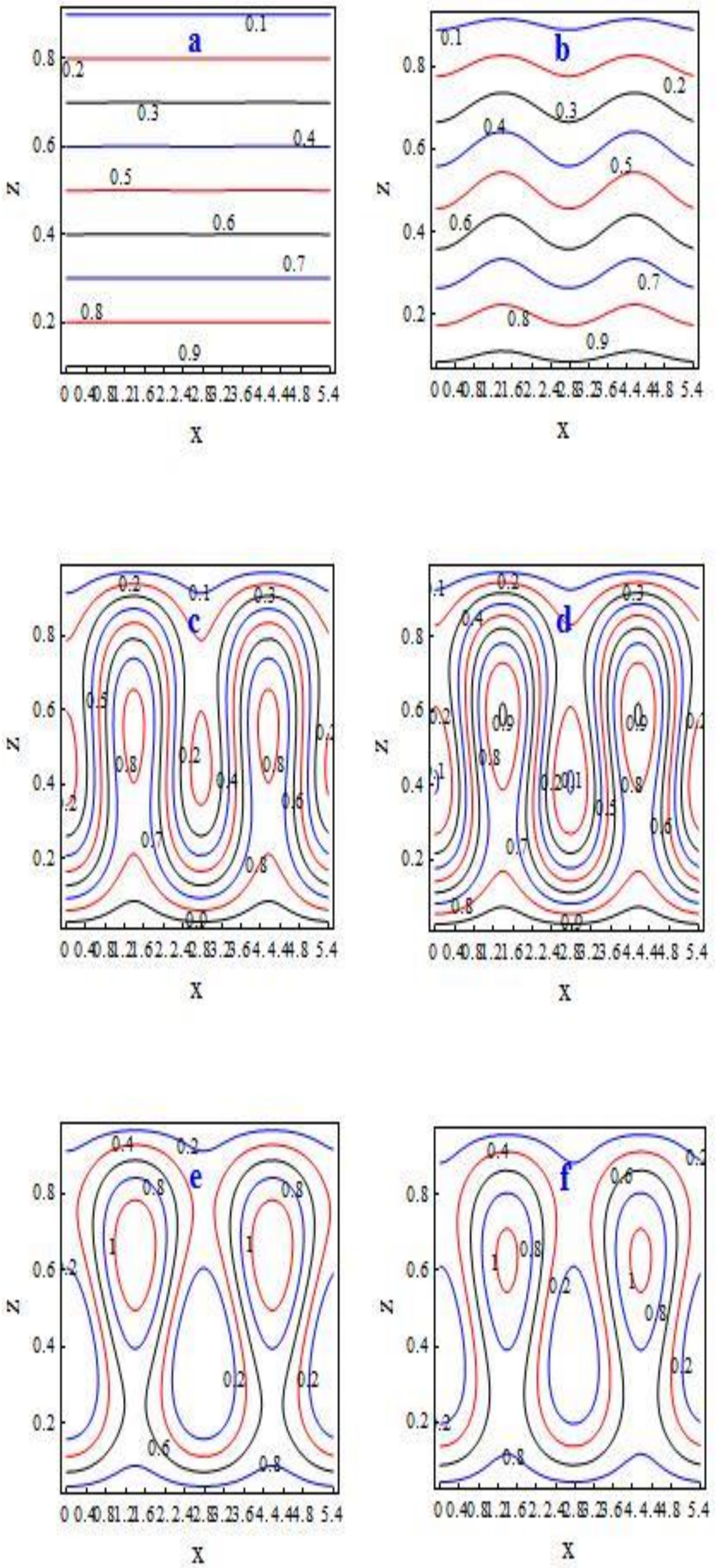

Fig 10 : Isotherms for different values of time (a) s = 0.0 (b) s = 1 (c) s = 1.5 (d) s = 2 (e) s = 3 (f) s = 6

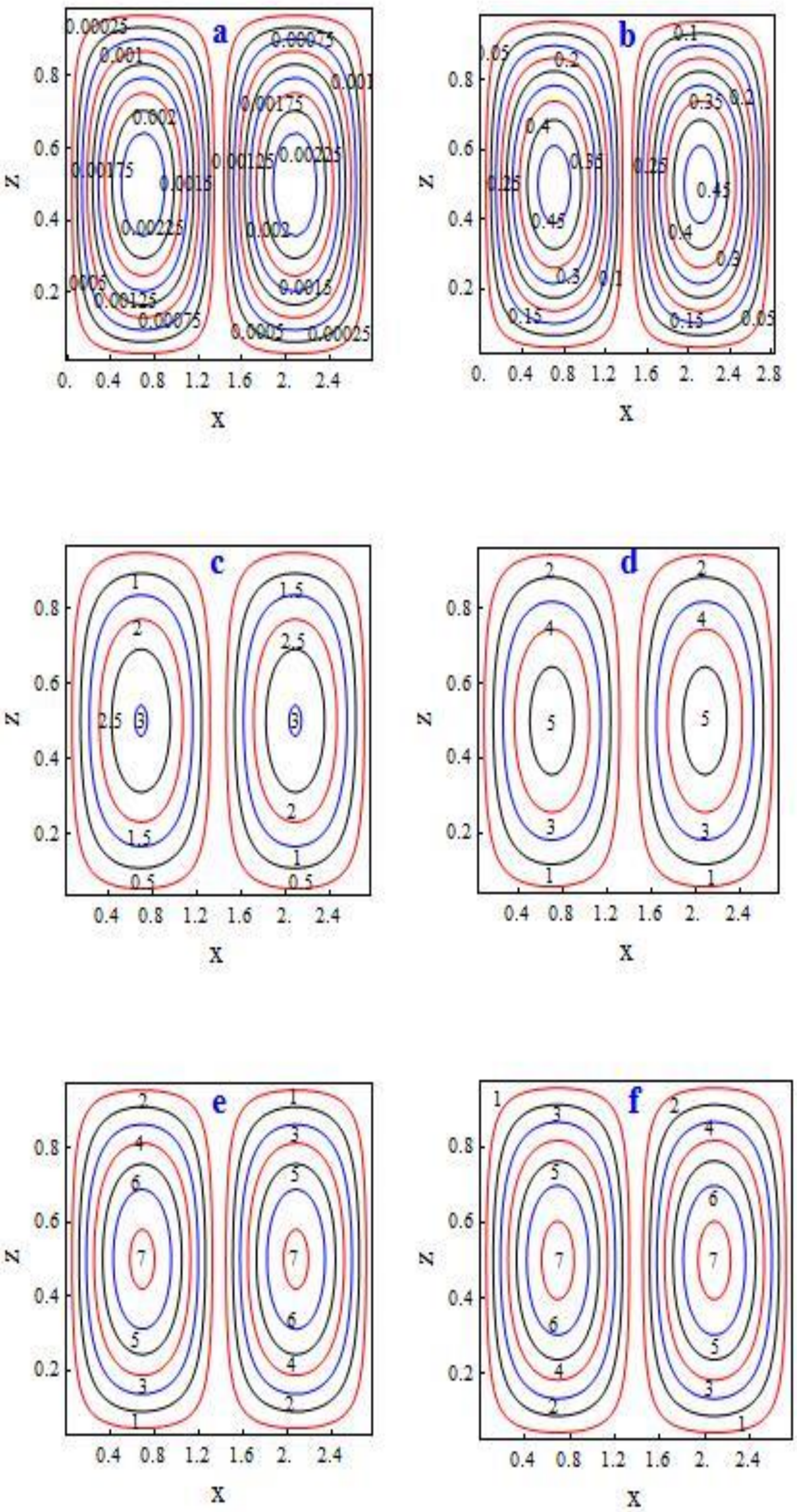

Fig 11 : Streamlines for different values of time (a) s = 0 (b) s = 1 (c) s = 1.5 (d) s = 2 (e) s = 3 (f) s = 6