Fuzzified Inventory Ordering Model for Green Environmental Sustainability under Inflation

Vikramjeet Singh(1), Seema Saxena(2)*, Rajesh Kumar Gupta(3), Pushpinder Singh(4) and Nitin Kumar Mishra(5)

(2,3,5) Department of Mathematics, Lovely Professional University, Phagwara 144411, Punjab(India)

(1) Departmemt of Mathematics, IKGPTU Campus, Batala,143506, Punjab(India)

(4)Department of Mathematics, Mata Gujri College, Fatehgarh Sahib, 140407,Punjab(India)

Abstract

In recent preceding years, it has been noticed, that on account of declension in the greenhouse outcome due to a decline in green natural resources and emission of the green houses gases, government is now paying more concern to the procedures and policy on this prominent issue. This research work introduces a concept of green technology in an inventory framework under an inflationary state of affairs. The major feature of adopting and environmental friendly measure is the sustainability towards the clean and green surrounding. This paper formulates a model incorporating the various green cost and determine the optimal measure for the total cost and replenishing schedule. It also incorporates the fuzzy sense for the inflation rate and the demand parameter there by including the uncertainty dwelling with the parameters. A comparative study between the crisp model and the fuzzified model is made to substantiate the study and validate the model.

Keywords: *Green technology, fuzzy logics, Inflation, Credit term, Inventory*

1. Introduction

The practice of involving the green concept of maintaining healthy surrounding and clean environment into the inventory train research is of utmost importance. In the past decades the operational managers and the workforce at the firms were responsible only at lower level for environment management. There are other firms which were held responsible for ensuring the product quality design, working for the waste administration during the product. But, presently the conditions have changed. The government has issued certain norms and policies for every firm, so as to have a proper hold over the environment management. With the revolution brought by the policies, it has become common to integrate the green environment management along with the running operations. The increase in the involvement of the green environment concept with the inventory operations have led several researchers to study and formulate various constraints and restrictions prevailing in the real-time problem into an inventory framework. The work done explains the inventory structure and the impact of the green inventory system on the cost and revenue of the inventory system. The significant interest to include the green inventory management is mainly driven by the deteriorating state of the environment due to overlapping of the waste and increase in the waste areas, reduction in the natural resources and increase in the pollution levels. Moreover including the uncertainty in the inventory model brings the problem to more realistic situation point the introduction to the manufacturing concepts over the recovery of the waste goods was done by [13]. He analyzed the ordering quantity model for the deterministic value of demand and return policy. Changes that are taking place in the market due to various financial and other reasons the model keeping in mind the various changes occurring in the product design for adopting to the new technologies was done by [1]. [4] surveyed literature and provided a comprehensive study for the remanufacturing of the goods. The manufacturer's location and retailer's location are far off and so during transportation the goods are likely to be damaged. Thus it becomes necessary for the retailer to screen the lot that is received. [9] formulated a model for the imperfect type of goods maximizing the profit. Deterioration was introduced by [5] in 1963. Thereafter several works done by [3],[21] depicts the importance of deteri- oration in the inventory model. [8] modeled a supply chain including deterioration with the credit period. A supply chain model for the deteriorated goods imperfect in nature was developed by [10] considering the demand as depending on the stock. [20] included the sales strategies under the admissible delay period to optimize the inventory model.

[15] model an inventory problem for the time varying deterioration rate. They included the admissible delay period in their model and optimize the total cost of the inventor problem. Trade credit has become an significant factor to be included in the inventory model. The supplier offers trade credit to the retailer on account of A reduction in his/her setup cost. Also the retailer generates profit on account of dual earning. [17] modeled an inventory problem including weibull deterioration and the green concept under trade credit to model the problem for optimizing the total cost.[11] modeled a problem for the affirmability of the environment in a supply chain with inflation induced cost parameters.

[16] developed an inventory model for the green inventory problem for a time quadratic demand in a finite planning horizon.

Inflation is one of the factors in prevailing in the market that cannot be ignored while framing and inventory model points as the time passes the money value of the amount decreases as it loses its power to buy more. Thus every cost related to inflation must be subjected to in inflation. [2] introduced the concept of inflation in the inventory model considering the money value. [6] analyzed inflation for weibull deterioration in the inventory model. [19] formulated an inventory model for partial back of backlogging and duration under inflation. [18] produced a replenishment schedule and inflation for the quadratic time varying demand and inflation. Considering supply chain and the distribution of the cost savings due to trade credit [12] represented an inventory problem with the credit factor used as a parameter for extra cost benefits realized from credit term.

Fuzzy logics are a must to be incorporated in the inventory problem, as it gives a scope for the inclusion of the uncertainity for the various parameters due to the market. [23] formulated the fuzzy inventory model making the various cost subject to fuzziness. [22] discussed the product design approach with the fuzziness. [14] included the fuzzy logic for parameters to model a realistic problem solution.[7] demonstrated a fuzzy model for an inventory problem.

The present paper analyses the green concept in the inventory model under an inflationary environment. The green concept includes re-manufacturing and Re utilizing the defective goods and again transported back in the system. Also uncertainty is kept in mind thus fuzzifying the rate of inflation and the time varying demand reaches to a more realistic solution for the inventory problem.

2. Presumptions and Annotations

2.1 Presumptions

The study incorporates following assumptions:

(1) Inflation rate r is applied to every inventory related cost.
(2) Deterioration of the inventory at hand occurs at a constant rate.
(3) The model is formulated for one retailer and supplier
(4) The goods received contain a percentage of defectives which are screened at retailer's end and are taken by the supplier.
(5) At the beginning of the model numbers of units assumed are u units.

2.2 Annotations:

1. $X_{1q}(t)$, $X_{2q}(t)$and$X_{3q}(t)$ are the amount of inventory at three levels.
2. Difference between the inflation and the discount rate k = (d-r)
3. Fuzzifying the difference between the inflation and the discount rate $= \tilde{k}$
4. $\beta_1(t)$ = rate of deterioration.
5. Q_{cr} = Quatity ordered in crisp ordering model.
6. T_{cr}^{R} and T_{cr}^{S} = Present estimate of cost value in the crisp environment for the retailer and the supplier.
7. A_r, H_g, D_g, P_g, S_g, TR, DEC, RM, S_u are the cost of ordering, holding, deterioration, purchasing, screening, transportation, dissembly, re-manufacturing and set- up.
8. $\widetilde{T_{cr}^{R}}$ = Fuzzified cost for retailer

3. Model Analysis

The two frames are considered while developing the model:

3.1 Frame 1 : Crisp Model

The supplier and the retailer follows the refilling plan according to the comfort and the total cost is derived under inflationary environment following green strategy. Each parameter involved in the inventory system is having a certain value.

3.2 Frame 2: Fuzzy Model

Due to the uncertainities prevailing in the market, parameters as the inflation rate, the discount rate and the time varying demand are fuzzified. Pentagonal fuzzy number is used followed by defuzzification of the total cost.

Model Characterization

The formulation of the model is based on the concept of re-manufacturing the imperfect goods that are present in the order received at the retailer's end. The imperfect lot of items are transported back for the reworking and are supplied back to be absorbed in the market. The initial number of units received is u units having the imperfect products percentage as p. The three differential equations depicts the amount of inventory required during each course of the time interval. The model behavior is figured below.

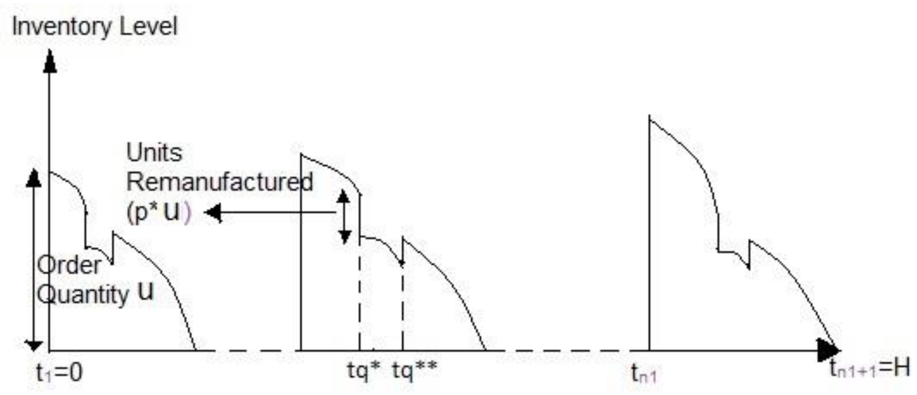

FIGURE 1. Graphical figuration of inventory framework

Solving the model with differential equations, we get the following results:

$$X_{1q}(t) = w_1(t_{q+1} - t) + \frac{w_1\beta_1 + w}{2}({t_{q+1}}^2 - t^2) + \frac{w.\beta_1}{3}(t_{q+1}^3 - t^3) + (p.\beta_1 t_q^* - p.\beta_1 t_q^{**}. \left(w_1(t_{q+1} - t_q) + \frac{w}{2}(t_{q+1}^2 - t^2)\right) - \beta_1.t.(w_1(t_{q+1} - t) + \frac{w}{2}(t_{q+1}^2 - t^2),$$

$t_q \le t \le t_q^*$,
(1)

$$X_{2q}(t) = (1 - \beta_1.t).\left(w_1(t_{q+1} - t) + \frac{w}{2}(t_{q+1}^2 - t^2)\right) + \frac{w_1\beta_1}{2}({t_{q+1}}^2 - t^2) + \frac{w.\beta_1}{3}(t_{q+1}^3 - t^3) - p.\frac{w_1\beta_1}{2}({t_{q+1}}^2 - t_q^2) + \frac{w.\beta_1}{3}(t_{q+1}^3 - t^3) - (p.(1 + \beta_1 t_q^{**} - \beta_1.t - \beta_1.t_q).(w_1(t_{q+1} - t_q) + \frac{w}{2}(t_{q+1}^2 - t_q^2) + p^2.(\beta_1 t_q^{**} - \beta_1 t_q^*),\ t_q^* \le t \le t_q^{**}, \{q = 1,2,\ldots.,n_1\}$$
(2)

$$X_{3q}(t) = (1 - \beta_1.t).\left(w_1(t_{q+1} - t) + \frac{w}{2}(t_{q+1}^2 - t^2)\right) + \frac{w_1\beta_1}{2}({t_{q+1}}^2 - t^2) + \frac{w.\beta_1}{3}(t_{q+1}^3 - t^3), t_q^{**} \le t \le t_{q+1},\ \{q = 1,2,\ldots\ldots.n_1\}$$
(3)

Amount of inventory during $[t_q, t_{q+1}]$ is

$$R_{cr} = \int_{t_q}^{t_q^*} X_{1q}(t)dt + \int_{t_q^*}^{t_q^{**}} X_{2q}(t)dt + \int_{t_q^{**}}^{t_{q+1}} X_{3q}(t)dt,$$
$\{q = 1,2,\ldots.. n_1\}$
(4)

The quantity ordered is =
$Q_{cr} = \sum_{q=1}^{n_1} u_{cr}$
(5)

Inflated cost for the retailer T_{cr}^R includes the current measure of all the relevant cost to the inventory

$$T_{cr}^{R} = \sum_{q=1}^{n_1} (PMAC + PMHC + PMDC + PMPC + PMSC)$$

$$= \sum_{q=1}^{n_1} e^{k.t_q}.n_1.A_r + H_g.\left(\int_{t_q}^{t_q^*} e^{k.t}.X_{1q}(t)dt + \int_{t_q^*}^{t_q^{**}} e^{k.t}.X_{2q}(t)dt + \int_{t_q^{**}}^{t_{q+1}} e^{k.t}.X_{3q}(t)dt\right) + D_g.\left(\int_{t_q}^{t_q^*} e^{k.t}.\beta_1.X_{1q}(t)dt + \int_{t_q^*}^{t_q^{**}} e^{k.t}.\beta_1.X_{2q}(t)dt + \int_{t_q^{**}}^{t_{q+1}} e^{k.t}.\beta_1.X_{3q}(t)dt\right) + (P_g + S_g).e^{k.t_q}.u$$

(6)

To derive the optimality of the total cost in the crisp nature the values from the above mentioned equation is derived by equating its derivative to zero.

$$\frac{\partial T_{cr}^{R}}{\partial t_q} = 0, \qquad n_1^{cr}, t_0, t_1^{cr}, t_2^{cr}, \ldots .. t_{n_1+1}^{cr} = H, being\ the\ values\ for\ optimality.$$

Similarly the total cost of the supplier is determined by including the costs that are incurred due to transporting back the imperfect goods and re-manufacturing them.

$$T_{cr}^{S} = n_1^{cr}.e^{k.t_q}.Su + P_{gs}.e^{k.t_q}.u + H_{gs}.\int_{t_q^*}^{t_q^{**}} e^{kt}.p.u.dt + (TR + DEC + RM).p.e^{kt}.u$$

(7)

The optimized quantity to be ordered is:

$$Q_{cs} = \sum_{q=1}^{n_1^{cr}} u_{cr}$$

(8)

Frame 2: Fuzzy Inventory Problem Model.

The discount rate , inflation rate and the time varying demand factors are taken as the pentagonal fuzzy numbers

$d = d_1, d_2, d_3, d_4, d_5, r = r_1, r_2, r_3, r_4, r_5$ *and* $b = b_1, b_2, b_3, b_4, b_5$

$$\widetilde{T_{cr}^{R}} = \sum_{q=1}^{n_1} e^{\tilde{k}.t_q}.n_1.A_r + H_g.\left(\int_{t_q}^{t_q^*} e^{\tilde{k}.t}.X_{1q}(t)dt + \int_{t_q^*}^{t_q^{**}} e^{\tilde{k}.t}.X_{2q}(t)dt + \int_{t_q^{**}}^{t_{q+1}} e^{\tilde{k}.t}.X_{3q}(t)dt\right) + D_g.\left(\int_{t_q}^{t_q^*} e^{\tilde{k}.t}.\beta_1.X_{1q}(t)dt + \int_{t_q^*}^{t_q^{**}} e^{\tilde{k}.t}.\beta_1.X_{2q}(t)dt + \int_{t_q^{**}}^{t_{q+1}} e^{\tilde{k}.t}.\beta_1.X_{3q}(t)dt\right) + (P_g + S_g).e^{\tilde{k}.t_q}.u$$

Defuzzification of the total cost is given by the signed distance method as follows:

$$d(\widetilde{T_{cr}^{R}}) = \frac{A+3B+4C+3D+E}{12}$$

4. Optimality Evaluation and Solution

The formulated mathematical model aims at convexity of the total cost for both the scenarios i.e., the crisp and the fuzzy environment. The following theorem states the convex nature of the total cost explained through the graph.

Theorem 1: The solution to the cost function in the green fuzzified model exists and is unique following convexity.

The nature of the cost in the two different environments follows a convex pattern. This is shown by the two graphs in both the situation for a value of the demand parameter w_1.

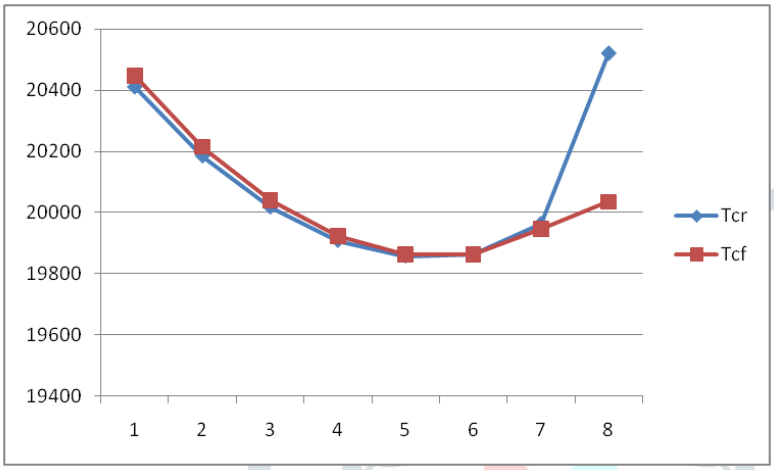

Figure 2 : Convexity of Total Cost

5. Numerical Example

The model is elucidated with the numerical examle and the potential of the green strategy along with the fuzzy logic in the inflationary conditions. The demand parameter w_1 is varied to observe its nature in crisp and fuzzy model.

Example 1: w_1 = 20 items/year, w = 5 items/year, β_1 = 0.05 items/year, P_g = 4$/item, γ = 0.01, t_1 = 0, A_r = 25 $/purchase order, S_u = 30 $/arrangement, H = 4,$P_s$ = 0.01 $/item, C_c = 2.5 $/item/year , H_g = 100 $/item/year, d = 0.1, r = 0.05, D_g = 70, CT = 0.1, TR=0.01 $/item, RM=0.01 $/item, DEC=0.01 $/item.

Table:1. Green Cost for retailer in crisp sense under inflation

w_1↓ n_1→	1	2	3	4
10	20410	20185	19908.6	19857.5
20	29222	28964.4	28728.6	28568.6
30	38034	37708	37440	37229.8

	5	6	7	8
10	19857.5	19864.5	19962	20520.4
20	28466.4	28422.2	28524.6	28704.8
30	37077.3	36982	37102.8	37282

Table:2. Supplier Cost for demand parameter ’w_1’

Value	T_{cs}^{S}	Q^{cs}
10	1304.33	78.25
20	1913.9	118.725
30	2125.41	158.275

Including the pentagonal fuzzy number the values for the fuzzy variable are classified as follows:

$d_1 = 0.01, d_2 = 0.06, d_3 = 0.1, d_4 = 0.15, d_5 = 0.18, r_1 = 0.03, r_2 = 0.04, r_3 = 0.05, r_4 = 0.06, r_5 = 0.07$,
$b_1 = 3, b_2 = 4, b_3 = 5, b_4 = 6, b_5 = 7$

Table:3. Green Cost for retailer in fuzzy sense under inflation

w_1↓ n_1→	1	2	3	4
10	20447	20214.5	20039.4	19922.2
20	29275	28990	28762.3	28592.3
30	38103.6	37766.8	37487.5	37265.8

	5	6	7	8
10	19863	19861.7	19945.5	20034.8
20	28479.9	28425.3	28550.7	28724.8
30	37101.3	36994.9	37202.8	37402.8

6. Logical perceptions

From Table 1 and Table 3, the following insights are developed:

(1) The numerical solutions demonstrate that the cost value in the crisp problem is less than the cost value obtained in the fuzzy problem.

(2) This increase in the cost value shows that the values to the parameters cannot be taken in the fixed nature. The values to these parameters change according to the market situations. Hence the cost derived keeping the parameters in fixed form does not project the correct solution to the problem.

(3) Fuzzifying the parameters, gives an increase in the cost, there by showing the realistic approach to the problem. The increase clearly mentions the significance of the fuzzy logics in the inventory problem.

The following graph figuratively shows the comparison between the fixed nature of cost and the fuzzy sense of cost. It is observed that the fuzzy cost is more than the fixed cost.

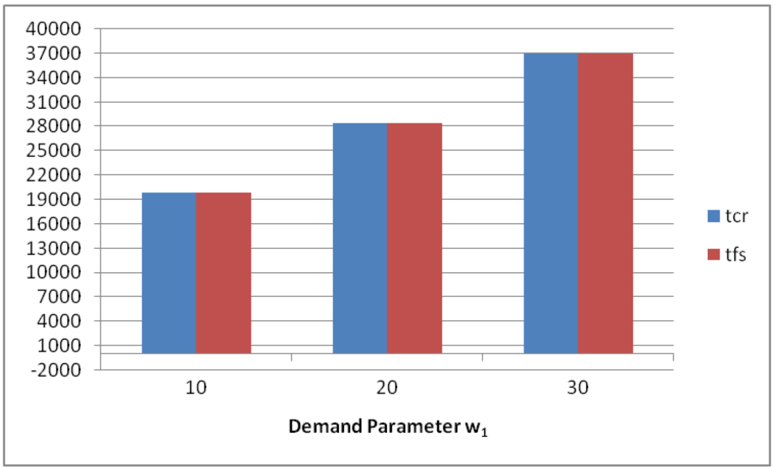

Figure 3: Cost Comparison

7. CONCLUSION

The work done in the paper demonstrates the green technology incorporated in the inventory problem. In addition to above the model is framed in an inflation based back-drop. The fuzziness in the various parameters that dwell due to the market conditions are also taken care by fuzzifying the significant parameters as the discounted value and the time depending demand factor. It is determined from the numerical elucidated along-with the theorem that the fuzzified cost is more than the cost value in the crisp model which clearly makes it evident that the fuzzy logics should be applied while deriving any inventory problem model. Pentagonal fuzzy number is used to fuzzify and signed distance method is used to defuzzify the cost value.

8.References

[1] Maurice Bonney, Svetan Ratchev, and Idir Moualek. The changing relationship between production and inventory examined in a concurrent engineering context. International Journal of Production Economics, 81, **(2003),**243–254.

[2] JA Buzacott. Economic order quantities with inflation. Journal of the Operational Research Society, 26(3), **(1975),** 553–558.Richard P Covert and George C Philip. An eoq model for items with weibull distribution deteriora- tion. AIIE transactions, 5(4), **(1973),** 323–326.

[3] Marisa MP de Brito. Managing reverse logistics or reversing logistics management? Number ERIM PhD Series; EPS-**2004**-035-LIS.

[4] P. M. Ghare and G. F. Schrader. (1963) A Model for an Exponential Decaying Inventory. Journal of Industrial Engineering, 14,**(1963),** 238-243.

[5] Sh-Tyan Lo, Hui-Ming Wee, and Wen-Chang Huang. An integrated production-inventory model with imperfect production processes and weibull distribution deterioration under inflation. International Journal of Production Economics, 106(1), **(2007),** 248–260.

[6] Hui-Ming Wee. Economic production lot size model for deteriorating items with partial back-ordering. Computers & Industrial Engineering, 24(3), **(1993),** 449–458.

[7] M Rameswari and R Uthayakumar. An integrated inventory model for deteriorating items with price-dependent demand under two-level trade credit policy. International Journal of Systems Science: Operations & Logistics, 5(3), **(2018),** 253–267.

[8] Biswajit Sarkar, Waqas Ahmed, Seok-Beom Choi, and Muhammad Tayyab. Sustainable inventory management for environmental impact through partial backordering and multi-trade-credit-period. Sustainability, 10(12), **(2018),**4761.

[9] T Sarkar, SK Ghosh, and KS Chaudhuri. An optimal inventory replenishment policy for a deterio-rating item with time-quadratic demand and time-dependent partial backlogging with shortages in all cycles. Applied Mathematics and Computation, 218(18), **(2012),** 9147–9155.

[10] Seema Saxena, Vikramjeet Singh, Rajesh Kumar Gupta, Nitin Kumar Mishra, and Pushpinder Singh. Green inventory supply chain model with inflation under permissible delay in finite planning horizon.

[11] Mojdeh Younesi and Emad Roghanian. A framework for sustainable product design: a hybrid fuzzy approach based on quality function deployment for environment. Journal of Cleaner Production, 108, **(2015),**385–394.

[12] David A Schrady. A deterministic inventory model for reparable items. Naval Research Logistics Quarterly, 14(3), **(1967),** 391–398.

[13] Pushpinder Singh, Nitin Kumar Mishra, Manoj Kumar, Seema Saxena, and Vikramjeet Singh. Risk analysis of flood disaster based on similarity measures in picture fuzzy environment. Afrika Matematika, 29(7-8), **(2018),** 1019–1038.

[14] Pushpinder Singh, Nitin Kumar Mishra, Vikramjeet Singh, and Seema Saxena. An eoq model of time quadratic and inventory dependent demand for deteriorated items with partially backlogged shortages under trade credit. In AIP Conference Proceedings, volume 1860, **(2017),** 020-037. AIP Publishing.

[15] Vikramjeet Singh, Seema Saxena, Rajesh Kumar Gupta, Nitin Kumar Mishra, and Pushpinder Singh. A supply chain model with deteriorating items under inflation. In 2018 4th International Conference on Computing Sciences (ICCS), **(2018),** 119–125. IEEE.

[16] Vikramjeet Singh, Seema Saxena, Pushpinder Singh, and Nitin Kumar Mishra. Replenishment policy for an inventory model under inflation. In AIP Conference Proceedings, volume 1860, **(2017),** 020-035. AIP Publishing.

[17] Zhihong Wang and Shaofeng Liu. Supply chain coordination under trade credit and quantity discount with sales effort effects. Mathematical Problems in Engineering, **2018**.

[22] Hans-Jürgen Zimmermann. Fuzzy set theoryand its applications. Springer Science & Business Media, **2011**.