Review on Recent Trends of Facility Location Problem with Uncertainty

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Abstract

In this modern world, Supply Chain Management has become an integral part of every enterprise because of the high satisfaction rate for customer's incoming demand and gain for the company itself. Facility Location Problem is prominent for the same as it is one of the most important components of Supply Chain Management. In this paper, a review of literature is presented. Different types of problem are studied which are having predetermined and/or uncertain parameters. It is seen that under uncertain environment, fuzzy and stochastic approaches provide better optimal results than deterministic problem though these models are much complex, certain well-known approaches such as Lagrangian Relaxation (LR), Metaheuristic, genetic algorithm any many more are there to obtain desired solution along with considering the necessary modification as per the constraints or parameters.

1. Introduction

In today's open market, company's main function is to successfully compete in this interconnected competitive global economy. It is believed, customers are linchpin for any company so, their prime objective is to fulfil incoming customer's need at their satisfaction level in the best possible manner, along with obtaining maximum gain for itself. Certain factors, decisions or stages are responsible in this process. Facility Location Problem (FLP) or Facility Location decision is one of them. In real-life problems, evaluation of optimistic location and allocation decisions is critical, in existing network design. It is done on the basis of the quality which service facilities have and customer's demand for those facilities. Also, the decisions related to transportation and inventory can be modified if there is any change in availability of raw material from supplier, transportation or shipment cost, inventory carrying cost, production cost as these are the basic components of the cost of goods or services. A successful facility location decision leads to reduction of cost with improved customers service levels. The variation in FLP depends upon the model or constraints of it. As the facilities provided by the company may or may not have fixed capacities, FLP is classified into Capacitated Facility Location Problem (CFLP) and Un-capacitated Facility Location Problem (UFLP). A UFLP model is a general formulation of FLP in which provided facilities can have, produce and ship unlimited capacities or quantities of goods. Mathematically,

For each facility $i \in I$, let $Y_i \in \{0,1\}$: if facility i is open, then $Y_i = 1$, otherwise $Y_i = 0$.

For each facility $i \in I$ and for each client $j \in J$, let $X_{ij} \in \{0,1\}$: if client j is assigned to facility i, then $X_{ij} = 1$, otherwise $X_{ij} = 0$.

The objective function:

$$Min\ Z = \sum_{i \in I}^{n} F_i\, Y_i + \sum_{i \in I}^{n} \sum_{j \in J}^{m} C_{ij} X_{ij} \qquad (1.1)$$

Subject to

$$\sum_{i \in I}^{n} \sum_{j \in J}^{m} X_{ij} = 1 \qquad (1.2)$$

$$X_{ij} \le Y_i, \forall\, j \in J, i \in I \qquad (1.3)$$

$$0 \le X_{ij}, Y_i \le 1, \forall\, j \in J, i \in I \qquad (1.4)$$

Here, $J = \{1, \dots, m\}$ is a set of clients (or demands); $I = \{1, \dots, n\}$ is a set of facilities and

The equation (1.1) represents, for each client $j \in J$ and facility $i \in I$, there is a cost C_{ij} to assign a client j to facility i; and there is a cost F_i associated with each facility $i \in I$. The demand constraint (1.2), ensures that each client is assigned to exactly one facility, while (1.3) shows that the clients are assigned to the facilities that are open. However, CFLP model exhibit that each facility has limited capacity or demand which are considered in deterministic parameters. In other words, if a located facility has a least cost for a demanded service or node, it may not be able to serve it because of limited capacity.

Mathematically,

The objective function:

$$Min\ Z = \sum_{i \in I}^{n} F_i\, Y_i + \sum_{i \in I}^{n} \sum_{j \in J}^{m} C_{ij} X_{ij} \qquad (1.5)$$

Subject to

$$\sum_{i \in I}^{n} \sum_{j \in J}^{m} X_{ij} = 1 \qquad (1.6)$$

$$AX_{ij} \le BY_i, \forall\, j \in J, i \in I \qquad (1.7)$$

$$0 \le X_{ij}, Y_i \le 1, \forall\, j \in J, i \in I \qquad (1.8)$$

Here, the capacity constraint (1.8), ensures that each client's demand served by the facility do not exceed its capacity. Changes in parameters and constraints change the shape of model as well as result.

2. Literature review

Many different techniques are used to obtain optimized result for the models of these to variants of FLP under deterministic parameters but in practical situations, some non-deterministic factors such as cost, demand, capacity or others may highly volatile due to lack of historical data, unpredictable events or due to some other reasons. While planning for facility location decision, inherent uncertainty exists and it is difficult for the manager to adjust and improve. An optimization approach gives optimal or reliable result to the FLP under predetermined parameters but in case of under uncertainty, the obtained result by same technique is less accurate and optimal because affecting factors may fluctuate widely (Gao, Averbakh, Snyder [1-3]) As a result, formulation of FLP under uncertain condition gains attention of researcher working in logistic and stochastic areas. Huge number of approaches are available, to obtain highly optimized result for an uncertain FLP model. Further this uncertainty is categorized into probabilistic/stochastic environment and fuzzy environment. Under stochastic FLP, the uncertainty of all parameters is assumed by means of probability distribution in order to have good result. In case of fuzzy uncertainty, all parameters are assumed as fuzzy random variables and then suitable optimization approaches are used for evaluations. Vast literature is available for these cases. Pirkul and Jayaraman [4] introduced a PLANWAR model to solve a multi-commodity, multi-plant CFLP and this mixed integer linear programming model achieved optimal value through the heuristic approach. To deal with the case of fuzzy demand in capacitated location/allocation problem, Zhou and Liu [5] defined a set of three fuzzy models: minimization of fuzzy expected cost, fuzzy α-cost, and maximization of creditability. For solution, Simplex and genetic algorithm are integrated. Hinojosa et al. [6] formulated a dynamic two echelon multi-commodity CFLP to cover the issue related to all costs in the model. And in order to minimize all costs, LR approach is used. Chen and Ting [7] compared the performance of two distinct methods: Multiple Ant Colony Systems (MACS) and Lagrangian heuristic and ant colony system to find the effective and optimal solution for Single-Source Capacitated Facility Location Problem (SSCFLP). Kucukdeniz et al. [8] involved fuzzy c-means and convex programming to find the solution of CFLP. Yang et al. [9] discussed a hybrid approach based on three distinct techniques to minimize the cost objective of two stage CFLP while Wang and Watada [10] studied the same model with fuzzy costs and demand. Arana-Jimenez and Blanco [11] formulated a framework of fuzzy capacitated facility location problem as minimax multiple objective mixed integer programming problem having all the parametric values as triangular fuzzy number. Klincewicz and Luss [12] showed the conversion in constraints of capacitated and uncapacitated facility location problem by using LR and heuristic approach which makes the model easy to implement in real life situations. Al-Sultan and Al-Fawzan [13] worked with fuzzy uncapacitated facility location (UFLP) problem and used tabu search algorithm for its solution. Barahona and Chudak et al. [14] solved large scale UFLP model by heuristic algorithm for feasible integer solution. Averbakh et al. [15] generalized UFLP model with two objectives: to minimize the service cost by the company and to maximize the profit. The tree typed network is solved by a polynomial-time dynamic programming algorithm. Kratica et al. [16] considered a multi-level UFLP in mixed integer programming model whose optimal value is obtained by CPLEX and Gurobi solvers. Xu and Xu [17] used UFLP problem with penalties and obtain a solution from a three-constant approximation algorithm while Wang and Xu [18] extended the same model and solve it by a three-approximate cross-monotonic cost-sharing method respectively. Hsieh [19] identified the demand, lead-time in trapezoidal fuzzy sense under inventory model having risk. Rezaei and Davoodi [20] discussed a multi-item fuzzy inventory model where costs, storage space, and number of orders are imprecise in nature. The model is solved by Genetic algorithm. Wen and Iwamura [21] designed a facility location/allocation model in uncertain scenario where demand is fuzzy and defined under Hurwicz criterion the solution of such model is obtained by comprising three distinct approaches: a simplex algorithm, fuzzy stimulation and a genetic algorithm. Whereas Handfield et al. [22] showed that demand, lead-time, supplier yield and penalty cost assumed to be in triangular fuzzy distribution in order to manage risk factors in inventory models. The model is solved by Genetic algorithm. A multi-facility location problem is studied by Esnaf and Kucukdeniz [23] in which capacity constraints are in fuzzy sense and clusters are made to treat the model as single facility location model. Shavandi and Bozorgi [24] used the concept of fuzzy demand in location-inventory or transportation model and resolve it by genetic algorithm. Phruksarphanrat and Tanthatemee [25] presented a lot-size inventory model under uncertain market condition where demand, supply is considered in fuzzy nature. Wang and Watada [26] originated a facility location model including risks with fuzzy cost and demand to solved by swarm PSO optimization approach. Altinel et al. [27] worked on capacitated multi-facility location problem for any distance function in which bivariate distribution is used and probabilistic extension of deterministic model is presented which is to be solved by heuristic approach. Fernández et al. [28] solved a discontinuous problem by using probabilistic choice rule and modified metaheuristic approach in which demand is the factor due to which customers were patronized to all existing facilities. Lee and Han [29] investigated FLP related to refuelling of electric vehicle. For allocation of sites for charging, probabilistic travel range is considered along with factors affecting the model and hence developed a novel Bender and Price approach which is faster than other considerable approaches. Ji et al. [30] used stochastic approach in fault tolerant FLP. The model has deterministic LP-round values, also two stage stochastic version provide same approximated ratio to random and derandomized form of the model. Markovic et al. [31] considered FLP for traffic flow case with restriction on time i.e. time is finite. In order to find appropriate solution, multi-stage stochastic program is formulated. Further LR reformulate the whole problem into two distinct models such as deterministic knapsack problem and sum of time decoupled single period stochastic program which are to be solved separately. A single source CFLP is considered by Lin [32] in which capacitated facilities have stochastic form of demand. The model is presented in deterministic and non-deterministic/ non-linear programming problem by using Poisson distribution and Normal distribution for demand function. The solution of the problem is obtained by hybrid heuristic approach of LR. For the first time, Jalali et al. [33] presented a bi-objective reliable FLP under three echelon supply chain management in which capacity levels are multiple and stochastic programming is used for constraints under uncertainty. Araya-Sassi et al introduced two distinct multi commodity inventory-location model with stochastic capacity constraints under a single supplier problem. Good quality gaps and near optimal solution is obtained in very less time by LR.

3. Conclusion

Facility Location Problem (FLP) is crucial in order to satisfy customer's need and to have high profit for a company itself. This paper has a review of literature related to variants of real-world FLP under various situations. Different types of problem are studied which are having predetermined and/or uncertain parameters. It is seen that under uncertainty and vagueness, fuzzy and stochastic approaches provide better optimal results than deterministic problem though these models are much complex, certain well-known approaches such as Lagrangian Relaxation (LR), Metaheuristic, genetic algorithm any many more are there to obtain desired solution along with considering the necessary modification as per the constraints or parameters. Future directions could be extending these approaches to three or more echelon model.

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