# Effect Of Imposed Shear Stress On The Growth Rate Of LL Mode Instabilities

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#### **Abstract**

We studied the linear stability of two liquid layers flowing down an inclined rigid surface in presence of insoluble surfactant at GL interface and imposed shear stress also on GL interface for high viscosity ratio ( $\mu_r = 50$ ). The growth rate of unstable LL mode is suppressed in longwave limit when the value of applied shear stress is increased but the growth rate suddenly increased in high wave limit for high value of imposed shear stress. It can be applied to produce either stable or unstable configurations as per our requirement. For solving linear governing equation, we used MATLAB for initial guesses and C coding for producing final results or numerical results.

**Key words:** LL mode, growth rate, perturbations, angle, rigid surface, viscosity ratio, Instability, surfactant mode.

# Introduction

Two Newtonian and incompressible liquid layered configurations is widely studied problem due to its large number of applications in lubrications, heat transfer [1] and in coatings [2]. It also has the applications in biological [3, 4] flows like blood flows in arteries and veins.

Many research papers related to the important effect of soluble or insoluble surfactant on the instability of various type of interfacial flow configurations have been studied [5, 6, 7, 8, 9, 10]. The numbers of research paper have been published on interfacial flow on inclined and vertical rigid surfaces. Flow in microfluidic devices and lung airways [11] have also been studied. In lung airways, researcher found the effect of natural surfactant and liquid layer lining on the liquid flow. The application of lung-airways is in disease like asthma attack. Time of asthma attack can be delayed by using up to certain percentages by using inherently present surfactant on the lung airways. Pozrikidis et al [3] also studied the problem concerning the fluid flow on inclined solid surfaces in presence of insoluble surfactant and they found the effect of surfactant is stabilizing on the marangoni mode as well as on GL mode perturbations. Wei [12] studied the problem of single liquid layer flowing down an inclined rigid surface in presence of insoluble surfactant on GL interface and in presence of shear stress at GL interface as well. Our research problem based on basic above study with

different configuration and effects. In this article, we have shown the effect of angle on the growth rate of perturbations of surfactant and LL mode for various viscosity ratios.

#### **Problem Formulation**

Two Newtonian and incompressible liquid layers are considered which fall down a rigid surface that is making some angle from horizontal. The viscosity of layer A and B is  $\mu_a$  and  $\mu_b$  and the density  $\rho_a$  and  $\rho_b$ respectively. The angle of inclination of rigid plane from horizontal is  $\theta$ . The surfactant which is introduced at LL interface has a concentration  $\Gamma^*$  (x, t) and surface tension is denoted by  $\gamma^*$ . This flow is governed by the equation of continuity and equation of motions and one constitutive equation for monolayer of insoluble surfactant which is present at LL interface.

$$\nabla^* \cdot \mathbf{v}^* = 0, \tag{1}$$

$$\rho[\partial_t^* \mathbf{v}^* + \mathbf{v}^* \cdot \nabla^* \mathbf{v}^*] = \nabla^* \cdot \mathbf{T}^* + \rho \mathbf{g}. \tag{2}$$

where  $\mathbf{v}^*$  and  $p^*$  are the velocity and pressure fields in the liquid layer;  $\mathbf{T}^* = -p^*\mathbf{I} + \mu[\mathbf{\nabla}^*\mathbf{v}^* + (\mathbf{\nabla}^*\mathbf{v}^*)^T]$  is the total stress tensor for the liquid layer.

Equation (1) is the continuity which generally based on the principle of conservation of mass menas mass neither created nor destroyed it can change or converted from one form to another form. This continuity equations shows the steady state condition and written for incompressible Newtonian fluid. Equations (ii) is the momentum conservation equation in tensor form. This equation can be expanded for all component like x, y and z. These two equations are the major equation which mainly governs the fluid flow.

## **Result and Discussion**

In this research paper, we study the effect of imposed shear stress on the growth rate of gas-liquid mode or GL instabilities. Here, we have verified and validated our code with Wei [12] and some other research paper as well. Results of this article are also matching with the analytical solution (not shown here) or long wave solution. Two liquid layers with different viscosities are flowing down an inclined surface in absence of interfacial insoluble surfactant and presence of shear stress at GL interface.

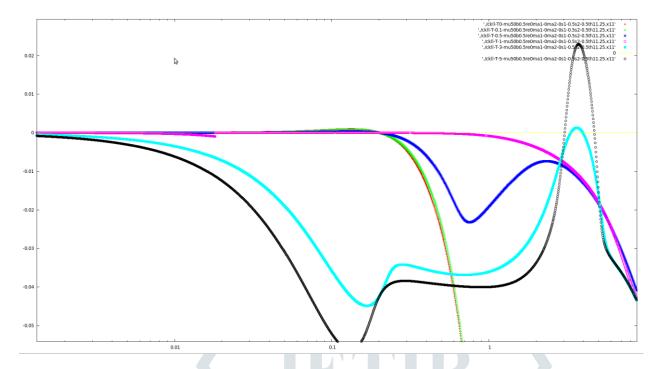


Figure 1: Plotted between growth rate (y-axis) and wavenumber (x-axis) for LL mode.

The figure 1 is plotted between growth rate of LL mode and wavenumber (k) for various values of imposed shear stress on GL interface. Remaining data are as follows:  $\mu_r = 50$ ,  $Ma_1 = 0$ ,  $Ma_2 = 0$ , Re = 0,  $\theta = 11.25^0$ . In this study we show the effect of imposed shear stress on the growth rate of LL mode instabilities for angle of inclination  $\theta = 11.25^0$  and viscosity ratio 50 that very high, it means lower liquid layer is less viscous than upper liquid layer. In this case, we consider fluid flows on inclined rigid surface instead of deformable solid surface. In absence of imposed shear stress both mode GL and LL remain stable in low wave range but becomes unstable at high wave range. On the other hand Marangoni mode remains stable for all wavenumber for Re = 0. Growth rate of LL mode instability decreases with increasing applied shear stress at GL interface but when the value of shear stress increases above a critical value than also LL mode remain stable in longwave range but suddenly becomes unstable in finite wave range. As a result we can predict that for generating stable mode we should keep the value of wave number low, high value of applied shear stress does not produce the stable solution in finite wave limit that is important and useful thing.

# Conclusion

We studied the Newtonian fluid flow down an inclined rigid surface in presence of shear stress at GL interface for high value of viscosity ration when we use two liquid layer flow in also presence of insoluble surfactant at gas-liquid interface. Here, we find that LL mode instabilities can be suppressed in low wave limit by increasing the value of shear stress at GL interface but LL mode becomes unstable in finite wave limit for creeping flow ( $Re \sim 0$ ). This research can be applied to produce smooth coating surface or reducing the instability in open inclined surface.

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