Study of Torsion of Rectangular Shaft Using Finite Element Analysis and Analytical Approach

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Abstract
A shaft is a structural member used to transmit torque. For most of the application, especially those involving high torque, shafts with circular cross-section are used as stress distribution is uniform along the radius in all the directions. However, in electronic devices, due to ease in power transmission and assembly shafts of non-circular sections, mostly square, are also used. This paper analyses the torsion of a rectangular shaft finite element method and compares it with analytical results.

1. Introduction
Non-circular shafts find their presence in stationary torque applications [1]. For transmitting power in machines and automobiles, circular shafts are used. However, there are many applications where non-circular members experience due to direct or indirect loading. Shafts used in motorized robotic arms, 3d printers are usually having square cross-section as the square cross-section makes the assembly simple. Chassis of automobiles is also made up of rectangular members and during turning, they also experience torsional moments. So, analysis of stress and deformation in non-circular members subjected to torque becomes an important aspect while designing them.

The torsion theory assumes that plane sections remain plain after deformation and this assumption holds true only for circular section. For members non-circular sections, the section deforms out of plane and the type of deformation is commonly referred to as warping. Figure 1 (a) and (b) shows the undeformed and deformed view of the rectangular view of the shaft when subjected to torque about the axis passing through the centroid.

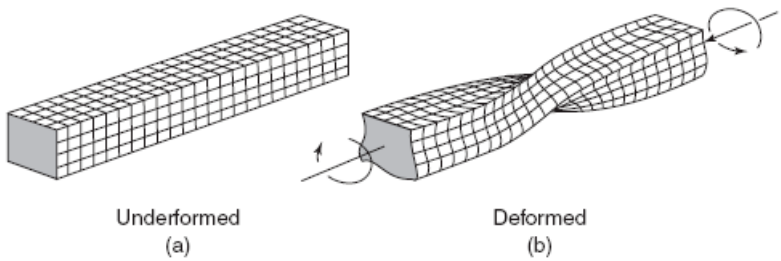

Figure 1 Deformation of a rectangular shaft [1]

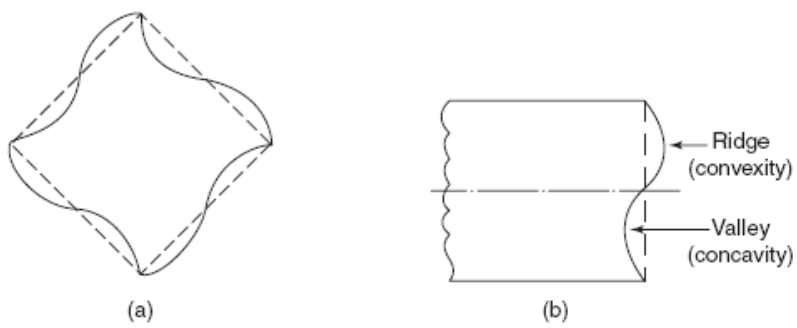

Figure 2 Changes in cross-section [1]

Figure 2 (a) and (b) shows different orthographic views of the section. Figure 2 (a) shows the deformation in plane of section and 2 (b) shows the deformation in a plane perpendicular to cross-section. As represented in Figure 2 (b) some point move out from the cross-sectional plane and move in the opposite direction. This type of deformation is called as warping.

This paper provides a comparative analysis between the finite element and analytical solution of a beam made up of a rectangular cross-section. For analytical solution, Saint Venant's Semiinverse method [2] and finite element analysis is performed using popular FE package Ansys Workbench [3].

2. Related Work

Design of shaft is based on torsion theory. The torsion of homogenous elastic members has been studies by many researchers [4]. The solution of irregular section using Fredholm integral equation on an artificial circle, which just encloses the bar's cross-section has been proposed by Chein-Shan Liu [5]. The author has used a meshless regularized integral equation method to attain the solution of the problem. Semi-analytical method along with conjugate gradient method has been used by author and using few Fourier terms calculations have been completed. Hematiyan and Doostfatemeh [6],[7] have presented an approach for torsional analysis of hollow non-circular sections based on Prandtl's stress function.

3. Finite Element Analysis

Popular FEA software Ansys Workbench Academic [3] has been used to carry out finite element modeling and determine stresses and deformation. The modeling has been done in Ansys Design Modeler where a box of specified size is made. The length of shaft has been taken along z-axis.

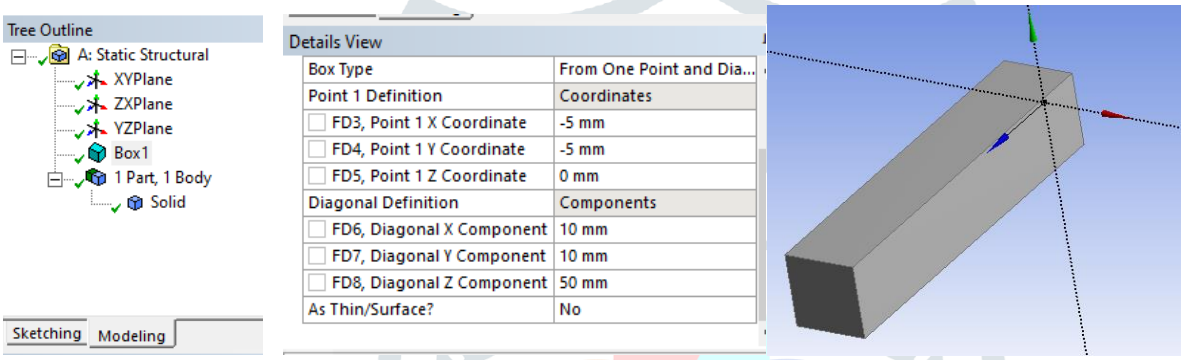

Figure 3 Modeling of Shaft

After modeling, meshing was performed using Cartesian mesh with linear and quadratic elements of different mesh sizes. The mesh size was chosen as 2 mm and 3 mm. One end of the shaft was fixed using fixed support option and other end was subjected to a moment about z-axis. The option of components was chosen for applying the moment.

Displacement along z-axis and shear stress result was requested during calculation. For obtaining the angular rotation, APDL commands were inserted. A remote point was inserted in the model and its position was extracted using APDL command

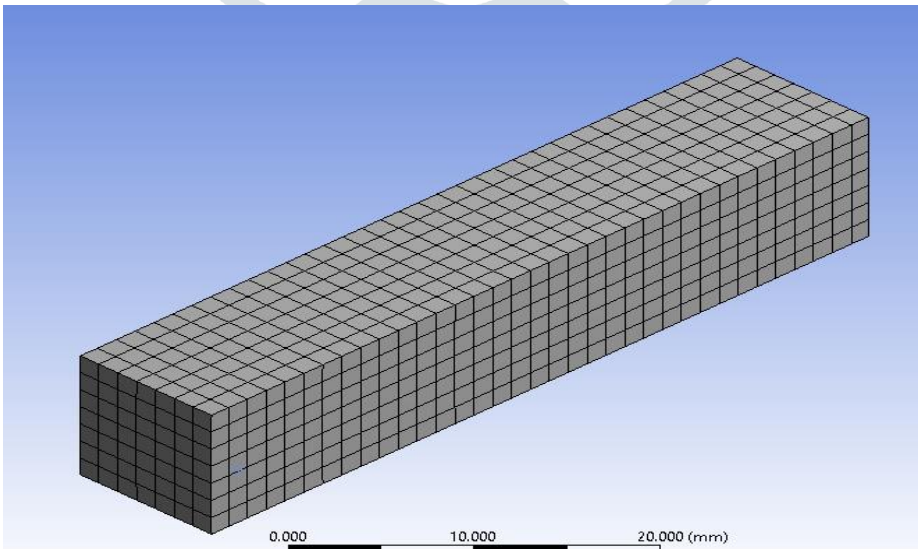

Figure 4 Discretized Geometry

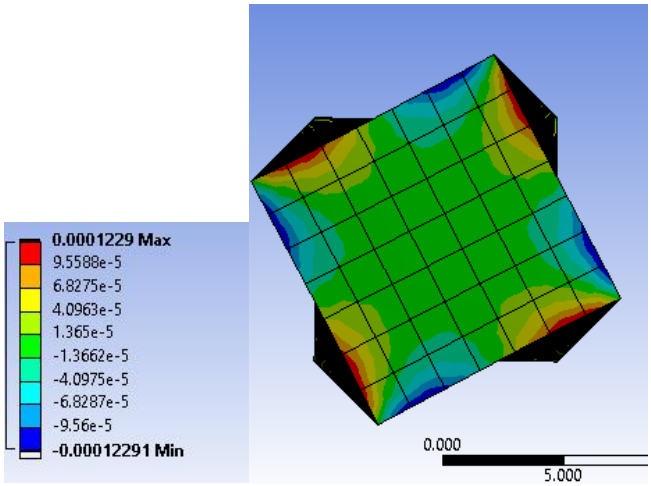

Figure 5 Displacement along z-direction (2-d view)

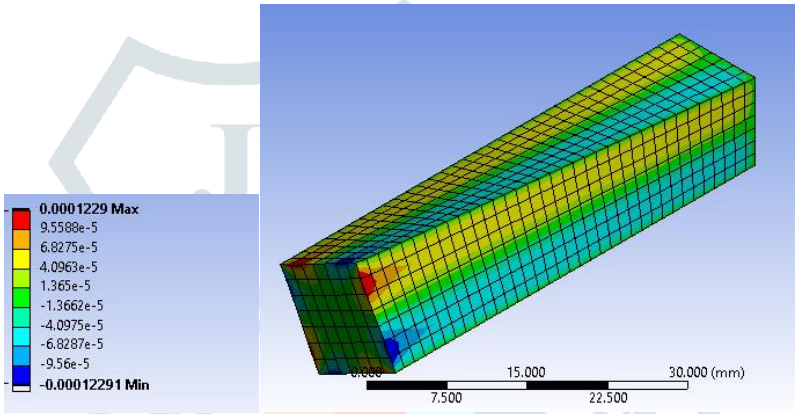

Figure 6 Displacement along z-direction (3-d view)

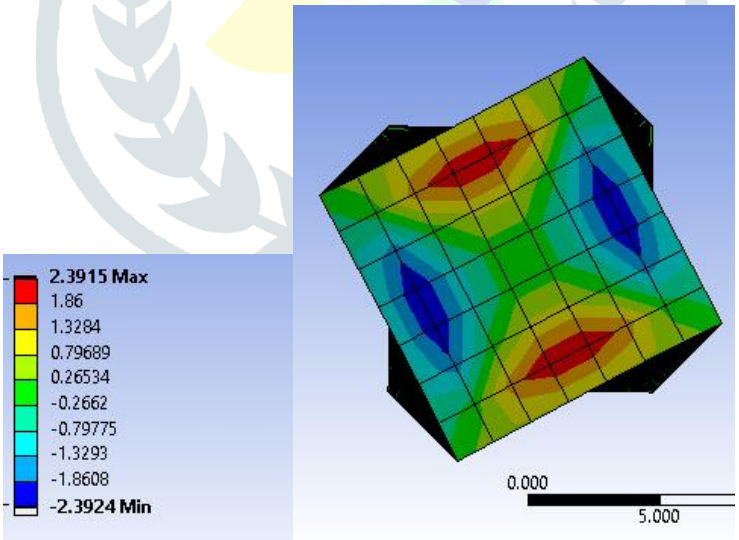

Figure 7 Shear Stress in the Cross-Section

4. Analytical Solution

The analytical solution is based on Semi-inverse method. For analysis, rectangular section as shown in Figure 9 is considered. The torsion problem is defined in terms of Prandtl Stress function as follow:

$$\nabla^2 \emptyset = -2G\theta$$

with the boundary condition $\emptyset = 0$ over the boundary. The solution [2] as provided for the shaft is:

$$J = k_1(2b)(2h)^3$$

$$\theta = \frac{T}{GJ}$$

$$\tau_{max} = 2G\theta h \frac{k_1}{k_2}$$

Substituting values for selected section,

2b = 10 mm = 0.01 m
2h = 10 mm = 0.01 m
G = 76.9 GPa = 79.6 × 10^9 Pa
T = 2 N-m
J = 1.41 × 10^{-9}
θ = 0.0184 rad/m
θ_{end} = 9.2226 × 10^{-4} rad = 0.0528°
τ = 0.48077 MPa

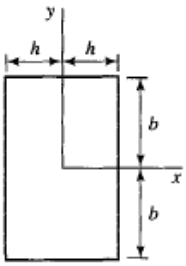

Figure 8 Cross-section of member

5. Results and Discussion

The results as obtained from Ansys Workbench are listed in table 1. As observed, there is some deviation in maximum stress in FEM result and analytical result. As mesh size is decreased, the value of displacement and stress is increased. Also use of higher order element result in higher value of displacement. Maximum displacement in z-direction is 0.12 µm when quad elements are used and 0.11 mm when linear elements are used.

Table 1 FEM Results

Element Size (mm)	Element Order	Max z Displacement (µm)	Max Displacement (µm)	Max Shear Stress (MPa)	Max Von Mises Stress (MPa)	Rotation (degree)
3	quadratic	0.1262	6.8318	1.0134	18.563	0.056
3	linear	0.0515	6.1851	0.143	15.305	0.050
2	quadratic	0.1229	6.7207	2.3915	18.031	0.055
2	linear	0.0981	6.5858	1.31	17.099	0.054
1	linear	0.1091	6.6319	1.867	17.887	0.054

The value of twist of end as obtained by FEM and analytical method is 0.055° for 2 mm quad, 0.054° for 1 mm linear and 0.052° respectively. The values of twist are approximately same. However, there is a significant difference between maximum shear stress obtained. For highest level of meshing done (1-mm linear and 2-mm quadratic), value of shear stress is 1.867 MPa and 2.39 MPa, which when compared to 0.48 MPa gives an error of 388% and 497%. The error in shear stress can be due to interpolation and computation resultant shear stress.

It is concluded that effect of mesh refinement is not significant when higher order elements are used. The twist determined by FEM is approximately same.

References

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