

THE NONSPLIT TREE DOMINATION NUMBER OF A GRAPH

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Abstract: Let $G = (V, E)$ be a connected graph. A subset D of V is called a dominating set of G if $N[D] = V$. The minimum cardinality of a dominating set of G is called the domination number of G and is denoted by $\gamma(G)$. A tree dominating set D of a graph G is a nonsplit tree dominating set (nstd - set) if the induced subgraph $\langle V - D \rangle$ is connected. The nonsplit tree domination number $\gamma_{\text{nstd}}(G)$ of G is the minimum cardinality of a nonsplit tree dominating set. The connectivity $\kappa(G)$ of G is the minimum number of vertices whose removal results in a disconnected or trivial graph. A partition $\{V_1, V_2, V_3, \dots, V_n\}$ of $V(G)$, in which each V_i is a nstd - set in G is called a nonsplit tree domatic partition of simply nstd - partition of G . The maximum order of a nstd - partition of G is called the nonsplit tree domatic number of G and is denoted by $d_{\text{nstd}}(G)$. In this paper, bounds for $\gamma_{\text{nstd}}(G)$ and its exact values for some particular classes of graphs and some special graphs are found and an upper bound for the sum of the nonsplit tree domination number and connectivity of a graph and bounds for $d_{\text{nstd}}(G)$ and its exact value for some particular classes of graphs are obtained.

Keywords: Domination number, connected domination number, split domination number, nonsplit domination number, tree domination number, domatic number, connectivity.

Mathematics Subject Classification: 05C69

1. INTRODUCTION

The graph $G = (V, E)$, we mean a finite, undirected, connected simple graph. The order and size of G are denoted by n and m respectively. If $D \subseteq V$, then $N(D) = \bigcup_{v \in D} N(v)$ and $N[D] = N(D) \cup D$ where $N(v)$ is the set of vertices of G which are adjacent to v . The connectivity $\kappa(G)$ of G is the minimum number of vertices whose removal results in a disconnected or trivial graph. A subset D of V is called a dominating set of G if $N[D] = V$. The minimum cardinality of a dominating set of G is called the domination number of G and is denoted by $\gamma(G)$. An excellent treatment of the fundamentals of domination is given in the book by Haynes et al. [2]. A survey of several advanced topics in domination is given in the book edited by Haynes et al. [3]. Sampathkumar and Walikar [8] introduced the concept of connected domination in graphs.

A dominating set D of G is called a connected dominating set, if the induced subgraph $\langle D \rangle$ is connected. The minimum cardinality of a connected dominating set of G is called the connected domination number of G and is denoted by $\gamma_c(G)$. Cockayne et al. Xuegang Chen, Liang Sun and Alice McRac [9] introduced the concept of tree domination in graphs. A dominating set D of G is called a tree dominating set, if the induced subgraph $\langle D \rangle$ is a tree. The minimum cardinality of a tree dominating set of G is called the tree domination number of G and is denoted by $\gamma_{tr}(G)$. A dominating set D of a graph G is a split dominating set if the induced subgraph $\langle V-D \rangle$ is disconnected. The split domination number $\gamma_{sd}(G)$ of G is the minimum cardinality of a split dominating set. A tree dominating set D of a graph G is a split tree dominating set if the induced subgraph $\langle V-D \rangle$ is disconnected. The split tree domination number $\gamma_{std}(G)$ of G is the minimum cardinality of a split tree dominating set. A dominating set D of a graph G is a non split dominating set if the induced subgraph $\langle V-D \rangle$ is connected. The non split domination number $\gamma_{nsd}(G)$ of G is the minimum cardinality of a non split dominating set. A tree dominating set D of a graph G is a split tree dominating set if the induced subgraph $\langle V-D \rangle$ is disconnected. The split tree domination number $\gamma_{std}(G)$ of a graph G equals the minimum cardinality of a split tree dominating set. A domatic partition of G is a partition $\{V_1, V_2, V_3, \dots, V_n\}$ of $V(G)$, in which each V_i is a dominating set of G . The maximum order of a domatic partition of G is called the domatic number of G and is denoted by $d(G)$.

In this paper, nonsplit tree domination number $\gamma_{nstd}(G)$ of a connected graph is defined and bounds for $\gamma_{nstd}(G)$ and its exact values for some particular classes of graphs and some special graphs are found and an upper bound for the sum of the nonsplit tree domination number and connectivity of a graph and bounds for $d_{nstd}(G)$ and its exact value for some particular classes of graphs are obtained.

2. PRIOR RESULTS

Theorem 2.1: [2] For any graph G , $\kappa(G) \leq \delta(G)$.

Theorem 2.2: [9] For any connected graph G with $n \geq 3$, $\gamma_{tr}(G) \geq n - 2$.

Theorem 2.3: [9] For any connected graph G with $\gamma_{tr}(G) \geq n - 2$ iff $G \cong P_n$ (or) C_n .

Theorem 2.4: [6] If T is a tree which is not a star, then, $\gamma_{ns}(G) \leq n - 2$.

3. MAIN RESULTS

In this section, a new parameter called nonsplit tree domination number is defined, bounds and some exact values of this parameter are found.

Definition 3.1:

A tree dominating set D of a graph G is a nonsplit tree dominating set if the induced subgraph $\langle V-D \rangle$ is connected. The nonsplit tree domination number $\gamma_{nstd}(G)$ of a graph G equals the minimum cardinality of a nonsplit tree dominating set.

The nonsplit tree domination number does not exist for some graphs. If the nonsplit tree domination number does not exist for a given connected graph G , then $\gamma_{nstd}(G)$ is defined to be zero.

Example 3.1:

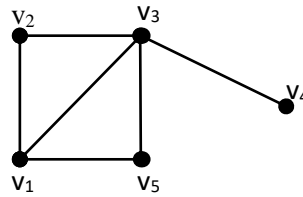


Figure 3.1.

In the graph given in Figure 3.1, minimum nonsplit tree dominating set is $\{v_3, v_4\}$ and $\gamma_{nstd}(G) = 2$.

Remarks 3.1:

1. Since $\langle D \rangle$ is a tree for any nstd - set D of a connected graph G , $|D| \geq 1$.
2. For any connected graph G , $\gamma(G) \leq \gamma_{nstd}(G)$, since every nstd - set is a dominating set. Further, every nonsplit tree dominating set D with $|D| \geq 1$ is a nonsplit connected dominating set.
3. The non split tree dominating set does not exist, when G is a path and corona graph of any connected graph.

Bounds and some exact values of nonsplit tree domination number:

Observation: 3.1

For any connected graph G , $\gamma(G) \leq \gamma_{nstd}(G)$.

Example: 3.2

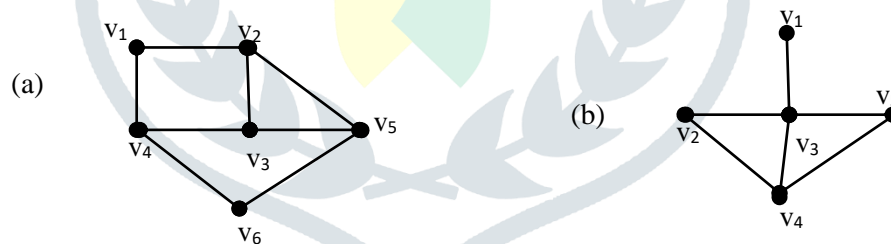


Figure 2.2

In the graph given in Figure 2.2(a), $D_1 = \{v_3, v_4\}$ is a γ - set and a γ_{nstd} - set and $\langle V - D_1 \rangle \cong P_4$. $\gamma(G) = \gamma_{nstd}(G) = 2$.

In the graph given in Figure 2.2 (b), $D_2 = \{v_1, v_3\}$ is a γ_{nstd} - set and $D_3 = \{v_3\}$ is a γ - set $\langle V - D_2 \rangle \cong P_3$. $\gamma(G) = 1$ and

$\gamma_{nstd}(G) = 2$. Therefore, $\gamma(G) < \gamma_{nstd}(G)$.

Observation: 3.2

For any spanning subgraph H of G , $\gamma_{nstd}(G) \leq \gamma_{nstd}(H)$. This is illustrated by following examples.

Example: 3.3



Figure 2.3

In the graph given in Figure 2.3, H is a spanning subgraph of G, $\gamma_{nstd}(G) \leq \gamma_{nstd}(H)$.

Example: 3.4



Figure 2.4

In the graph given in figure 2.4, $\gamma_{nstd}(G) = 1$ and $\gamma_{nstd}(H) = 1$

Remarks 3.2:

1. For any graph G, $\gamma(G) \leq \gamma_{ns}(G) \leq \gamma_{nstd}(G)$.
2. For any graph G, $\gamma(G) \leq \gamma_{tr}(G) \leq \gamma_{nstd}(G)$. There are illustrated by following examples.

Example: 3.5

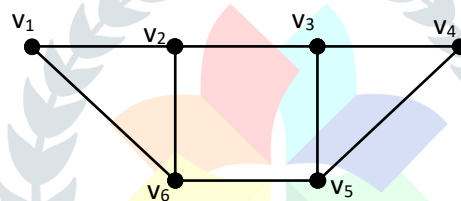


Figure 2.5

In the graph given in Figure 2.5, $\gamma(G) = \gamma_{ns}(G) = \gamma_{tr}(G) = \gamma_{nstd}(G) = 2$.

Example: 3.6

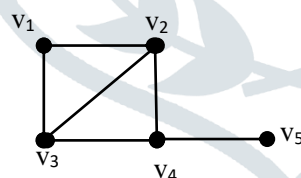


Figure 2.6

In the graph given in Figure 2.6, $\gamma(G) = 2$, $\gamma_{ns}(G) = 3$, $\gamma_{tr}(G) = 2$, $\gamma_{nstd}(G) = 3$. Here $\gamma(G) < \gamma_{ns}(G)$ and $\gamma_{tr}(G) < \gamma_{nstd}(G)$.

Theorem 3.1: For any connected graph G with n vertices, $\gamma_{nstd}(G) = 1$ if and only if $G \cong H+K_1$, where H is a connected graph with (n-1) vertices.

Proof:

Assume $G \cong H+K_1$. Then $D = V(K_1)$ is a nonsplit tree dominating set of G. Thus $\gamma_{nstd}(G) = 1$. Conversely, if $\gamma_{nstd}(G) = 1$, then there exist a nonsplit tree dominating set D of G, with $|D| = 1$. Since $\langle V-D \rangle$ is a connected graph and each vertex in $\langle V-D \rangle$ is adjacent to v in D. Therefore, $G \cong \langle V-D \rangle + K_1$. If the graph $\langle V-D \rangle$ is denoted by H, then $G \cong H+K_1$.

Remarks 3.3:

If G is one of the following graphs, then $\gamma_{\text{nstd}}(G) = 1$.

(1) The Diamond graph is a planar undirected cubic graph with 4 vertices and 5 edges. It contains of a complete graph K_4 minus one edge. For this diamond graph, nonsplit tree domination number is 1.

(2) For any wheel W_n ($n \geq 3$), $\gamma_{\text{nstd}}(W_n) = 1$.

(3) For the complete graph K_n with n vertices, $\gamma_{\text{nstd}}(K_n) = 1$.

Theorem 3.2: For any cycle C_n on n vertices, $\gamma_{\text{nstd}}(C_n) = n - 2$, $n \geq 3$.

Proof:

Let $V(C_n) = \{v, v_1, v_2, \dots, v_n\}$. Then $D = V(C_n) - \{v_{n-1}, v_n\}$ is a nonsplit tree dominating set of C_n . The induced subgraph $\langle D \rangle$ is a tree and $\langle V-D \rangle \cong P_2$. Therefore, $\gamma_{\text{nstd}}(C_n) = |D| = V(C_n) - \{v_{n-1}, v_n\} = n - 2$. Also a subset of $V(C_n)$ containing less than $n - 2$ vertices is not a nonsplit tree dominating set. Therefore, $\gamma_{\text{nstd}}(C_n) = n - 2$.

Theorem 3.3: $\gamma_{\text{nstd}}(\overline{C_n}) = 2$, for $n > 5$, where $\overline{C_n}$ is the complement of C_n .

Proof:

Let $V(\overline{C_n}) = \{v_1, v_2, \dots, v_n\}$. In $\overline{C_n}$, the set $\{v_i, v_{i+3}\}$, $1 \leq i \leq \lceil n/2 \rceil$, is a nonsplit tree dominating set. Thus, $\gamma_{\text{nstd}}(\overline{C_n}) = |D| = 2$.

Remark 3.4:

If $n = 5$, $G \cong \overline{C_5} \cong C_5$, then $\gamma_{\text{nstd}}(\overline{C_5}) = 3$.

Theorem 3.4: $\gamma_{\text{nstd}}(\overline{P_n}) = 2$, for $n \geq 5$, where $\overline{P_n}$ is the complement of P_n .

Proof:

Let $V(\overline{P_n}) = \{v_1, v_2, \dots, v_n\}$. In $\overline{P_n}$, there exists two adjacent vertices $\{v_1, v_n\}$ ($n \geq 5$) is a nonsplit tree dominating set and $\langle D \rangle$ is a tree and $\langle V-D \rangle$ is connected graph. Thus, $\gamma_{\text{nstd}}(\overline{P_n}) = |D| = 2$.

Remark 3.5:

If $G \cong P_n \circ K_1$, then $D = V(P_n)$ is a tree dominating set of G and $\langle D \rangle \cong P_n$ and $\langle V-D \rangle \cong nK_1$. Since $\langle V-D \rangle$ is a disconnected graph. Therefore, the non split tree dominating set does not exist.

Theorem 3.5: $\gamma_{\text{nstd}}(K_{r,s}) = 2$, $r, s \geq 2$.

Proof:

Let $G = K_{r,s}$. Let V_1, V_2 be a bipartition of G , such that $|V_1| = r$ and $|V_2| = s$. A non split tree dominating set is $D = \{u_1, v_1\}$. Also $V - D = \{u_1, u_2, \dots, u_m, v_1, v_2, \dots, v_n\}$. Here the induced sugraph $\langle V-D \rangle$ is connected. Therefore, $\gamma_{\text{nstd}}(K_{r,s}) = |D| = 2$.

The nonsplit tree domination number of some special graphs:

Definition 3.2: Frucht graph:

The *Frucht graph* is a 3- regular graph with 12 vertices and 18 edges and no non trivial symmetries.

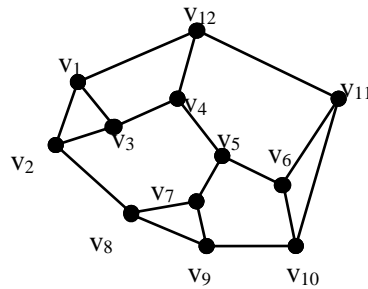


Figure 3.7

Theorem 3.6: If G is a Frucht graph, then $\gamma_{nstd}(G) = 5$.

Proof:

Let $V(G) = \{v_1, v_2, v_3, \dots, v_{12}\}$, the set $D = \{v_3, v_4, v_5, v_6, v_7\}$ is a dominating set and the induced subgraph $\langle V-D \rangle \cong C_n$. Therefore, the set D is a non split tree dominating set of G. Hence, $\gamma_{nstd}(G) = 5$.

Definition 3.3: Durer graph

The *Durer graph* is an undirected cubic graph with 12 vertices and 18 edges.

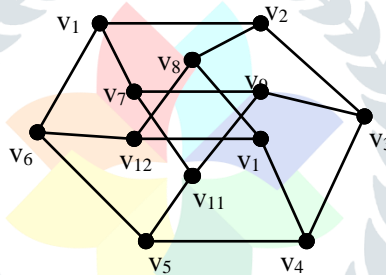


Figure 3.8

Theorem 3.7: If G is a Durer graph, then $\gamma_{nstd}(G) = 6$.

Proof:

Let $V(G) = \{v_1, v_2, v_3, \dots, v_{12}\}$, the set $D = \{v_1, v_2, v_4, v_5, v_8, v_{10}\}$ is a dominating set and the induced subgraph $\langle V-D \rangle \cong P_n$. Therefore, the set D is a non split tree dominating set of G. Hence, $\gamma_{nstd}(G) = 6$.

4. NONSPLIT TREE DOMINATION NUMBER AND CONNECTIVITY OF GRAPHS:

Theorem 4.1: Let G be a connected graph. Then $\gamma_{nstd}(G) + \kappa(G) = 2n - 3$ ($n \geq 3$) if and only if G is isomorphic to C_3 .

Proof:

By Theorem 2.1 and 3.2, $\gamma_{nstd}(G) + \kappa(G) \leq n - 2 + \delta(G) \leq n - 2 + n - 1 = 2n - 3$.

If $G \cong C_3$, then $\gamma_{nstd}(G) = 1$ and $\kappa(G) = 2$ and $\gamma_{nstd}(G) + \kappa(G) = 3 = 2n - 3$.

Conversely, assume $\gamma_{\text{nstd}}(G) + \kappa(G) = 2n - 3$, for $n \geq 3$. Then the following cases are to be considered.

- (i) $\gamma_{\text{nstd}}(G) = n$ and $\kappa(G) = n - 3$
- (ii) $\gamma_{\text{nstd}}(G) = n - 1$ and $\kappa(G) = n - 2$
- (iii) $\gamma_{\text{nstd}}(G) = n - 2$ and $\kappa(G) = n - 1$.

Since $\gamma_{\text{nstd}}(G) \leq n - 2$, the case (iii) alone be considered. $\gamma_{\text{nstd}}(G) = n - 2$ and $\kappa(G) = n - 1$.

$\gamma_{\text{nstd}}(G) = n - 2$ if and only if $G \cong C_n$ and $\kappa(C_n) = 2 = n - 1$. Therefore $G \cong C_3$.

Theorem 4.2: Let G be a connected graph. Then $\gamma_{\text{nstd}}(G) + \kappa(G) = 2n - 4$ ($n \geq 4$) if and only if G is isomorphic to C_4 and K_4 .

Proof:

Assume $\gamma_{\text{nstd}}(G) + \kappa(G) = 2n - 4$, $n \geq 4$. Then the following cases are to be considered.

- (i) $\gamma_{\text{nstd}}(G) = n$ and $\kappa(G) = n - 4$
- (ii) $\gamma_{\text{nstd}}(G) = n - 1$ and $\kappa(G) = n - 3$
- (iii) $\gamma_{\text{nstd}}(G) = n - 2$ and $\kappa(G) = n - 2$
- (iv) $\gamma_{\text{nstd}}(G) = n - 3$ and $\kappa(G) = n - 1$

There is no connected graph G with $\gamma_{\text{nstd}}(G) = n$, $\kappa(G) = n - 4$ and $\gamma_{\text{nstd}}(G) = n - 1$, $\kappa(G) = n - 3$.

Case(i): $\gamma_{\text{nstd}}(G) = n - 2 = \kappa(G)$

$\gamma_{\text{nstd}}(G) = n - 2$ if and only if $G \cong C_n$ and $\kappa(C_n) = 2 = n - 2$, implies $n = 4$. Therefore, $G \cong C_4$.

Case(ii): $\gamma_{\text{nstd}}(G) = n - 3$ and $\kappa(G) = n - 1$

If $\kappa(G) = n - 1$, then $G \cong K_n$, $n \geq 3$. But $\gamma_{\text{nstd}}(K_n) = 1 = n - 3$, implies $n = 4$. Therefore $G \cong K_4$.

Theorem 4.3: Let G be a connected graph. Then $\gamma_{\text{nstd}}(G) + \kappa(G) = 2n - 5$ ($n \geq 5$) if and only if $G \cong C_5$, K_5 and K_{4-e} .

Proof:

Assume $\gamma_{\text{nstd}}(G) + \kappa(G) = 2n - 5$, $n \geq 5$. Then the following cases are to be considered.

- (i) $\gamma_{\text{nstd}}(G) = n$ and $\kappa(G) = n - 5$
- (ii) $\gamma_{\text{nstd}}(G) = n - 1$ and $\kappa(G) = n - 4$
- (iii) $\gamma_{\text{nstd}}(G) = n - 2$ and $\kappa(G) = n - 3$
- (iv) $\gamma_{\text{nstd}}(G) = n - 3$ and $\kappa(G) = n - 2$
- (v) $\gamma_{\text{nstd}}(G) = n - 4$ and $\kappa(G) = n - 1$

There is no connected graph G with $\gamma_{\text{nstd}}(G) = n$, $\kappa(G) = n - 5$ and $\gamma_{\text{nstd}}(G) = n - 1$, $\kappa(G) = n - 4$.

Case(i): $\gamma_{\text{nstd}}(G) = n - 2$ and $\kappa(G) = n - 3$

$\gamma_{\text{nstd}}(G) = n - 2$ if and only if $G \cong C_n$ and $\kappa(C_n) = 2 = n - 3$, implies $n = 5$. Therefore G is isomorphic to C_5 .

Case(ii): $\gamma_{\text{nstd}}(G) = n - 3$ and $\kappa(G) = n - 2$

Since $\kappa(G) \leq \delta(G)$, $\delta(G) \geq n - 2$. If $\delta(G) > n - 2$, then $G \cong K_n$, $n \geq 3$. Therefore $\gamma_{\text{nstd}}(G) = 1 = n - 3$, which gives $n = 4$. Thus $G \cong K_4$. But $\kappa(G) = 3 \neq n - 2$. Assume $\delta(G) = n - 2$. Then G is isomorphic to $K_n - Y$ where Y is a matching in K_n , $n \geq 3$ and $\gamma_{\text{nstd}}(G) \leq 2$.

If $\gamma_{\text{nstd}}(G) = 2 = n - 3$ then $n = 5$. Therefore $G \cong K_5 - e$, $K_5 - 2e$. If $G \cong K_5 - e$, then $\gamma_{\text{nstd}}(G) = 1 \neq n - 3$. If $G \cong K_5 - 2e$, then $\gamma_{\text{nstd}}(G) = 1 \neq n - 3$. If $\gamma_{\text{nstd}}(G) < 2$, then $n = 4$.

Therefore $G \cong K_4 - e$, C_4 . If $G \cong K_4 - e$, then $\gamma_{\text{nstd}}(G) = 1 = n - 3$ and $\kappa(G) = 2 = n - 2$. If $G \cong C_4$, then $\gamma_{\text{nstd}}(G) = 2 \neq n - 3$.

Case(iv): $\gamma_{\text{nstd}}(G) = n - 4$ and $\kappa(G) = n - 1$

If $\kappa(G) = n - 1$, then $G \cong K_n$, $n \geq 3$. Therefore $\gamma_{\text{nstd}}(G) = 1 = n - 4$, which gives $n = 5$.

Thus $G \cong K_5$.

5. NONSPLIT TREE DOMATIC NUMBER

In this section a new parameter known as nonsplit tree domatic number of a connected graph is defined and studied.

Definition 5.1:

A partition $\{V_1, V_2, V_3, \dots, V_n\}$ of $V(G)$, in which each V_i is a tr - set in G is called a tree domatic partition of simply tr- partition of G . The maximum order of a tr- partition of G is called the tree domatic number of G and is denoted by $d_{\text{tr}}(G)$.

Definition 5.2:

A partition $\{V_1, V_2, V_3, \dots, V_n\}$ of $V(G)$, in which each V_i is a nsd - set in G is called a nonsplit domatic partition of simply nsd - partition of G . The maximum order of a ns - partition of G is called the nonsplit domatic number of G and is denoted by $d_{\text{nsd}}(G)$.

Definition 5.3:

A partition $\{V_1, V_2, V_3, \dots, V_n\}$ of $V(G)$, in which each V_i is a nstd -set in G is called a nonsplit tree domatic partition of simply nstd - partition of G . The maximum order of a nstd - partition of G is called the nonsplit tree domatic number of G and is denoted by $d_{\text{nstd}}(G)$.

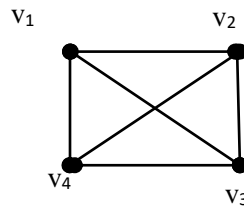
Example 5.1:

Figure 5.1

In the graph given in Figure 5.1, $D_1 = \{v_1\}$, $D_2 = \{v_2\}$, $D_3 = \{v_3\}$, $D_4 = \{v_4\}$ and $d_{\text{nstd}}(G) = 4$.

Remarks 5.1:

- 1) Since any nonsplit tree domatic partition of G is also a nonsplit domatic partition of G , $d_{\text{ns}}(G) \leq d(G)$ and $d_{\text{nstd}}(G) \leq d(G)$ and $d_{\text{ns}}(G) \leq d_{\text{nstd}}(G) \leq d(G)$.
- 2) Let $u \in V(G)$ and $d(u) = \delta$. Let $\{V_1, V_2, V_3, \dots, V_{d_{\text{nstd}}}\}$ be a nonsplit tree domatic partition of G . Since $|V_k| \geq \gamma_{\text{nstd}}(G)$ for each k , $\gamma_{\text{nstd}}(G) \cdot d_{\text{nstd}}(G) \leq n$.

Example 5.2:

- 1) In wheel graph W_6 , $d_{\text{nstd}}(G) < d(G)$.
- 2) In cycle C_4 , $\gamma_{\text{nstd}}(G) \cdot d_{\text{nstd}}(G) = n$.

Theorem 5.1: If $\gamma_{\text{nstd}}(G) > 0$, then $d_{\text{nstd}}(G) \leq \frac{n}{\gamma_{\text{nstd}}(G)}$ and the bound is sharp.

Proof:

Let $\{D_1, D_2, \dots, D_k\}$ is a partition of $V(G)$ into k nonsplit tree dominating sets, such that $d_{\text{nstd}}(G) = k$. Since each $\langle D_i \rangle$ is a nonsplit tree dominating set, it follows that $\gamma_{\text{ntr}}(G) \leq |D_i|$ for $1 \leq i \leq k$.

$$\text{Thus, } n = \sum_{i=1}^k |D_i| \geq \gamma_{\text{ntr}}(G) \cdot k$$

$$d_{\text{nstd}}(G) \leq \frac{n}{\gamma_{\text{nstd}}(G)}$$

Observation 5.1:

- 1) $d_{\text{ntr}}(W_n) = 2$, $n \geq 4$.
- 2) $d_{\text{ntr}}(K_{m,n}) = \min\{m, n\}$, $m, n \geq 1$.
- 3) $d_{\text{ntr}}(K_n) = n - 1$, $n \geq 3$.

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