Complementary Tree Domination in Jahangir Graph $J_{2,m}$

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Abstract

A set *D* of a graph $G = (V,E)$ is a dominating set if every vertex in $V - D$ is adjacent to some vertex in *D*. The domination number $\gamma(G)$ of *G* is the minimum cardinality of a dominating set. A dominating set *D* is called a complementary tree dominating set if the induced sub graph $< V - D >$ is a tree. The minimum cardinality of a complementary tree dominating set is called the complementary tree domination number of *G* and is denoted by $\gamma_{ctd}(G)$. In this paper, results on complementary tree domination number γ_{ctd}, total complementary tree domination number γ_{tctd} and connected complementary tree γ_{cctd} in Jahangir graph $J_{2,m}$ are found

Keywords: Complementary tree domination number, total, connected complementary tree domination number, Jahangir graphs.

1 Introduction

Graphs discussed in this paper are undirected and simple graphs. For a graph *G*, let *V(G)* and *E(G)* denote its vertex set and edge set respectively. The concept of domination was first studied by ore [6]. A set $D \subseteq V$ is said to a dominating set of *G*, if every vertex in $V - D$ is adjacent to some vertex in *D*. The minimum cardinality of a dominating set is called the domination number of *G* and is denoted by $\gamma(G)$. The concept of complementary tree domination was into deiced by S. Muthammai M. Bhanumathi and P. Vidhya in [5]. A dominating set $D \subseteq V$ is called a complementary tree dominating (ctd) set, if the sub graph $< V - D >$ is a tree. The minimum cardinality of a complementary tree dominating set is called the complementary tree domination number of *G* and is denoted by $\gamma_{ctd}(G)$.

A dominating set D_t is called a total complementary tree dominating set of every vertex $v \in V$ is adjacent to an element of D_t and $< V - D_t >$ is a tree. The minimum cardinality of a total complementary tree dominating set (*tctd*) is called the total complementary tree domination number of *G* and is denoted $\gamma_{tctd}(G)$.

A dominating set D_c is called complementary tree dominating set (*cctd*) if the induced sub graph $< D_c >$ is connected. The connected complementary tree domination number $\gamma_{cctd}(G)$ is the minimum cardinality of *cctd* set. In this paper, complementary tree domination number(γ_{ctd}), total and connected complementary tree domination number in Jahangir graphs are found

Definition 1 *Jahangir graphs $J_{n,m}$ for $m \geq 3$, is a graph on $nm + 1$ vertices i.e., a graph consisting of a cycle C_{nm} with one additional vertex which is adjacent to m vertices of C_{nm} distance n to each other on C_{nm}.*

Example 1 *Figure 1 shows Jahangir graph $J_{2,8}$. It appears on Jahangir's tomb in his mausoleum. It lies in 5 kilometer north-wsit of Lahore, Pakistan across the river Ravi [1].*

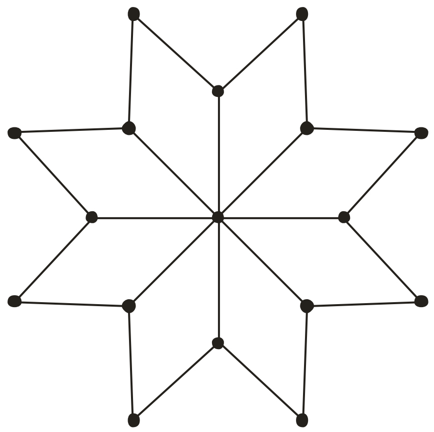

Figure 1: $J_{2,8}$

Example 2

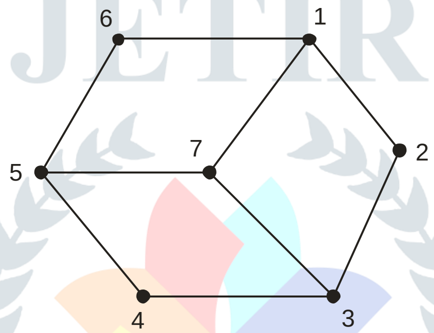

Figure 2: $J_{2,m}$

In Figure 2 $\gamma_{ctd}(J_{2,m}) = 2$ where $D = \{3, 6\}$.

2 Complementary Tree Domination Number, Total and Connected Domination Number of $J_{2,m}$

In this section, we study γ_{ctd}, γ_{tctd} and γ_{cctd} in Jahangir graphs $J_{2,m}$.

Remark 1 *Let v_{2m+1} be the label of the center vertex and $v_1, v_2, \ldots, v_{2m}$ be the label v of the vertices that incident clockwise on cycle C_{2m} so that $deg(v_1) = 3$.*

Theorem 2.1 For $m \geq 3$ the γ_{ctd}, $(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil$

Proof. Let v_{2m+1} be the label of the center vertex and $v_1, v_2, \ldots, v_{2m}$ be the label of the vertices that incident clockwise on cycle C_{2m} so that $deg(v_1) = 3$. Let D be a minimum *ctd* set of $J_{2,m}$. Therefore $|D| = \gamma_{ctd}(J_{2,m})$.

Case (i) *$2m = 0 \pmod 3$*

Let $D = \{v_3, v_6, v_9, \ldots, v_{3i}\}$ where $i = 1, 2, \ldots, m$ Then D is a minimum ctd-set of $J_{2,m}$ and $3i = 2m$. Since v_3 dominates v_2 and v_4, v_6 dominates v_5 and v_7 etc and v_{3i} dominates v_{3i-1} and v_1 and odd label vertices dominated by the central vertex v_{2m+1}. Also $< V - D > \cong T$, Where T is a tree.

$$\text{Therefore } |D| = \gamma_{ctd}(G) = \left\lceil \frac{2m}{3} \right\rceil$$

Let $D = [v_1, v_4, \ldots., v_{3i-2}]$ is a minimum ctd-set of $J_{2,m}$.

If $3i - 2 = 2m$, then $|D| = \gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil$

If $3i - 2 \neq 2m$, then v_{2m} is a isolate vertex in $< V - D >$ so that $v_{2m} \in D$.

$$\text{Therefore } |D| = \left\lceil \frac{2m}{3} + 1 \right\rceil$$

$$= \left\lceil \frac{2m}{3} \right\rceil + 1$$

Which contradicts the minimality

Let $D = [v_2, v_5, \ldots., v_{3i-1}]$ is a minimum ctd-set of $J_{2,m}$.

If $3i - 1 = 2m$, then $|D| = \gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil$

If $3i - 2 \neq 2m$, then v_{2m} is a dominated by v_{3i-1} and v_1 is dominated by v_2

$$\text{Therefore } |D| = \gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil$$

Case (ii) *2m = 1 (mod 3)*

Let $D = \{v_3, v_6, \ldots., v_{3i}\}$ is a minimum ctd-set of $J_{2,m}$. Hence v_1 is not dominated by D so that $v_{2m} \in D$.

$$\text{Therefore } |D| = \left\lceil \frac{2m}{3} + 1 \right\rceil$$

$$= \left\lceil \frac{2m}{3} \right\rceil$$

Let $D = [v_1, v_4, v_7 \ldots., v_{3i-2}]$ is a minimum ctd-set of $J_{2,m}$.

If $3i - 2 = 2m$, then D is a minimal ctd-set of $J_{2,m}$ then $|D| = \gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil$

If $3i - 2 \neq 2m$, then v_{2m} is a isolate in $<V - D>$, so that $v_{2m} \in D$.

$$\text{Therefore } |D| = \left\lceil \frac{2m}{3} + 1 \right\rceil$$

$$= \left\lceil \frac{2m}{3} \right\rceil + 1$$

Which contradicts the minimality

Let $D = [v_2, v_5, v_8, \ldots, v_{3i-1}]$

If $3i - 1 = 2m$, then $|D| = \left\lceil \frac{2m}{3} \right\rceil$

If $3i - 2 \neq 2m$, then v_{2m} is a dominated by v_{3i-1} and v_1 is dominated by v_2

$$\text{Therefore } |D| = \left\lceil \frac{2m}{3} \right\rceil$$

Case (iii) *2m = 1 (mod 3)*

Let $D = \{v_1, v_4, \ldots, v_{3i-2}\}$

If $3i - 2 = 2m$ or $3i - 2 = 2m - 2$ then $|D| = \left\lceil \frac{2m}{3} \right\rceil$

Let $D = \{v_2, v_3, \ldots, v_{3i-1}\}$

If $3i - 1 = 2m$ or $3i - 1 = 2m - 1$ then $|D| = \frac{2m}{3} = \left\lceil \frac{2m}{3} \right\rceil$

From the above three cases

$$\gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{3} \right\rceil, \text{ for } m \geq 3$$

Total Complementary tree domination of $J_{2,m}$

Theorem 2.2 For m>3, $\gamma_{tctd}(J_{2,m}) > \gamma_{ctd}(J_{2,m})$

Proof:

Let D_t be a minimum tctd-set of $J_{2,m}$.

Case (i) *m is odd*

Let $D_t = \{v_1, v_2, v_5, v_6, \ldots, v_{2m-1}, v_{2m}\}$

$$\therefore |D_t| = m + 1$$

$$i.e., \quad = \frac{2m}{2} + 1$$

$$= \left\lceil \frac{2m}{2} \right\rceil + 1$$

$$= \left\lceil \frac{2m}{2} \right\rceil$$

Case (ii) *m is even*

Let $D_t = \{v_1, v_2, v_5, v_6, \ldots, v_{2m-1}, v_{2m}\}$

$$\therefore |D_t| = m$$

$$= \left\lceil \frac{2m}{2} \right\rceil$$

Which is minimum tctd set of $J_{2,m}$.

$$\therefore \gamma_{ctd}(J_{2,m}) = \left\lceil \frac{2m}{2} \right\rceil > \left\lceil \frac{2m}{3} \right\rceil$$

$$= \gamma_{ctd}(J_{2,m})$$

Connected complementary tree domination of $J_{2,m}$.

Theorem 2.3 For $m \geq 3$, $\gamma_{cctd}(J_{2,m}) = 2m - 2$

Proof. We know $\gamma_{cctd}(C_m) = m - 2 \text{ for } m \geq 3$

Let $D_c = \{v_1, \ldots, v_{2m-3}, v_{2m+1}\}$. Where D_c is the minimum ctd-set of c.

Here v_{2m} is dominate by v_1, v_{2m-3} dominate v_{2m-2} and v_{2m-1} is dominate by v_{2m+1}.

$$\therefore \quad |D_c| = 2m - 3 + 1 = 2m - 2$$

$$\therefore \gamma_{cctd}(J_{2,m}) = 2m - 2$$

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