MAX- MIN MATRICES USING FERMAT NUMBER

Dr. N. Elumalai[1], Mrs. R. Muthamizh Selvi[2],
[1]Associate professor, [2]Assistant Professor
[1]Department of Mathematics, A.V.C. College (Autonomous), Mannampandal, TamilNadu.
[2]Department of Mathematics, Saradha Gangadharan Arts & Science College, Puducherry.

Abstract:

Let T be a finite number of multiple set of real numbers taken as increasing order of numbers. The purpose of this article is to study the different properties of MIN matrix and MAX matrix of the set T with min (x_i, x_j) and max (x_i, x_j) as their (i, j) entries, respectively. We are going to do this by interpreting these matrices as Fermat max and min matrices and applying the determinant formulae and the inverse formulae for Fermat MIN matrices and Fermat MAX matrices.

Keywords:

MIN matrix, MAX matrix, meet matrix, join matrix, Fermat min matrix and Fermat max matrix.

1 Introduction:

MIN and MAX matrices are simple-structured matrices that appear in many contexts in mathematics and statistics. As is pointed out in the next section, in some cases MIN matrices have an interpretation as covariance matrices of certain stochastic processes. Bhatia [1] shows that the MIN matrix [min(i, j)] is infinitely divisible, and in [2] he gives a more comprehensive treatment to this subject. Moyé studies the covariance matrix of Brownian motion, which appears to be a certain MIN matrix. Motivated by Moyé's work, Neudecker, Trenkler and Liu [3] defined a more general matrix

$$A = \begin{bmatrix} a_1 & a_1 & a_1 & & a_1 \\ a_1 & a_2 & a_2 & & a_2 \\ a_1 & a_2 & a_3 & & a_3 \\ & & \vdots & & \\ & & \vdots & & \\ & & \vdots & & \\ a_1 & a_2 & a_3 & & a_n \end{bmatrix}$$

(a_i are real numbers for all i = 1, . . . , n), and proposed the following problems:

– find a necessary and sufficient condition for A to be positive definite;
– find the determinant of A;
– find the inverse of A when A is nonsingular.

Pierre de Fermat [4] conjectured that all numbers $F_m = 2^{2^m} + 1$ for $m = 0, 1, 2, \ldots$ are prime. Nowadays we know that the first five members of this sequence are prime and that F_m is composite for $5 \le m \le 30$. The numbers F_m are called Fermat numbers. If F_m is prime, we say that it is a Fermat prime.

Until 1796 Fermat numbers were most likely a mathematical curiosity. The interest in the Fermat primes dramatically increased when C. F.Gauss [5] stated that there is a remarkable connection between the Euclidean construction (i.e., by ruler and compass) of regular polygons and the Fermat numbers. In particular, he proved that if the number of sides of a regular polygon is of the form $2^k F_{m_1} \ldots F_{m_r}$ where $k \ge 0$, $r \ge 0$, and F_{m_i} are distinct Fermat primes, then this polygon can be constructed by ruler and compass. The converse statement was established later by Wantzel.

As we are going to see, there is a very natural and straight forward way to interpret MIN and MAX matrices as meet and join matrices, whose properties are well studied. On the other hand, because of the simple structure of MIN and MAX matrices it is easy to apply basically any result related to meet and join matrices to MIN and MAX matrices. At the same time we give some thoughts about how difficult it would be to verify these formulas by using only elementary linear algebra. The reader is also very welcome to amuse herself/himself by trying to answer the same question.

2 Preliminaries:

We begin by presenting the definition of MIN and MAX matrices.
Let T = $\{x_1, x_2, x_3, \dots\dots\dots, x_n\}$ be a finite multiple set of real numbers, where $x_1 \le x_2 \le \dots \le x_n$ (in some cases, however, we need to assume that $x_1 < x_2 < \cdots < x_n$). The MIN matrix $(T)_{min}$ of the set T has min (x_i, x_j) as its (i,j) entry, whereas the MAX matrix of the set T has max (x_i, x_j) as its (i,j) entry and is denoted by $[T]_{max}$. Both matrices are clearly square and symmetric and they may be written explicitly as

$$(T)_{min} = \begin{bmatrix} x_1 & x_1 & x_1 & & x_1 \\ x_1 & x_2 & x_2 & & x_2 \\ x_1 & x_2 & x_3 & & x_3 \\ & & \vdots & & \\ & & \vdots & & \\ & & \vdots & & \\ x_1 & x_2 & x_3 & & x_n \end{bmatrix} \text{ and } [T]_{max} = \begin{bmatrix} x_1 & x_2 & x_3 & & x_n \\ x_2 & x_2 & x_3 & & x_n \\ x_3 & x_3 & x_3 & & x_n \\ & & \vdots & & \\ & & \vdots & & \\ & & \vdots & & \\ x_n & x_n & x_n & & x_n \end{bmatrix}$$

2.1 Remark:

Here it is convenient to assume that the elements of T are listed in increasing order, since this assumption does not affect most of the basic properties of the matrices $(T)_{min}$ and $[T]_{max}$. Rearranging the indexing of the elements of the set T corresponds to multiplying the matrices $(T)_{min}$ and $[T]_{max}$ from left by a certain permutation matrix Q and from right by the matrix Q^T. Properties like determinant and positive definiteness remain invariant in this operation.
An interesting special case of MIN matrices is obtained by setting T = {1, 2, . . . n}. In this case we have

$$(T)_{min} = \begin{bmatrix} 1 & 1 & 1 & & 1 \\ 1 & 2 & 2 & & 2 \\ 1 & 2 & 3 & & 3 \\ & & \vdots & & \\ & & \vdots & & \\ & & \vdots & & \\ 1 & 2 & 3 & & n \end{bmatrix} \text{ and } [T]_{max} = \begin{bmatrix} 1 & 2 & 3 & & n \\ 2 & 2 & 3 & & n \\ 3 & 3 & 3 & & n \\ & & \vdots & & \\ & & \vdots & & \\ & & \vdots & & \\ n & n & n & & n \end{bmatrix}$$

The matrix $(T)_{min}$ is, up to a positive scalar, the covariance matrix of a stochastic process with increments which possess the same variance and are uncorrelated. Bhatia provided six alternative proofs for its positive definiteness. This same matrix is also studied in a recent book about matrices in statistics, see [6]. Next we review some basic concepts of lattice theory. A partially ordered set (poset) is a pair (P, ⪯), where P is a nonempty set and ⪯ is a reflexive, antisymmetric and transitive relation. A closed interval [x, y] in P is the set

$$[x, y] = \{z \in P / x \preceq z \preceq y\}, \qquad x, y \in P.$$

Poset (P, ⪯) is said to be locally finite if the interval [x, y] is finite for all x, y ∈ P. Poset (P, ⪯) is a chain if x ⪯ y or y ⪯ x for all x, y ∈ P. A lattice is a poset, where the infimum x ∧ y and the supremum x ∨ y exist for all x, y ∈ P. It is easy to see that every chain is a lattice with x ∧ y = min(x, y) and x ∨ y = max(x, y).

For example, the set of real numbers equipped with the usual ordering is a lattice and a chain, but it is not locally finite. The set of positive integers equipped with the divisibility relation is a locally finite lattice with x ∧ y = gcd (x, y) and x ∨ y = lcm (x, y), but this poset is not a chain.

Next we need to define meet and join matrices. Let (P, ⪯) be a locally finite lattice. Moreover, let S = $\{x_1, x_2, x_3, \dots\dots\dots, x_n\}$ be a finite subset of P with distinct elements such that $x_i \preceq x_j \Rightarrow i \le j$ (in other words, the indexing of the elements $x_i \in S$ is a linear extension[7]). Finally, let f be a function on P to R (or to C). The meet matrix $(S)_f$ of the set S with respect to the function f is the n × n matrix with f $(x_i \wedge x_j)$ as its (i,j) entry. Similarly, the join matrix $[S]_f$ of the set S with respect to f is the n × n matrix with f $(x_i \vee x_j)$ as its (i,j) entry.

Like MIN and MAX matrices, meet and join matrices are square and (complex) symmetric as well. A proper way to describe meet and join matrices might be to say that in meet and join matrices the entries are determined partly by the function f and partly by the set S and the underlying lattice structure (P, ⪯).

3 Some important results for meet and join matrices:

In our study of MIN and MAX matrices we are going to make use of a couple of known results for meet and join matrices. The first one is about the structure of $(S)_f$. For any two subsets S = $\{x_1, x_2, \dots\dots\dots x_n\}$ and T = $\{y_1, y_2, \dots\dots\dots y_m\}$ of P, let E(S,T) = (e_{ij}) denote the nxm incidence matrix defined as

$$e_{ij} = \begin{cases} 1 & \text{if } y_j \leqslant x_i \\ 0 & \text{otherwise.} \end{cases}$$

Proposition 3.1.

[8], Let T = $\{y_1, y_2, \ldots \ldots \ldots y_m\}$ be a meet closed subset of P containing S = $\{x_1, x_2, \ldots \ldots \ldots x_n\}$ (m≥n). Then

$$(S)_f = E\wedge E^T = AA^T,$$

where E = E(S,T), ∧ = diag $\left(\psi_{T,f}(y_1), \ldots \ldots \ldots \psi_{T,f}(y_m)\right)$, A = $E\wedge^{\frac{1}{2}}$ and $\psi_{T,f}$ is defined recursively as

$$\psi_{T,f}(y_j) = f(y_j) - \sum_{y_j < y_j} \psi_{T,f}(y_j)$$

The main idea of this factorization can be generalized for join matrices and even for meet and join matrices on two sets. Furthermore, the other things, to find the following determinant and inverse formulas for meet and join matrices. In Propositions 3.3 and 3.5 the function $\Phi_{S,f}$ is again the Möbius inversion of f, but in this case the inversion is executed from above. In other words,

$$\Phi_{S,f}(x_k) = f(x_k) - \sum_{x_k < x_v} \Phi_{S,f}(x_v).$$

Proposition 3.2.

If S is meet closed [9], then

$$\det(S)_f = \prod_{v=1}^{n} \psi_{S,f}(x_v) = \prod_{v=1}^{n} \sum_{\substack{z \leqslant x_v \\ z \nleq x_t \\ t < v}} \sum_{w \leqslant z} f(w)\mu_P(w,z).$$

Proposition 3.3.

If S is join closed [10], then

$$\det[S]_f = \prod_{v=1}^{n} \sum_{x_v < x_t} f(x_t)\mu_s(x_v, x_t) = \prod_{v=1}^{n} \sum_{\substack{x_v \leqslant z \\ x_t \nleq z \\ v < t}} \sum_{z \leqslant w} f(w)\mu_P(z,w).$$

Proposition 3.4.

Suppose that S is meet closed [9]. If $(S)_f$ is invertible, then the inverse of $(S)_f$ is the n × n matrix B = (b_{ij}), where

$$b_{ij} = \sum_{k=1}^{n} \frac{(-1)^{i+j}}{\psi_{S,f}(x_v)} \det E(S_i^k) \det E(S_j^k)$$

Where $E(S_i^k)$ is the (n − 1) × (n − 1) sub matrix of E(S) obtained by deleting the i^th row and the k^th column of E(S), or

$$b_{ij} = \sum_{x_i \vee x_j \leqslant x_k} \frac{\mu_S(x_i, x_j)\mu_S(x_j, x_k)}{\psi_{S,f}(x_k)}$$

where μ_S is the Möbius function of the poset (S, ⪯).

Proposition 3.5.

Suppose that S is join closed [10]. If $[S]_f$ is invertible, then the inverse of $[S]_f$ is the $n \times n$ matrix $B = (b_{ij})$, where

$$b_{ij} = \sum_{k=1}^{n} \frac{(-1)^{i+j}}{\Phi_{S,f}(x_k)} \det E(S_k^i) \det E(S_k^j)$$

Where $E(S_i^k)$ is the $(n-1) \times (n-1)$ sub matrix of E(S) obtained by deleting the k^{th} row and the i^{th} column of E(S), or

$$b_{ij} = \sum_{x_k \preccurlyeq x_i \wedge x_j} \frac{\mu_S(x_k, x_i)\mu_S(x_k, x_j)}{\Phi_{S,f}(x_k)}$$

where μ_S is the Möbius function of the poset $(S, \preceq)$.

4 MIN and MAX matrices as meet and join matrices

The most straight forward attempt to interpret MIN and MAX matrices as meet and join matrices would be to set $(P, \preceq) = (R, \leq)$. This, however, cannot be done since the set of real numbers is not locally finite (meet and join matrices are usually studied via Möbius inversion, which requires the local finiteness property). Nevertheless, there is a way around the problem. We set $P = \{1, 2, \ldots, n\}$, $\preceq$ is the usual ordering $\leq$ of the integers and S = P. Since in this case $(P, \leq)$ is a chain with n elements, it is trivially a locally finite lattice. Moreover, by defining $f : P \to R$ by $f(i) = z_i$ for all $i = 1, 2, \ldots, n$ we obtain $(S)_f = (T)_{min}$ and $[S]_f = [T]_{max}$.

Executing the Möbius inversion is now easy due to the simple chain-structure of the poset $(P, \leq)$ (general information about Möbius inversion and Möbius functions on posets can be found). For the Möbius function of the chain $(P, \leq)$ we have for $i, j \in P$ that

$$\mu_P(j,i) = \begin{cases} 1 & \text{if } i = j \\ -1 & \text{if } i = j+1 \\ 0 & \text{otherwise.} \end{cases}$$

The function μ_P can then be used to define two other functions ψ_P and Φ_P as

$$\psi_P(1) = x_1, \qquad \psi_P(i) = \sum_{1 \le j \le i} \mu_P(i,j) x_i = x_i - x_{i-1} \qquad for\ 1 < i \le n.$$

$$\Phi_P(n) = x_n, \qquad \Phi_P(n) = \sum_{i \le j \le n} \mu_P(i,j) x_j = x_i - x_{i+1} \qquad for\ 1 < i \le n.$$

It turns out that the values of the functions ψ_P and Φ_P characterize many key properties of the matrices $(T)_{min}$ and $[T]_{max}$ by[11].

4.1 Remark:

Similarly defined functions $\psi_{P,S,f}$ and $\Phi_{P,S,f}$ are also used in the study of more general meet and join matrices, but here these functions take particularly simple forms due to the simple chain-structure of the set P.

Meet and join matrices and their special cases GCD and LCM matrices have been studied in dozens of research papers and their basic properties are rather well known. In this article we are going to formulate these general results for MIN and MAX matrices. Since most of the results presented in this paper follow directly from some stronger theorem for meet and join matrices, it would not be absolutely necessary to reprove these statements. However, we are going to see that in many cases it is still interesting and useful to find simpler proofs that are also accessible to those who are not so familiar with the methods used in the study of meet and join matrices.

5 SOME DEFINITIONS:

5.1 MINIMUM AND MAXIMUM MATRICES:

Let S = $\{x_1, x_2, x_3, \dots\dots\dots, x_n\}$ be ordered set of distinct positive integers. The $n \times n$ matrix $[S] = (S_{ij})$, where $S_{ij} = (x_i, x_j)$ the minimum number of x_i and x_j is called the *MINIMUM (MIN) matrix.*

Let S = $\{x_1, x_2, x_3, \dots\dots\dots, x_n\}$ be ordered set of distinct positive integers. The $n \times n$ matrix $[S] = (S_{ij})$, where $S_{ij} = (x_i, x_j)$ the maximum number of x_i and x_j is called the *MAXIMUM (MAX) matrix.*

5.2 FERMAT MATRICES:

Let S = $\{x_1, x_2, \dots\dots\dots x_n\}$ be a set of distinct positive integers and the nxn matrix $[S] = (s_{ij})$, where $s_{ij} = 2^{2^{(x_i,x_j)}} + 1$, call it to be *Fermat matrix* on S [12].

5.3 FERMAT MINIMUM MATRICES:

Let M = $\{x_1, x_2, \dots\dots\dots x_n\}$ be a set of distinct positive integers and the nxn matrix and $[M] = (m_{ij})$, where $m_{ij} = 2^{2^{\min(x_i,x_j)}} + 1$, call it to be *Fermat MIN matrix* on S.

5.4 FERMAT MAXIMUM MATRICES:

Let M = $\{x_1, x_2, \dots\dots\dots x_n\}$ be a set of distinct positive integers and the nxn matrix and $[M] = (m_{ij})$, where $m_{ij} = 2^{2^{\max(x_i,x_j)}} + 1$ call it to be *Fermat MAX matrix* on S.

6 DETERMINANTS OF FERMAT MIN AND FERMAT MAX MATRICES:

Theorem 6.1:

We consider the determinants of the matrices

$$\det(T)_{\min} = \psi_p(1)\psi_p(2)\dots\dots\dots\psi_p(n) = x_1(x_2 - x_1)(x_3 - x_2)\dots\dots\dots(x_n - x_{n-1}).$$

$$\det[T]_{\max} = \Phi_p(1)\Phi_p(2)\dots\dots\dots\Phi_p(n) = (x_1 - x_2)(x_2 - x_3)\dots\dots\dots(x_{n-1} - x_n)x_n.$$

Proof: These determinant formulas follow directly from Proposition 3.2 and Proposition 3.3.

Theorem 6.2:

Next we consider the determinants of the fermat min and max matrices

$$\text{Det fer }(T)_{\min} = \psi_p(1)\psi_p(2)\dots\dots\dots\psi_p(n) = x_1(x_2 - x_1)(x_3 - x_2)\dots\dots\dots(x_n - x_{n-1}),$$
$$\text{where } x_i = 2^{2^{\min(x_i,x_j)}} + 1 \quad j = 1,2,\dots\dots n.$$

$$\text{Det fer }[T]_{\max} = \Phi_p(1)\Phi_p(2)\dots\dots\dots\Phi_p(n) = (x_1 - x_2)(x_2 - x_3)\dots\dots\dots(x_{n-1} - x_n)x_n,$$
$$\text{where } x_i = 2^{2^{\max(x_i,x_j)}} + 1 \quad j = 1,2,\dots\dots n.$$

Example 1

If S = {2,3} is a lower closed set. Consider 2x2 Fermat Min matrix on S is

$$\text{Fer(S)}_{\min} = \begin{bmatrix} 17 & 17 \\ 17 & 257 \end{bmatrix}$$

Det Fer(S) $_{\min}$ = $x_1(x_2 - x_1)$ = 17x240 = 4080

Consider 2x2 Fermat Max matrix on S is

$$Fer(S)_{max} = \begin{bmatrix} 17 & 257 \\ 257 & 257 \end{bmatrix}$$

$$Det\ Fer(S)_{max} = (x_1 - x_2)x_1 = (-240) \times 257 = -61680$$

Example 2

If S = {1, 2, 3} is a lower closed set.

Consider 3x3 Fermat Min matrix on S is

$$Fer(S)_{min} = \begin{bmatrix} 5 & 5 & 5 \\ 5 & 17 & 17 \\ 5 & 17 & 257 \end{bmatrix}$$

$$Det\ Fer(S)_{min} = x_1(x_2 - x_1)(x_3 - x_2) = 5x12x240 = 14400.$$

Consider 3x3 Fermat Max matrix on S is

$$Fer(S)_{max} = \begin{bmatrix} 5 & 17 & 257 \\ 17 & 17 & 257 \\ 257 & 257 & 257 \end{bmatrix}$$

$$Det\ Fer(S)_{max} = (x_1 - x_2)(x_2 - x_3)x_3 = (-12) \times (-240) \times 257 = 740160.$$

Example 3

If S = {1, 2, 3, 4} is a lower closed set.

Consider 4x4 Fermat Min matrix on S is

$$Fer(S)_{min} = \begin{bmatrix} 5 & 5 & 5 & 5 \\ 5 & 17 & 17 & 17 \\ 5 & 17 & 257 & 257 \\ 5 & 17 & 257 & 65537 \end{bmatrix}$$

$$Det\ Fer(S)_{min} = x_1(x_2 - x_1)(x_3 - x_2)(x_4 - x_3) = 5 \times 12 \times 240 \times 65280 = 94,00,32,000.$$

Consider 4x4 Fermat Max matrix on S is

$$Fer(S)_{max} = \begin{bmatrix} 5 & 17 & 257 & 65537 \\ 17 & 17 & 257 & 65537 \\ 257 & 257 & 257 & 65537 \\ 65537 & 65537 & 65537 & 65537 \end{bmatrix}$$

$$Det\ Fer(S)_{max} = (x_1 - x_2)(x_2 - x_3)(x_3 - x_4)x_4 = (-12) \times (-240) \times (-65280) \times 65537$$
$$= -1,23,21,37,54,36,800.$$

7 INVERSES OF FERMAT MIN AND FERMAT MAX MATRICES:

Under the assumption that the elements of the set T are distinct the MIN and MAX matrices of the set T are usually invertible. Next we shall find their inverses.

Theorem 7.1:

Suppose that the elements of the set T are distinct. If $x_1 \neq 0$, then the MIN matrix is invertiable and the inverse matrix is the nxn tridiagonal matrix $B = (b_{ij})$, where

$$B_{ij} = \begin{cases} 0 & if\ |i-j| > 1 \\ \frac{x_2}{x_1(x_2-x_1)} & if\ i=j=1 \\ \frac{1}{x_i - x_{i-1}} + \frac{1}{x_{i+1}-x_i} & if\ 1 < i = j < n \\ \frac{1}{x_n - x_{n-1}} & if\ i = j = n \\ \frac{-1}{|x_i - x_j|} & if\ |i-j| = 1. \end{cases}$$

where $x_i = 2^{2^{x_i}} + 1 \quad i = 1,2,\ldots\ldots n.$

Theorem 7.2:

If $x_n \neq 0$, then the inverse of the MAX matrix is invertiable and the inverse matrix is the nxn tridiagonal matrix C = (C_{ij}), where

$$C_{ij} = \begin{cases} 0 & if\ |i-j| > 1 \\ \frac{1}{(x_1-x_2)} & if\ i=j=1 \\ \frac{1}{x_{i-1} - x_i} + \frac{1}{x_i - x_{i+1}} & if\ 1 < i = j < n \\ \frac{1}{x_{n-1} - x_n} + \frac{1}{x_n} & if\ i = j = n \\ \frac{1}{|x_i - x_j|} & if\ |i-j| = 1. \end{cases}$$

where $x_i = 2^{2^{x_i}} + 1 \quad i = 1,2,\ldots\ldots n.$

Proof: The inverse formulas follow straight from Proposition 3.4 and Proposition 3.5. An elementary approach would be to construct the supposed inverse matrices and multiply them with the matrices (T) min and [T] max.

Example 4

(S) is a Mersenne Min matrix on lower closed set S = {2, 3}. Then by definition 6.1

$$(S)^{-1} = B = (b_{ij})$$

Therefore since $(S)^{-1} = B$ is the symmetric we have

$$(S)^{-1} = B = \begin{bmatrix} \frac{257}{4080} & \frac{-1}{240} \\ \frac{-1}{240} & \frac{1}{240} \end{bmatrix}$$

(S) is a Mersenne Max matrix on lower closed set S = {2, 3}. Then by definition 6.2

$$(S)^{-1} = C = (c_{ij})$$

Therefore since $(S)^{-1} = C$ is the symmetric we have

$$(S)^{-1} = C = \begin{bmatrix} \frac{-1}{240} & \frac{1}{240} \\ \frac{1}{240} & \frac{-17}{61680} \end{bmatrix}$$

Example 5

(S) is a Mersenne Min matrix on lower closed set S = {1, 2, 3}. Then by definition 6.1

$$(S)^{-1} = B = (b_{ij})$$

Therefore since $(S)^{-1} = B$ is the symmetric tridiagonal we have

$$(S)^{-1} = B = \begin{bmatrix} \frac{17}{60} & \frac{-1}{12} & 0 \\ \frac{-1}{12} & \frac{7}{80} & \frac{-1}{240} \\ 0 & \frac{-1}{240} & \frac{1}{240} \end{bmatrix}$$

(S) is a Mersenne Max matrix on lower closed set S = {2, 4, 6}. Then by definition 6.2

$$(S)^{-1} = C = (c_{ij})$$

Therefore since $(S)^{-1} = C$ is the symmetric tridiagonal we have

$$(S)^{-1} = C = \begin{bmatrix} \frac{-1}{12} & \frac{1}{12} & 0 \\ \frac{1}{12} & \frac{-7}{80} & \frac{1}{240} \\ 0 & \frac{1}{240} & \frac{-17}{61680} \end{bmatrix}$$

Example 6

(S) is a Mersenne Min matrix on lower closed set S = {1, 2, 3, 4}. Then by definition 6.1

$$(S)^{-1} = B = (b_{ij})$$

Therefore since $(S)^{-1} = B$ is the symmetric tridiagonal we have

$$(S)^{-1} = B = \begin{bmatrix} \frac{17}{60} & \frac{-1}{12} & 0 & 0 \\ \frac{-1}{12} & \frac{7}{80} & \frac{-1}{240} & 0 \\ 0 & \frac{-1}{240} & \frac{91}{21760} & \frac{-1}{65280} \\ 0 & 0 & \frac{-1}{65280} & \frac{1}{65280} \end{bmatrix}$$

(S) is a Mersenne Max matrix on lower closed set S = {1, 2, 3, 6}. Then by definition 6.2

$$(S)^{-1} = C = (c_{ij})$$

Therefore since $(S)^{-1} = C$ is the symmetric tridiagonal we have

$$(S)^{-1} = C = \begin{bmatrix} \frac{-1}{12} & \frac{1}{12} & 0 & 0 \\ \frac{1}{12} & \frac{-7}{80} & \frac{1}{240} & 0 \\ 0 & \frac{1}{240} & \frac{-91}{21760} & \frac{1}{65280} \\ 0 & 0 & \frac{1}{65280} & \frac{-257}{4278255360} \end{bmatrix}$$

CONCLUSION:

In this paper, the different properties of MIN and MAX matrices of the set T with min (x_i, x_j) and max (x_i, x_j) as their (i, j) entries like determinant value and inverse of MIN and MAX matrices have been studied. The study is carried out by applying known results of meet and joins matrices to Fermat min and Fermat max matrices.

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5. Conclusion

Furthermore, we give the exact determinants and the inverse matrices of Fermat and Mersenne left circulant matrix. Meanwhile, the non singularity of these special matrices is discussed. On the basis of circulant matrices technology, we will develop solving the problems in [19–22].

Some of the most important

properties of Fermat GCD matrices are presented in terms of meet matrices.