CONTRA *b* AND *b** OPEN MAPS IN INTUITIONISTIC TOPOLOGICAL SPACES

A. VASUKI[1], J. SATHIYARAJ[2], A. VADIVEL[3] and O. UMA MAHESWARI[4]
[1]Research Scholar , Department of Mathematics, Government Arts College(Autonomous), Karur.
[2, 3]Assistant Professor, Department of Mathematics, Government Arts College(Autonomous), Karur.
[4]Assistant Professor, Department of Mathematics, J. J. College of Arts and Science(Autonomous), Pudhukkottai.
E-mail: [1] vasumice@gmail.com, [2]sjsathiyaa@gmail.com, [3]avmaths@gmail.com, [4]ard_uma@yahoo.com.sg

Abstract. In this paper, some new class of functions, called intuitionistic *b* and *b**-open (resp. closed) functions, intuitionistic contra *b* and *b**-open(resp. closed) functions are introduced and studied their properties in intuitionistic topological space.
Key Words and Phrases: Intuitionistic *b*-open, intuitionistic *b**-open, intuitionistic *b*- closed, intuitionistic *b**- closed, intuitionistic contra *b*-open, intuitionistic contra *b**-open, intuitionistic contra *b*-closed, intuitionistic contra *b**- closed .
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1. Introduction

The concept of intuitionistic fuzzy sets and fyzzy topological space were defined by Atanassov[4]. Later in 1996, Coker [5, 6, 7 and 8] defined and studied intuitionistic sets, intuitionistic points and intuitionistic topological spaces. Also, he defined the closure and interior operators in intuitionistic topological spaces and established their properties. This is a discrete form of intuitionistic fuzzy sets where all the sets are crisp sets. Many different forms of open sets have been introduced over the years in general topology. Andrijevic [2] introduced and studied about b-open sets in general topology. Andrijevic[3] introduced and discussed some more properties of semi preopen set in topological space and Abd EI. Monsef et.al[1] introduced β -open sets and β -continuous mapping and discussed some basic properties. Gnanambal Ilango and Singaravelan [10,13] introduced the concepts of intuitionistic β-continuous and irresolute functions in 2017. In this paper, is to define and study about intuitionistic contra b and b*-open (closed) functions in intuitionistic topological space. Also some properties of these are discussed.

2. Preliminaries

The following definitions and results are essential to proceed further.

Definition 2.1: [6] Let X be a non empty fixed set. An intuitionistic set (briefly. *IS*) A is an object of the form $A=(X, A_1, A_2)$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \varnothing$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

The family of all *IS*'s in X will be denoted by $IS(X)$. Every crisp set A on a non-empty set X is obviously an intuitionistic set.

Definition 2.2 [6] Let X be a non-empty set, $A=(X, A_1, A_2)$ and $B=(X, B_1, B_2)$ be intuitionistic sets on X ,then

1. $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $B_2 \subseteq A_2$.
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
3. $A \subset B$ if and only if $A_1 \cup A_2 \supseteq B_1 \cup B_2$.
4. $\overline{A} = (X, A_2, A_1)$.
5. $A \cup B = (X, A_1 \cup B_1, A_2 \cap B_2)$.
6. $A \cap B = (X, A_1 \cap B_1, A_2 \cup B_2)$.
7. $A - B = A \cap \overline{B}$
8. $\tilde{\phi} = (X, \phi, X)$ and $\tilde{X} = (X, X, \phi)$

Corollary 2.1 [3] Let A, B, C and A_i be IS's in X. Then

1. $A_i \subseteq B$ for each i implies that $\bigcup A_i \subseteq B$.
2. $B \subseteq A_i$ for each i implies that $B \subseteq \bigcap A_i$.
3. $\overline{\bigcup A_i} = \bigcap \overline{A_i}$ and $\overline{\bigcap A_i} = \bigcup \overline{A_i}$.
4. $A \subseteq B \Leftarrow \overline{B} \subseteq \overline{A}$.
5. $\overline{(\overline{A})} = A$, $\overline{\tilde{\phi}} = \tilde{X}$ and $\overline{\tilde{X}} = \tilde{\phi}$

Definition 2.3 [8] An intuitionistic topology (briefly IT) on a non-empty set X is a family τ of IS's in X satisfying the following axioms

1. $\tilde{\phi}, \tilde{X} \in \tau$
2. $A \cap B \in \tau$ for any $A, B \in \tau$.
3. $\bigcup A_i \in \tau$ for an arbitrary family in τ

In this case the pair (X, τ) is called intuitionistic topological space (briefly ITS) and the IS's in τ are called the intuitionistic open set in X denoted by $I^{(\tau)}O$ and the complement of an $I^{(\tau)}O$ is called Intuitionistic closed set in X denoted by $I^{(\tau)}C$. The family of all $I^{(\tau)}O$ (resp. $I^{(\tau)}C$) sets in X will be denoted by $I^{(\tau)}O(X)$ (resp. $I^{(\tau)}C(X)$.)

Definition 2.4 [7] Let (X, τ) be an ITS and $A \in IS(X)$. Then the intuitionistic interior (resp. intuitionistic closure) of A are defined by $int(A) = \bigcup\{K : K \in I^{(\tau)}O(X) \text{ and } K \subseteq A\}$ (resp. $cl(A) = \bigcap\{K : K \in I^{(\tau)}C(X) \text{ and } A \subseteq K\}$.)

In this study we use $I^{(\tau)}i(A)$ (resp. $I^{(\tau)}c(A)$) instead of $int(A)$ (resp. $cl(A)$).

Definition 2.5 [12] Let (X, τ) be an ITS and an IS A in X is said to be intuitionistic b-open (briefly $I^{(\tau)}bO$) if $A \subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A))$ and intuitionistic b-closed (briefly $I^{(\tau)}bC$) if $I^{(\tau)}i(I^{(\tau)}c(A)) \cap I^{(\tau)}c(I^{(\tau)}i(A)) \subseteq A$.

The family of all $I^{(\tau)}bO$ (resp $I^{(\tau)}bC$) sets in X will be denoted by $I^{(\tau)}bO(X)$ (resp. $I^{(\tau)}bC(X)$.)

Definition 2.6 [12] Let (X, τ) be an ITS and A be an $IS(X)$, then

1. intuitionistic b-interior of A is the union of all $I^{(\tau)}bO(X)$ contained in A, and is denoted by $I^{(\tau)}bi(A)$. i.e. $I^{(\tau)}bi(A) = \bigcup\{G : G \in I^{(\tau)}bO(X) \text{ and } G \subseteq A\}$.
2. intuitionistic b-closure of A is the intersection of all $I^{(\tau)}bC(X)$ containing A, and is denoted

by $I^{(\tau)}bc(A)$.i.e. $I^{(\tau)}bc(A) = \square\{G : G \in I^{(\tau)}bC(X) \text{ and } G \supseteq A\}$.

Definition 2.7[8] Let X be a non empty set and p∊ X. Then the IS $\tilde{p}$ defined by $\tilde{p}$ = (X, {p}, $\{p\}^c$) is called an intuitionistic point (IP for short) in X. The intuitionistic point $\tilde{p}$ is s aid to be contained in A = (X, A_1, A_2) (i.e $\tilde{p}$ ∊ A) if and only if $\tilde{p}$ ∊ A_1.

Definition 2.8 [8] Let $f: (X, \tau) \to (Y, \sigma)$ be a function. If A = (X, A_1, A_2) is an intuitionistic set in X, then the image of A under f, denoted by f(A), is an intuitionistic set in Y defined by f(A) = (Y, f(A_1), f_(A_2)), where f_(A_2) = $(f(A_2)^c)^c$.

Definition 2.9 [8] Let $f: (X, \tau) \to (Y, \sigma)$ be a function. If A = (Y, A_1, A_2) is an intuitionistic set in Y, then the preimage of A under f, denoted by f^{-1} (A), is an intuitionistic set in X defined by f^{-1} (A) = (X, f^{-1} (A_1), f^{-1} (A_2)).

Definition 2.10[6, 8] Let $A,\ A_i (i \in J)$ be IS's in X, $B,\ B_j (j \in K)$ IS's in Y and $f: (X, \tau) \to (Y, \sigma)$ be a function. Then

(a). $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$
(b). $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
(c). $A \subseteq f^{-1}(f(A))$ and if f is one to one, then $A = f^{-1}(f(A))$
(d). $f(f^{-1}(B)) \subseteq B$ and if f is onto, then $f(f^{-1}(B)) = B$
(e). $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
(f). $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
(g). $f(\cup A_i) = \cup f(A_i)$
(h). $f(\cap A_i) \subseteq \cap f(A_i)$ and if f is one to one, then $f(\cap A_i) = \cap f(A_i)$
(i). $f^{-1}(\tilde{Y}) = \tilde{X}$
(j). $f^{-1}(\tilde{\phi}) = \tilde{\phi}$
(k). $f(\tilde{X}) = \tilde{Y}$ if f is onto
(l). $f(\tilde{\phi}) = \tilde{\phi}$
(m). If f is onto, then $\overline{f(A)} \subseteq f(\bar{A})$: and if furthermore, f is $1-1$, we have $\overline{f(A)} \subseteq f(\bar{A})$ and
(n). $f^{-1}(\bar{B}) = \overline{f^{-1}(B)}$
(o). $B_1 \subset B_2 \Rightarrow f^{-1}(B_1) \subset f^{-1}(B_2)$

Definition2.11[8] Let (X, τ) and (Y, δ) be two intuitionistic topological spaces and $f: (X, \tau) \to (Y, \delta)$ be a function. Then f is said to be intuitionistic continuous if and only if the preimage of every intuitionistic open set in Y is intuitionistic open in X.

Definition 2.12[6] A map $f: (X, \tau) \to (X, \sigma)$ is called intuitionistic open(closed) if the image $f(A)$ is intuitionistic open(closed) in Y for every intuitionistic open(closed) set in X.

3. Intuitionistic b and b^*-open(closed) functions

Definition 3.1. A map $f: (X, \tau) \to (Y, \sigma)$ is called intuitionistic b-open function (in short - Ib open) if the image $f(A)$ is $I^{(\sigma)}bO(Y)$ for every intuitionistic open set in X.

Definition 3.2 A map $f: (X, \tau) \to (Y, \sigma)$ is called intuitionistic b-closed (in short Ib-closed) if the image $f(A)$ is $I^{(\sigma)}bC(Y)$ for every intuitionistic closed set in X.

Example3.1. Let $X = \{a,b,c\} = Y$, $\tau = \{\tilde{\phi}, \tilde{X}, (X,\{a\},\{b\})\}$ $\sigma = \{\tilde{\phi}, \tilde{Y}, (Y,\{b\},\phi), (Y,\{b\},\{c\})\}$ and $f: (X, \tau) \to (Y, \sigma)$ be a function such that $f(a) = b,\ f(b) = c,\ f(c) = a$. Then the function f is both Ib-open and Ib-closed.

Definition 3.3 Let $\tilde{p}$ be an IP in X. A subset N of X is said to be Ib-neighborhood of $\tilde{p}$ in X, if there exists $I^{(\tau)}bO$ set G belongs to (X, τ) such that $\tilde{p}$ belongs to $G \subseteq N$. We shall denote the set of all Ib-neighborhoods of $\tilde{p}$ by $Ib - N(\tilde{p})$.

Definition 3.4 Let (X, τ) and (Y, δ) be two ITS and $f: (X, \tau) \to (Y, \delta)$ be a function. Then f is said to be intuitionistic b-continuous (In short. Ib-continuous) if and only if the preimage of every intuitionistic open set in Y is intuitionistic b-open in X.

Theorem 3.1 A function $f: (X, \tau) \to (Y, \sigma)$ is Ib-open if and only if for any intuitionistic subset B of (Y, σ) and for any $I^{(\tau)}C$ set S containing $f^{-1}(B)$, there exists an $I^{(\sigma)}bC$set A of (Y, σ) containing B such that $f^{-1}(A) \subseteq S$.

Proof. Suppose that is an Ib-open map. Let B be any intuitionistic subset of (Y, σ) and S be an $I^{(\tau)}C$ set of (X, τ) such that

$$f^{-1}(B) \subset S \qquad (1)$$

$\Rightarrow S^c \subseteq f^{-1}(B)$

$\Rightarrow f(S^c) \subseteq B$

$\Rightarrow (f(S^c))^c \supseteq B$

$\Rightarrow B \subseteq (f(S^c))^c = A$

Hence $f^{-1}(B) \subseteq f^{-1}(A) \subseteq S$(using equation 1)

Conversely, let B be an $I^{(\tau)}C$ set of (X, τ) and S be an $I^{(\sigma)}bO(Y)$. Then $f^{-1}(f(B^c)) \subset B^c$ and B^c is $I^{(\tau)}O(X)$. By assumption, there exists an $I^{(\sigma)}bC$set A of (Y, σ) such that $f(B^c) \subset A$ and $f^{-1}(A) \subset B^c$ and so $B \subset (f^{-1}(S^c))$.

Hence $S^c \subset f(B)$

$\subset f(f^{-1}(S)^c)$

$\subset S^c$

$\Rightarrow f(B) = S^c$.

Since S^c is $I^{(\sigma)}bC$ in (Y, σ), $f(B)$ is $I^{(\sigma)}bC$ in (Y, σ) and therefore f is Ib-open.

Theorem 3.2. A map $f: (X, \tau) \to (Y, \sigma)$ is Ib-closed if and only if for each intuitionistic subset A of (Y, σ) and for each intuitionistic open set U containing $f^{-1}(A)$ there is an $I^{(\sigma)}bO$ set V of (Y, σ) such that $A \subset V$ and $f^{-1}(A) \subset U$.

Proof. Suppose that f is an Ib-closed map. Let $A \subset Y$ and U be an $I^{(\tau)}O(X)$such that $f^{-1}(A) \subset U$. Then $V = (f(U^c))^c$ is an $I^{(\sigma)}bO$ set containing A such that $f^{-1}(A) \subset U$.

Conversely, let A be an $I^{(\tau)}C$ set of (X, τ). Then $f^{-1}(f(A^c)) \subset A^c$ and A^c is intuitionistic open. By assumption, there exists an $I^{(\sigma)}bO$set V of (Y, σ) such that $f(A^c) \subset V$ and $f^{-1}(V) \subset A^c$ and so $A \subset (f^{-1}(V))^c$.

Hence $V^c \subset f(A)$

$\subset f(f^{-1}(V)^c)$

$V^c \subset f(A) \subset V^c$

$\Rightarrow f(A) = V^c$. Since V^c is $I^{(\sigma)}bC(Y)$, $f(A)$ is $I^{(\sigma)}bC(Y)$ and therefore f is Ib-closed map.

Theorem 3.3. If $f: (X, \tau) \to (Y, \sigma)$ be Ib-open, then $f(I^{(\tau)}i(A)) \subset I^{(\sigma)}bi(f(A))$.

Proof. Let $A \subset X$ and $f: (X, \tau) \to (Y, \sigma)$ be Ib-open map. Since $f(I^{(\tau)}i(A)) \subseteq f(A)$

$$I^{(\sigma)}bi(f(I^{(\tau)}i(A))) \subseteq I^{(\sigma)}bi(f(A)) \qquad (2)$$

Since $f(I^{(\tau)}i(A))$ is $I^{(\sigma)}bO(Y)$,

$$I^{(\sigma)}bi(f(I^{(\tau)}i(A))) \subseteq f(I^{(\tau)}i(A)) \qquad (3)$$

From 2 and 3, we have $f(I^{(\tau)}i(A)) \subset I^{(\sigma)}bi(f(A))$.

Theorem 3.4. A map $f: (X, \tau) \to (Y, \sigma)$ is Ib-closed if and only if $I^{(\sigma)}bc(f(A)) \subset f(I^{(\tau)}c(A))$.

Proof. Let $A \subset X$ and $f: (X, \tau) \to (Y, \sigma)$ be Ib-closed, then $f(I^{(\tau)}c(A))$ is $I^{(\sigma)}bC(Y)$ which implies $I^{(\sigma)}bc(f^{(\tau)}cl(A))) = f(I^{(\tau)}c(A))$.

Since $f(A) \subset f(I^{(\tau)}c(A))$,

$\Rightarrow I^{(\sigma)}bc(f(A)) \subset I^{(\sigma)}bc(f(I^{(\tau)}c(A))) \subset f(I^{(\tau)}c(A))$ for every intuitionistic subset A of X.

Conversely, let A be any $I^{(\tau)}C$set in (X, τ). Then $A = I^{(\tau)}c(A)$and so

$$f(A) = f(I^{(\tau)}c(A)) \supseteq I^{(\sigma)}bc(f(A)) \qquad (1)$$

Since,

$$f(A) \subset I^{(\sigma)}bc(f(A)) \qquad (2)$$

From 1 and 2, $f(A) = I^{(\sigma)}bc(f(A))$.

ie., $f(A)$ is $I^{(\sigma)}bC(Y)$ and hence f is Ib-closed.

Theorem 3.5. For a bijective map $f: (X, \tau) \to (Y, \sigma)$ the following are equivalent.

(a). f is Ib-open

(b). f is Ib-closed

(c). $f^{-1}: (Y, \sigma) \to (X, \tau)$ is Ib-continuous.

Proof. (a)⇒(b): Let $A = (X, A_1, A_2)$ be $I^{(\tau)}C(X)$.Then $X - A = (X, A_2, A_1)$ is $I^{(\tau)}O(X)$. Since f is an Ib-open map, $f(X - A)$ is $I^{(\sigma)}bO(Y)$.

So, $f(X, A_2, A_1) = (Y, f(A_2), f(A_1))$

$= (Y, f(A_2), Y - f(X - A_1))$ is $I^{(\sigma)}bO(Y)$.

So, $(Y, Y - f(X - A_1), f(A_2))$ is $I^{(\sigma)}bC(Y)$.

Since $f$is bijective, $Y - f(X - A_1) = f(A_1)$,

$(Y, Y - f(X - A_1), f(A_2)) = (Y, f(A_1), f(A_2))$ is $I^{(\sigma)}bC(Y)$.

Hence f is an Ib-closed map.

(b)⇒ (c): Let A be an $I^{(\tau)}C(X)$. Since f is Ib-closed, $f(A)$ is $I^{(\sigma)}bC(Y)$. Since f is bijective $f(A) = (f^{-1})^{-1}(A)$, f^{-1} is Ib-continuous.

(c) ⇒ (a): Let A be $I^{(\tau)}O(X)$. By hypothesis, $(f^{-1})^{-1}(A)$is $I^{(\sigma)}bO(Y)$.ie., $f(A)$ is $I^{(\sigma)}bO(Y)$.

Definition 3.5 A map $f: (X, \tau) \to (Y, \sigma)$ is called intuitionistic b^*-open(briefly Ib^*-open) if $f(U)$ is $I^{(\sigma)}bO(Y)$.for every $I^{(\tau)}bO$set U of (X, τ).

Definition 3.6 A map $f: (X, \tau) \to (Y, \sigma)$ is called called intuitionistic b^*-closed (briefly Ib^*-closed) if $f(U)$ is $I^{(\sigma)}bC(Y)$ for every $I^{(\tau)}bC$ set U of (X, τ).

Example 3.2 Let $X = \{a, b, c\} = Y$, $\tau = \{\tilde{\phi}, \tilde{X}, (X, \{a\}, \{b\})\}$, and $\sigma = \{\tilde{\phi}, \tilde{Y}, (Y, \{b\}, \phi)\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b,\ f(b) = c,\ f(c) = a$. Then the map f is both Ib^*-open and Ib^*-closed.

Theorem 3.6 For a function $f: (X, \tau) \to (Y, \sigma)$ the following statements are equivalent:

1. f is an Ib^*-open.
2. The image of each $I^{(\tau)}b$-neighborhood of any intuitionistic point $\tilde{p}$ in X is $I^{(\sigma)}b$-neighborhood of $f(\tilde{p})$ in Y
3. $f(I^{(\tau)}bi(A)) \subseteq I^{(\sigma)}bi(f(A))$ for each $A \subseteq X$.
4. $I^{(\tau)}bi(f^{-1}(B)) \subseteq f^{-1}(I^{(\sigma)}bi(B))$ for each $B \subseteq Y$.

Proof. (i)⇒(ii): Let $\tilde{p}$ belongs to $I^{(\tau)}b\text{-}N(\tilde{p})$, then there exists D belongs to $I^{(\tau)}bO(X)$, such that $\tilde{p}$ belongs to $D \subseteq U$, so $f(\tilde{p})$ belongs to $f(D) \subseteq f(U)$ but $f(D)$ belongs to $I^{(\sigma)}bO(Y)$ hence $f(U)$ belongs to $I^{(\sigma)}b\text{-}N(f(\tilde{p}))$.

(ii) ⇒ (iii): For each $\tilde{p}$ belongs to X and U belongs $I^{(\tau)}b\ N(\tilde{p})$ by hypothesis. There exists D belongs to $I^{(\tau)}bO(X)$ such that $\tilde{p}$ belongs to $D \subseteq U$ and so $f(\tilde{p})$ belongs to $f(D) \subseteq f(U)$ which leads to $f(D)$ belongs to $I^{(\sigma)}bO(Y)$.

(iii) ⇒ (iv): is obvious.

(iv) ⇒ (i): Assume D belongs to $I^{(\tau)}bO(X)$. By (iv) we get,

$$I^{(\tau)}bi(f^{-1}(f(D))) \subseteq f^{-1}(I^{(\sigma)}bi(f(D))) \qquad (1)$$

Since, $D \subseteq f^{-1}(f(D))$

$\Rightarrow I^{(\tau)}bi(D) \subseteq I^{(\tau)}bi(f^{-1}(f(D))) \subseteq f^{-1}(I^{(\sigma)}bi(f(D)))$(using equation 1)

$\Rightarrow f(I^{(\sigma)}bi(D)) \subseteq I^{(\tau)}bi(f(D))$

$\Rightarrow f(D) \subseteq I^{(\tau)}bi(f(D))$.

Hence $f(D)$ belongs to $I^{(\sigma)}bO(Y)$. Thus f is Ib^*-open.

Theorem 3.7 For a function $f: (X, \tau) \to (Y, \sigma)$ the following statements are equivalent:

i. f is an Ib^*-open.

ii. $I^{(\sigma)}bc(f(A)) \subseteq f(I^{(\tau)}bc(A))$ for each $A \subseteq X$.

iii. $f^{-1}(I^{(\tau)}bc(f(B)) \subseteq I^{(\sigma)}bc(f^{-1}(B))$for each $B \subseteq Y$.

Proof. Similar to above theorem .

Theorem3.8 If a function $f: (X, \tau) \to (Y, \sigma)$ is Ib^*-open, then $I^{(\sigma)}bi(f(A)) \subseteq f(I^{(\tau)}bi(A))$ for every intuitionistic set A of X.

Proof. Suppose f is an Ib^*-open and A be any arbitrary intuitionistic subset of X. Since $I^{(\sigma)}bi(f(A))$ is an $I^{(\tau)}bO(X)$, $f(I^{(\tau)}bi(A))$is an $I^{(\sigma)}bO(Y)$ as f is an Ib^*-open function. Hence $I^{(\sigma)}bi(f(A)) \subseteq f(I^{(\tau)}bi(A))$.

Theorem 3.9 The Ib-continuous and Ib-open functions of an $ITS\ (X, \tau)$ into an $ITS\ (Y, \sigma)$ is an Ib^*-open function.

Proof. Let $f: (X, \tau) \to (Y, \sigma)$ be an Ib-continuous and Ib-open function, let U be an $I^{(\tau)}bO(X)$, then $f(U) \subseteq f(I^{(\tau)}i(I^{(\tau)}c(U)) \cup I^{(\tau)}c(I^{(\tau)}i(U)))$

$$\subseteq I^{(\tau)}i(f(I^{(\tau)}c(U))) \cup I^{(\tau)}c(f(I^{(\tau)}i(U)))$$

$$\subseteq I^{(\tau)}i(I^{(\tau)}c(f(U))) \cup I^{(\tau)}c(I^{(\tau)}i(f(U)))$$

Thus $f(U) \subseteq I^{(\tau)}i(I^{(\tau)}c(f(U))) \cup I^{(\tau)}c(I^{(\tau)}i(f(U)))$

and f is an Ib^*-open function.

Theorem 3.10 Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\mu)$ be two functions. Then

i. Each f and g are Ib^*-open, then their composition is also respectively.
ii. If f is an Ib-open and g is an Ib^*-open then gof is an Ib^*-open function.
iii. If f is onto Ib-continuous and gof is an Ib^*-open function, then g is an Ib-open.
iv. If gof is surjection Ib-continuous and f is an Ib^*-open, then g is an Ib-continuous functions.
v. If gof is strongly an Ib-continuous and f is an Ib-open, then g is an Ib^*-open.

Theorem 3.11 Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\mu)$ be two functions. Then

a) Each f and g are Ib^*-closed, then their composition is also respectively.
b) If f is an Ib-closed and g is an Ib^*-closed then gof is an Ib^*-closed function.
c) If f is onto Ib-continuous and gof is an Ib^*-closed function, then g is an Ib-closed.
d) If gof is surjection Ib-continuous and f is an Ib^*-closed, then g is an Ib-continuous functions.
e) If gof is strongly an Ib-continuous and f is an Ib-closed, then g is an Ib^*-closed.

4. Intuitionistic contra b and contra b^*-open functions

Definition 4.1 A map $f:(X,\tau)\to(Y,\sigma)$ is called contra Ib-open if the image $f(A)$ is $I^{(\sigma)}bC(Y)$ for every intuitionistic open set in X.

Definition 4.2 A map $f:(X,\tau)\to(Y,\sigma)$ is called contra Ib-closed if the image $f(A)$ is $I^{(\sigma)}bC(Y)$ for every intuitionistic open set in X.

Example 4.1 Let $X=\{a,b,c\}=Y$, with the topologies $\tau=\{\tilde{\phi},\tilde{X},(X,\phi,\phi),(X,\phi,\{b,c\})\}$, $\sigma=\{\tilde{\phi},\tilde{Y},(Y,\phi,\phi),(Y,\phi,\{a\}),(Y,\{b\},\{c\}),(Y,\{b\},\phi),(Y,\phi,\{c\}),(Y,\phi,\{a,c\})\}$ & $f:(X,\tau)\to(Y,\sigma)$ be a function such that $f(a)=c,\ f(b)=a,\ f(c)=b$. Then the function f is both contra Ib-open and contra Ib-closed.

Theorem 4.1 If $f:(X,\tau)\to(X,\sigma)$ be contra Ib-open, then $f(I^{(\tau)}i(A))\subset I^{(\sigma)}bc(f(A))$.

Proof. Let $A\subset X$ and $f:(X,\tau)\to(X,\sigma)$ be contra Ib-open map. Since $f(I^{(\tau)}i(A)\subseteq f(A)$
$\Rightarrow I^{(\sigma)}bc(f(I^{(\tau)}i(A))\subseteq I^{(\sigma)}bc(f(A))$. Since $f(I^{(\tau)}i(A))$ is $I^{(\sigma)}bC(Y)$,
$\Rightarrow I^{(\sigma)}bc(f(I^{(\tau)}i(A))\subset f(I^{(\tau)}i(A))$.
We have $f(I^{(\tau)}i(A))\subseteq I^{(\sigma)}bc(f(A))$.

Theorem 4.2 A map $f:(X,\tau)\to(Y,\sigma)$ is contra Ib-closed if and only if $I^{(\sigma)}bi(f(A))\subset f(I^{(\tau)}c(A))$.

Proof. Let $A\subset X$ and $f:(X,\tau)\to(Y,\sigma)$ be contra Ib-closed, then $f(I^{(\tau)}c(A))$is $I^{(\sigma)}bO(Y)$ which implies $I^{(\sigma)}bi(f(I^{(\tau)}c(A))=f(I^{(\tau)}c(A))$.
Since $f(A)\subseteq f(I^{(\tau)}c(A))$,

$I^{(\sigma)}bi(f(A)) \subset I^{(\sigma)}bc(f(I^{(\tau)}c(A))) \subset f(I^{(\tau)}c(A))$ for every intuitionistic subset A of X.
Conversely, let A be any intuitionistic closed set in (X, τ). Then $A = I^{(\tau)}c(A)$ and so $f(A) = f(I^{(\tau)}c(A)) \subseteq I^{(\sigma)}bi(f(A))$by hypothesis.
Since $f(A) \subset I^{(\sigma)}bi(f(A))$, $f(A) = I^{(\sigma)}bi(f(A))$, ie., $f(A)$ is $I^{(\sigma)}bO(Y)$ and hence f is contra Ib-closed.

Theorem 4.3 For a bijective map $f: (X, \tau) \to (Y, \sigma)$ the following are equivalent.

(a). f is contra Ib-open

(b). f is contra Ib-closed

(c). $f^{-1}: (Y, \sigma) \to (X, \tau)$ is contra Ib-continuous.

Proof. (a)⇒ (b): Let $A = (X, A_1, A_2)$ be intuitionistic closed in (X, τ). Then $X - A = (X, A_2, A_1)$ is intuitionistic open in X. Since f is contra Ib-open map, $f(X - A)$ is $I^{(\sigma)}bC(Y)$.

So, $f(X, A_2, A_1) = (Y, f(A_2), f(A_1))$

$= (Y, f(A_2), Y - f(X - A_1))$ is $I^{(\sigma)}bC(Y)$.

So, $(Y, Y - f(X - A_1), f(A_2))$ is $I^{(\sigma)}bO(Y)$.

Since $Y - f(X - A_1) = f(A_1)$,

$(Y, Y - f(X - A_1), f(A_2)) = (Y, f(A_1), f(A_2))$ is $I^{(\sigma)}bO(Y)$.

Hence f is contra Ib-closed map.

(b) ⇒ (c): Let A be intuitionistic closedset in (X, τ). Since f is contra Ib-closed, $f(A)$ is $I^{(\sigma)}bO(Y)$. Since f is bijective $f(A) = (f^{-1})^{-1}(A)$, f^{-1} is contra Ib-continuous.

(c) ⇒(a): Let A be intuitionistic open in (X, τ). By hypothesis, $(f^{-1})^{-1}(A)$is $I^{(\sigma)}bC(Y)$. ie., $f(A)$ is $I^{(\sigma)}bC(Y)$.

Definition 4.3 A map $f: (X, \tau) \to (Y, \sigma)$ is called intuitionistic contra b^*-open (in short contra Ib^*-open) if $f(U)$ is $IbO(Y)$ for every Ib-closed set U of (X, τ).

Example 4.2 Let $X = \{a, b, c\} = Y$, $\tau = \{\tilde{\phi}, \tilde{X}, (X, \{a\}, \{b\})\}$, and $\sigma = \{\tilde{\phi}, \tilde{Y}, (Y, \{b\}, \phi)\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then the map f is contra Ib^*-open.

Definition 4.4 A map $f: (X, \tau) \to (Y, \sigma)$ is called contra Ib^*-closed if $f(U)$ is $I^{(\sigma)}bC(Y)$ for every $I^{(\tau)}b$-open set U of (X, τ).

Example 4.4 Let $X = \{a, b, c\} = Y$, $\tau = \{\tilde{\phi}, \tilde{X}, (X, \{a\}, \{b\})\}$, and $\sigma = \{\tilde{\phi}, \tilde{Y}, (Y, \{b\}, \phi)\}$. Define $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then the map f is contra Ib^*-closed.

Theorem 4.4 For a function $f: (X, \tau) \to (Y, \sigma)$ the following statements are equivalent:

(i). f is an contra Ib^*-open.

(ii). $f(I^{(\tau)}bi(A)) \subseteq I^{(\sigma)}bc(f(A))$ for each $A \subseteq X$.

(iii). $I^{(\tau)}bi(f^{-1}(B)) \subseteq f^{-1}(I^{(\sigma)}bc(f(B))$for each $B \subseteq Y$.

Proof. ⇒(ii): Let $A \subset X$ and $f: (X, \tau) \to (X, \sigma)$ be contra Ib^*-open map then $f(I^{(\tau)}i(A))$is $I^{(\sigma)}bC(Y)$.

$\Rightarrow I^{(\sigma)}bc(f(I^{(\tau)}bi(A)) \subseteq I^{(\sigma)}bc(f(A))$, since $f(I^{(\tau)}bi(A))$is $I^{(\sigma)}bC(Y)$.

$\Rightarrow I^{(\sigma)}bc(f(I^{(\tau)}bi(A))) \supset f(I^{(\tau)}bi(A))$.

We have $f(I^{(\tau)}bi(A)) \subset I^{(\sigma)}bc(f(A))$.
(ii) ⇒ (iii): are obvious.
(iii) ⇒ (i): Assume D belongs to $I^{(\tau)}bO(X)$, then by (iii) we get

$$I^{(\tau)}bi(f^{-1}(D)) \subseteq f^{-1}(I^{(\sigma)}bc(f(D)) \subseteq f^{-1}(I^{(\tau)}bc(f(D))$$

so $f(D) \subseteq I^{(\sigma)}bc(f(D))$,
hence $f(D)$ belongs to $I^{(\sigma)}bC(Y)$.

Theorem 4.5 For a function $f:(X,\tau) \to (Y,\sigma)$ the following statements are equivalent:
(i). f is an contra Ib^*-open.
(ii). $I^{(\sigma)}bi(f(A)) \subseteq f(I^{(\tau)}bc(A))$ for each $A \subseteq X$.
(iii). $f^{-1}(I^{(\tau)}bi(B)) \subseteq (I^{(\sigma)}bc(f(B))$ *for each* $B \subseteq X$.

Proof. Similar to above theorem.

Theorem 4.6 If a function $f:(X,\tau) \to (Y,\sigma)$ is contra Ib^*-open, then $(I^{(\sigma)}bc(f(A)) \subseteq f(I^{(\tau)}bi(A))$ for every intuitionistic set A of X.

Proof. Suppose f is an contra Ib^*-open and A be any arbitrary intuitionistic subset of X. Since $I^{(\tau)}bi(A)$ is an $I^{(\tau)}bO(X)$, $f(I^{(\tau)}bi(A))$ is an $I^{(\sigma)}bC(Y)$ as f is an contra Ib^*-open function. Hence $I^{(\sigma)}bc(f(A)) \subseteq f(I^{(\tau)}bi(A))$.

Theorem 4.7 The contra *Ib*-continuous and contra *Ib*-open functions of an $ITS\ (X,\tau)$ into an $ITS\ (Y,\sigma)$ be contra Ib^*-open function.

Proof. Let $f:(X,\tau) \to (Y,\sigma)$ be an contra *Ib*-continuous and contra *Ib*-open function, let U be an contra $I^{(\tau)}bO(X)$, then

$$\begin{aligned} f(U) &\subseteq f(I^{(\tau)}i(I^{(\tau)}c(U)) \cap I^{(\tau)}c(I^{(\tau)}i(U))) \\ &\subseteq I^{(\tau)}i(f(I^{(\tau)}c(U))) \cap I^{(\tau)}c(f(I^{(\tau)}i(U))) \\ &\subseteq I^{(\tau)}i(I^{(\tau)}c(f(U))) \cap I^{(\tau)}c(I^{(\tau)}i(f(U))) \end{aligned}$$

Thus $f(U) \subseteq I^{(\tau)}i(I^{(\tau)}c(f(U))) \cap I^{(\tau)}c(I^{(\tau)}i(f(U)))$
and f is contra Ib^*-open function.

Theorem 4.8 Let $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\mu)$ be two functions. Then
(i). Each f and g are contra Ib^*-open, then their composition is also respectively.
(ii). If f is contra Ib-open and g is contra Ib^*-open then gof is contra Ib^*-open function.
(iii). If f is onto contra Ib-continuous and gof is contra Ib^*-open function, then g iscontra Ib-open.
(iv) If gof is surjection contra Ib-continuous and f is contra Ib^*-open, then g is contra Ib -continuous functions.
(v) If gof is contra Ib-continuous and f is contra Ib-open, then g is contra Ib^*-open.

Theorem 4.9 Let $f:(X,\tau) \to (Y,\sigma)$ and $g:(Y,\sigma) \to (Z,\mu)$ be two functions. Then
(i).Each f and g are contra Ib^*-closed, then their composition is also respectively.
(ii).If f is contra Ib-closed and g is contra Ib^*-closed then gof is contra Ib^*-closed function.
(iii). If f is onto contra Ib-continuous and gof is contra Ib^*-closed function, then g is

contra Ib-closed.

(iv). If gof is surjection $contraIb$-continuous and f is contra Ib^*-closed, then g is contra Ib-continuous functions.

(v). If gof is contra Ib-continuous and f is contra Ib-closed, then g is contra Ib^*-closed.

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