ON INTUITIONISTIC b CONTINUITY IN INTUITIONISTIC TOPOLOGICAL SPACES

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Abstract. In this paper, some new class of functions in intuitionistic topological spaces, called intuitionistic b continuous, intuitionistic b irresolute and intuitionistic b homeomorphisms are investigated and discussed their relations with existing functions in intuitionistic topological spaces.

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1.Introduction

From 1996Coker [[2], [3], [4]] defined and studied intuitionistic topological spaces, intuitionistic open sets, intuitionistic closed sets and compactness on intuitionistic topological spaces. Also, he defined the closure and interior operators in intuitionistic topological spaces and established their properties. Many different forms of continuousfunctions have been introduced over the years in general topology. In particular Ekici [5] introduced and studied various forms of continuous functions in topological spaces. Intuitionistic b-open sets and its properties are discussedby Prabhu et.al., [9]. Gnanambal and Singaravelan [7 and 10] introduced the concepts of intuitionistic β-continuous and irresolute functions in 2017. In this paper, some new class of functions in intuitionistic topological spaces, called as intuitionistic b continuous, intuitionistic b irresolute and intuitionistic b homeomorphisms are introduced and discussed their relations with existing functions in intuitionistic topological spaces.

2.Preliminaries

The following definitions and results are essential to proceed further.

Definition 2.1: [2] Let X be a non empty fixed set. An intuitionistic set (briefly. IS) A is an object of the form $A=(X,A_1,A_2)$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \varnothing$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

The family of all IS's in X will be denoted by $IS(X)$. Every crisp set A on a non-empty set X is obviously an intuitionistic set.

Definition 2.2 [2] Let X be a non-empty set, $A=(X,A_1,A_2)$ and $B=(X,B_1,B_2)$ be intuitionistic sets on X, then

1. $A \subseteq B$ if and only if $A_1 \subseteq B_1$ and $B_2 \subseteq A_2$.
2. $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
3. $A \subset B$ if and only if $A_1 \cup A_2 \supseteq B_1 \cup B_2$.
4. $\overline{A} = (X, A_2, A_1)$.
5. $A \cup B = (X, A_1 \cup B_1, A_2 \cap B_2)$.

6. $A \cap B = (X, A_1 \cap B_1, A_2 \cup B_2)$.
7. $A - B = A \cap \overline{B}$
8. $\tilde{\phi} = (X, \phi, X)$ and $\tilde{X} = (X, X, \phi)$

Corollary 2.1 [2] Let A, B, C and A_i be IS's in X. Then

1. $A_i \subseteq B$ for each i implies that $\bigcup A_i \subseteq B$.
2. $B \subseteq A_i$ for each i implies that $B \subseteq \bigcap A_i$.
3. $\overline{\bigcup A_i} = \bigcap \overline{A_i}$ and $\overline{\bigcap A_i} = \bigcup \overline{A_i}$.
4. $A \subseteq B \Leftarrow \overline{B} \subseteq \overline{A}$.
5. $\overline{(\overline{A})} = A$, $\overline{\tilde{\phi}} = \tilde{X}$ and $\overline{\tilde{X}} = \tilde{\phi}$

Definition 2.3[4] An intuitionistic topology (briefly IT) on a non-empty set X is a family τ of IS's in X satisfying the following axioms

1. $\tilde{\phi}, \tilde{X} \in \tau$.
2. $A \cap B \in \tau$ for any $A, B \in \tau$.
3. $\bigcup A_i \in \tau$ for an arbitrary family in τ.

In this case the pair (X, τ) is called intuitionistic topological space (briefly ITS) and the IS's in τ are called the intuitionistic open set in X denoted by $I^{(\tau)}O$ and the complement of an $I^{(\tau)}O$ is called Intuitionistic closed set in X denoted by $I^{(\tau)}C$. The family of all $I^{(\tau)}O$ (resp. $I^{(\tau)}C$) sets in X will be denoted by $I^{(\tau)}O(X)$ (resp. $I^{(\tau)}C(X)$.)

Definition 2.4[4]Let (X, τ) be an ITS and $A \in IS(X)$. Then the intuitionistic interior (resp. intuitionistic closure) of A are defined by $int(A) = \bigcup\{K : K \in I^{(\tau)}O(X) \text{ and } K \subseteq A\}$ (resp. $cl(A) = \bigcap\{K : K \in I^{(\tau)}C(X) \text{ and } A \subseteq K\}$.)

In this study we use $I^{(\tau)}i(A)$ (resp. $I^{(\tau)}c(A)$) instead of $int(A)$ (resp. $cl(A)$).

Definition 2.5 Let (X, τ) be an ITS and an IS A in X is said to be

1. intuitionistic regular-open [6] (briefly $I^{(\tau)}RO$) if $A = I^{(\tau)}i(I^{(\tau)}c(A))$ and intuitionistic *regular*-closed (briefly $I^{(\tau)}RC$) if $I^{(\tau)}c(I^{(\tau)}i(A)) = A$.

2. intuitionistic pre-open [6] (briefly $I^{(\tau)}PO$) if $A \subseteq I^{(\tau)}i(I^{(\tau)}c(A))$ and intuitionistic pre-closed (briefly $I^{(\tau)}PC$) if $I^{(\tau)}c(I^{(\tau)}i(A)) \subseteq A$.

3. intuitionistic semi-open [6] (briefly $I^{(\tau)}SO$) if $A \subseteq I^{(\tau)}c(I^{(\tau)}i(A))$ and intuitionistic semi-closed (briefly $I^{(\tau)}SC$) if $I^{(\tau)}i(I^{(\tau)}c(A)) \subseteq A$.

4. intuitionistic α-open [8] (briefly $I^{(\tau)}\alpha O$) if $A \subseteq I^{(\tau)}i(I^{(\tau)}c(I^{(\tau)}i(A)))$ and intuitionistic α-closed (briefly $I^{(\tau)}\alpha C$) if $I^{(\tau)}c(I^{(\tau)}i(I^{(\tau)}c(A))) \subseteq A$.

5. intuitionistic β-open [8] (briefly $I^{(\tau)}\beta O$) if $A \subseteq I^{(\tau)}c(I^{(\tau)}i(I^{(\tau)}c(A)))$ and intuitionistic β-closed (briefly $I^{(\tau)}\beta C$) if $I^{(\tau)}i(I^{(\tau)}c(I^{(\tau)}i(A))) \subseteq A$.

6. intuitionistic b-open [9] (briefly $I^{(\tau)}bO$) if $A \subseteq I^{(\tau)}i(I^{(\tau)}c(A)) \cup I^{(\tau)}c(I^{(\tau)}i(A))$ and intuitionistic b-closed (briefly $I^{(\tau)}bC$) if $I^{(\tau)}i(I^{(\tau)}c(A)) \cap I^{(\tau)}c(I^{(\tau)}i(A)) \subseteq A$.

The family of all $I^{(\tau)}RO$ (resp. $I^{(\tau)}RC$, $I^{(\tau)}PO$, $I^{(\tau)}PC$, $I^{(\tau)}SO$, $I^{(\tau)}SC$, $I^{(\tau)}\alpha O$, $I^{(\tau)}\alpha C$, $I^{(\tau)}\beta O$, $I^{(\tau)}\beta C$, $I^{(\tau)}bO$ and $I^{(\tau)}bC$) sets in X will be denoted by $I^{(\tau)}RO(X)$ (resp.

$I^{(\tau)}RC(X)$, $I^{(\tau)}PO(X)$, $I^{(\tau)}PC(X)$, $I^{(\tau)}SO(X)$, $I^{(\tau)}SC(X)$, $I^{(\tau)}\alpha O(X)$, $I^{(\tau)}\alpha C(X)$, $I^{(\tau)}\beta O(X)$, $I^{(\tau)}\beta C(X)$, $I^{(\tau)}bO(X)$ and $I^{(\tau)}bC(X)$.)

Definition 2.6 [6,8,9] Let (X,τ) be an ITS and A be an $IS(X)$, then

1. intuitionistic regular-interior (resp. intuitionistic pre- interior, intuitionistic semi-interior, intuitionistic α-interior and intuitionistic β-interior) of A is the union of all $I^{(\tau)}RO(X)$ (resp. $I^{(\tau)}PO(X)$, $I^{(\tau)}SO(X)$, $I^{(\tau)}\alpha O(X)$ and $I^{(\tau)}\beta O(X)$) contained in A, and is denoted by $I^{(\tau)}Ri(A)$ (resp. $I^{(\tau)}Pi(A)$, $I^{(\tau)}Si(A)$, $I^{(\tau)}\alpha i(A)$ and $I^{(\tau)}\beta i(A)$.)

i.e. $I^{(\tau)}Ri(A) = \bigcup\{G : G \in I^{(\tau)}RO(X) \text{ and } G \subseteq A\}$,

$I^{(\tau)}Pi(A) = \bigcup\{G : G \in I^{(\tau)}PO(X) \text{ and } G \subseteq A\}$,

$I^{(\tau)}Si(A) = \bigcup\{G : G \in I^{(\tau)}SO(X) \text{ and } G \subseteq A\}$,

$I^{(\tau)}\alpha i(A) = \bigcup\{G : G \in I^{(\tau)}\alpha O(X) \text{ and } G \subseteq A\}$,

$I^{(\tau)}\beta i(A) = \bigcup\{G : G \in I^{(\tau)}\beta O(X) \text{ and } G \subseteq A\}$.

2. intuitionistic regular-closure (resp. intuitionistic pre-closure, intuitionistic semi-closure, intuitionistic α-closure, intuitionistic β-closure) of A is the intersection of all $I^{(\tau)}RC(X)$ (resp. $I^{(\tau)}PC(X)$, $I^{(\tau)}SC(X)$, $I^{(\tau)}\alpha C(X)$, $I^{(\tau)}\beta C(X)$) containing A, and is denoted by $I^{(\tau)}Rc(A)$ (resp. $I^{(\tau)}Pc(A)$, $I^{(\tau)}Sc(A)$, $I^{(\tau)}\alpha c(A)$, $I^{(\tau)}\beta c(A)$.)

i.e. $I^{(\tau)}Rc(A) = \bigcap\{G : G \in I^{(\tau)}RC(X) \text{ and } G \supseteq A\}$,

$I^{(\tau)}Pc(A) = \bigcap\{G : G \in I^{(\tau)}PC(X) \text{ and } G \supseteq A\}$

$I^{(\tau)}Sc(A) = \bigcap\{G : G \in I^{(\tau)}SC(X) \text{ and } G \supseteq A\}$,

$I^{(\tau)}\alpha c(A) = \bigcap\{G : G \in I^{(\tau)}\alpha C(X) \text{ and } G \supseteq A\}$,

$I^{(\tau)}\beta c(A) = \bigcap\{G : G \in I^{(\tau)}\beta C(X) \text{ and } G \supseteq A\}$.

Definition 2.7 [2,4] *Let A, $A_i (i \in J)$ be IS's in X, B, $B_j (j \in K)$ IS's in Y and $f: (X, \tau) \to (Y, \sigma)$ be a function. Then*

(a). $A_1 \subseteq A_2 \Rightarrow f(A_1) \subseteq f(A_2)$
(b). $B_1 \subseteq B_2 \Rightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$
(c). $A \subseteq f^{-1}(f(A))$ *and if* f *is one to one, then* $A = f^{-1}(f(A))$
(d). $f(f^{-1}(B)) \subseteq B$ *and if* f *is onto, then* $f(f^{-1}(B)) = B$
(e). $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$
(f). $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$
(g). $f(\cup A_i) = \cup f(A_i)$
(h). $f(\cap A_i) \subseteq \cap f(A_i)$ *and if* f *is one to one, then* $f(\cap A_i) = \cap f(A_i)$
(i). $f^{-1}(\tilde{Y}) = \tilde{X}$
(j). $f^{-1}(\tilde{\phi}) = \tilde{\phi}$
(k). $f(\tilde{X}) = \tilde{Y}$ *if* f *is onto*
(l). $f(\tilde{\phi}) = \tilde{\phi}$
(m). *If* f *is onto, then* $\overline{f(A)} \subseteq f(\bar{A})$*: and if furthermore,* f *is* $1-1$, *we have* $\overline{f(A)} \subseteq f(\bar{A})$
(n). $f^{-1}(\bar{B}) = \overline{f^{-1}(B)}$
(o). $B_1 \subset B_2 \Rightarrow f^{-1}(B_1) \subset f^{-1}(B_2)$

Definition 2.8*[4] Let (X, τ) and (Y, σ) be two intuitionistic topological spaces and $f: X \to Y$ be a function. Then f is said to be intuitionistic continuous if and only if the preimage of every intuitionistic openset in Y is intuitionistic open in X.*

Definition 2.9*[11] Let (X,τ) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function. Then f is said to be intuitionistic semi continuous if for every intuitionistic set V of Y, $f^{-1}(V)$ is semi open in X.*

Definition 2.10*[11] Let (X,τ) and (Y,σ) be two ITS's and let $f: X \to Y$ is called intuitionistic regular continuous if for every intuitionistic open set V of Y, $f^{-1}(V)$ is regular open in X.*

Definition 2.11*[11] Let (X,τ) and (Y,σ) be two ITS's and let $f: X \to Y$ is called intuitionistic pre continuous if for every intuitionistic open set V of Y, $f^{-1}(V)$ is pre open in X.*

Definition 2.12*[11] Let (X,τ) and (Y,σ) be two ITS's and let $f: X \to Y$ is called intuitionistic α-continuous if for every intuitionistic open set V of Y, $f^{-1}(V)$ is α-open in X.*

Definition 2.13*[11] A mapping $f: (X, \tau) \to (Y, \sigma)$ is said to be intuitionistic β-continuous (briefly $I\beta$-continuous), if the inverse image of each intuitionistic open set in Y is $I\beta$-open in X.*

Definition 2.14*[10] A map $f: (X, \tau) \to (X, \sigma)$ is called intuitionistic open(closed) if the image $f(A)$ is intuitionistic open(closed) in Y for every intuitionistic open(closed) set in X.*

Definition 2.16[10] A bijection $f:(X,\tau)\to(Y,\sigma)$ is called intuitionistic homeomorphism if f is both intuitionistic continuous and intuitionistic open map.

Definition 2.17[10] A function $f:(X,\tau)\to(Y,\sigma)$ is called intuitionistic open (resp. closed) if the image $f(A)$ in Y is intuitionistic open (resp. closed) for every intuitionistic open (resp. closed) set in X.

Definition 2.18[12] A function $f:(X,\tau)\to(Y,\sigma)$ is called intuitionistic b open (resp. b closed) if the image $f(A)$ in Y is intuitionistic b open (resp. Ib closed) for every intuitionistic open (resp. closed) set in X.

3. On intuitionistic b continuous functions

Here intuitionistic b continuous functions are defined and its relations with other existing intuitionistic functions are studied. Also some basic properties are investigated.

Definition 3.1A mapping $f:(X,\tau)\to(Y,\sigma)$ is said to be intuitionistic b continuous (briefly Ib continuous), if the inverse image of every intuitionistic open set in Y is Ib open set in X .

Theorem 3.1 Every intuitionistic continuous function $f:(X,\tau)\to(Y,\sigma)$ is ib continuous.

Proof: Let A be a intuitionistic open set of (Y,σ). Since $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic continuous, then $f^{-1}(A)$ is intuitionistic open in (X,τ), we know that every intuitionistic open set is Ib-open set, then $f^{-1}(A)$ is Ib-open in (X,τ). Thus $f:(X,\tau)\to(Y,\sigma)$ is Ib-continuous.

The converse of the above theorem need not be true from the following example.

Example 3.1 Let $X=\{a,b,c\}=Y$, with the intuitionistic topologies $\sigma=\{\tilde{\phi},\tilde{Y},(Y,\{a,b\},\phi)\}$and $\tau=\{\tilde{\phi},\tilde{X},(X,\phi,\{a,b\}),(X,\phi,\{b\}),(X,\phi,\{b,c\}),(X,\{c\},\{a,b\}),(X,\{c\},\{b\}),(X,\{a,c\},\{b\})\}$. Define $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b, f(b)=c, f(c)=a.$ Let $A=(Y,\{a,b\},\phi)$ is intuitionistic open set but $f^{-1}(A)=(X,\{a,c\},\phi)$ is not an intuitionistic open in (X,τ). Therefore $f:(X,\tau)\to(Y,\sigma)$ is not intuitionistic continuous. But $f^{-1}(A)=(X,\{a,c\},\phi)$is an I b-open in (X,τ). Thus $f:(X,\tau)\to(Y,\sigma)$ is Ib-continuous.

Theorem 3.2 Every intuitionistic regular (resp. pre and α) continuous function $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic b continuous map.

Proof: Let A be a intuitionistic open set of (Y,σ). Since $f:(X,\tau)\to(Y,\sigma)$ is intuitionistic regular (resp. pre and α) continuous, then $f^{-1}(A)$ is intuitionistic regular (resp. pre and α) open in (X,τ), we know that every intuitionistic regular (resp. pre and α) open set is Ib-open set, then $f^{-1}(A)$ is Ib-open in (X,τ). Thus $f:(X,\tau)\to(Y,\sigma)$ is Ib-continuous.

The converse of the above theorem need not be true from the following example.

Example 3.2 Let $X = \{a,b,c\} = Y$, with the intuitionistic topologies $\sigma = \{\tilde{\phi}, \tilde{Y}, (Y, \{a,b\}, \phi)\}$ and $\tau = \{\tilde{\phi}, \tilde{X}, (X, \phi, \{a,b\}), (X, \phi, \{b\}), (X, \phi, \{b,c\}), (X, \{c\}, \{a,b\}), (X, \{c\}, \{b\}), (X, \{a,c\}, \{b\})\}$. Define $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = b, f(b) = c, f(c) = a$. Let $A = (Y, \{c\}, \{b\})$ is intuitionistic open set then $f^{-1}(A) = (X, \{b\}, \{a\})$ is an I b-open in (X,τ). Thus $f:(X,\tau) \to (Y,\sigma)$ is Ib-continuous, but $f^{-1}(A) = (X, \{b\}, \{a\})$ is neither an intuitionistic regular open nor an intuitionistic pre open nor an intuitionistic α open in (X,τ). Therefore $f:(X,\tau) \to (Y,\sigma)$ is neither an intuitionistic regular continuous nor an intuitionistic pre continuous nor an intuitionistic α continuous map.

Theorem 3.3 Every intuitionistic semi continuous function $f:(X,\tau) \to (Y,\sigma)$ is ib continuous.

Proof: Let A be a intuitionistic open set of (Y,σ). Since $f:(X,\tau) \to (Y,\sigma)$ is intuitionistic semi continuous, then $f^{-1}(A)$ is intuitionistic semi open in (X,τ), we know that every intuitionistic semi open set is Ib-open set, then $f^{-1}(A)$ is Ib-open in (X,τ). Thus $f:(X,\tau) \to (Y,\sigma)$ is Ib-continuous.

The converse of the above theorem need not be true from the following example.

Example 3.3 Let $X = \{a,b\} = Y$, with the intuitionistic topologies $\tau = \{\tilde{\phi}, \tilde{X}, (X, \{a\}, \phi)\}$ and $\sigma = \{\tilde{\phi}, \tilde{Y}, (Y, \{b\}, \{a\})\}$. Define $f:(X,\tau) \to (Y,\sigma)$ by $f(a) = b, f(b) = a$. Let $A = (Y, \{b\}, \{a\})$ is intuitionistic open set but $f^{-1}(A) = (X, \{a\}, \{b\})$ is not an intuitionistic semi open in (X,τ). Therefore $f:(X,\tau) \to (Y,\sigma)$ is not intuitionistic semi continuous. But $f^{-1}(A) = (X, \{a\}, \{b\})$ is an I b-open in (X,τ). Thus $f:(X,\tau) \to (Y,\sigma)$ is Ib-continuous.

The following diagram shows the relationship between then existing intuitionistic continuous function.

$$\begin{tikzcd}
 & & IP-continuous \arrow[dr] \arrow[drr] & & \\
IR-continuous \arrow[r] & I-continuous \arrow[r] & I\alpha-continuous \arrow[r] \arrow[u] \arrow[d] & Ib-continuous & I\beta-continuous \\
 & & IS-continuous \arrow[ur] \arrow[urr] & &
\end{tikzcd}$$

Note: A $\longrightarrow$ B represents A implies B but not conversely.

Theorem 3.4 Let $f:(X,\tau) \to (Y,\sigma)$ be a mapping, where X and Y are ITS, then the following are equivalent.

(i) The function f is Ib-continuous.

(ii) The inverse image of intuitionistic closed set of Y is $I^{(\tau)}b$-closed set in X.

(iii) $f(I^{(\tau)}bcl(A)) \subseteq I^{(\sigma)}cl(f(A))$ for intuitionistic set A of X.

(iv) $I^{(\tau)}bcl(f^{-1}(B)) \subseteq f^{-1}(I^{(\sigma)}cl(B))$ for each intuitionistic set of Y.

Proof. (i) $\Rightarrow$ (ii): Let A be intuitionistic closed subset of Y, then $Y - A$ is intuitionistic open in Y. Since f is Ib-continuous, $f^{-1}(Y - A) = X - f^{-1}(A)$, is $I^{(\tau)}b$-open in X, which implies that $f^{-1}(A)$ is $I^{(\tau)}b$-closed in X.

(ii) $\Rightarrow$ (iii): Let A be an intuitionistic set of X. The $I^{(\sigma)}cl(f(A))$ is intuitionistic closed in Y. By (ii) $f^{-1}(I^{(\sigma)}cl(f(A)))$ is $I^{(\tau)}b$-closed in X and

$$f^{-1}(I^{(\sigma)}cl(f(A))) = I^{(\tau)}bcl(f^{-1}(I^{(\sigma)}cl(f(A))))$$

since $A \subseteq f^{-1}(f(A))$

we have $I^{(\tau)}bcl(A) \subseteq I^{(\tau)}bcl(f^{-1}(f(A)))$

$$\subseteq I^{(\tau)}bcl(f^{-1}(I^{(\sigma)}cl(f(A))))$$
$$= f^{-1}(I^{(\sigma)}cl(f(A)))$$

$I^{(\tau)}bcl(A) \subseteq f^{-1}(I^{(\sigma)}cl(f(A)))$

$f(I^{(\tau)}bcl(A)) \subseteq I^{(\sigma)}cl(f(A))$.

(iii) ⇒ (iv): Let B be an intuitionistic set of Y. Then by (iii) we have

$f(I^{(\tau)}bcl(f^{-1}(B))) \subseteq I^{(\sigma)}cl(f(f^{-1}(B)))$.

Hence $I^{(\tau)}bcl(f^{-1}(B)) \subseteq f^{-1}(I^{(\sigma)}cl(f(f^{-1}(B))))$

$\subseteq f^{-1}(I^{(\sigma)}cl(B))$

$I^{(\tau)}bcl(f^{-1}(B)) \subseteq f^{-1}(I^{(\sigma)}cl(B))$.

(iv) ⇒ (i): Let B be an intuitionistic set of Y. Then $I^{(\sigma)}clB = C$ is intuitionistic closed subset in Y so that $I^{(\sigma)}cl(C) = C$. Now by(iv)

$I^{(\tau)}bcl(f^{-1}(C)) \subseteq f^{-1}(I^{(\sigma)}cl(C))$

$= f^{-1}(C)$ (since C is intuitionistic closed)

we have $f^{-1}(C) \supseteq I^{(\tau)}bcl(f^{-1}(C))$

$= (I^{(\tau)}bint(f^{-1}(C^c)))^c$.

Hence $f^{-1}(C^c)$ is $I^{(\tau)}b$-open in X. That is $f^{-1}(C)$ is $I^{(\tau)}b$-closed. Therefore f is Ib-continuous.

Theorem 3.5 Let $f: (X, \tau) \to (Y, \sigma)$ be a mapping, where X and Y are intuitionistic topological spaces, then the followings are equivalent.

(i) The function f is Ib-continuous.

(ii) For each subset A of Y, $f^{-1}(I^{(\sigma)}int(A)) \subseteq I^{(\tau)}bint(f^{-1}(A))$.

Proof. (i) ⇒ (ii): Let $A = (X, A_1, A_2)$ be any intuitionistic set of Y. $I^{(\sigma)}int(A)$ is open set in Y and $f^{-1}(I^{(\sigma)}int(A))$ is a $I^{(\tau)}b$-open set in X, since f is Ib-continuous. As $f^{-1}(I^{(a)}int(A)) \subseteq f^{-1}(A)$and $f^{-1}(I^{(\sigma)}int(A)) \subseteq I^{(\tau)}bint(f^{-1}(A))$.

(ii) ⇒ (i): Let A be any intuitionistic open set of Y, so that $I^{(\sigma)}int(A) = A$.

$$f^{-1}(I^{(\sigma)}int(A)) = f^{-1}(A) \quad (1)$$

By (ii) $f^{-1}(I^{(\sigma)}int(A)) \subseteq I^{(\tau)}bint(f^{-1}(A))$

using equation (1)$f^{-1}(A) \subseteq I^{(\tau)}bint(f^{-1}(A))$.

Hence $f^{-1}(A)$ is $I^{(\tau)}b$-open, where A is intuitionistic open in Y. Therefore f is Ib-continuous.

Theorem 3.6 Let $f: (X, \tau) \to (Y, \sigma)$ be a single valued function then the followings are equivalent.

(i) The function f is Ib-continuous.

(ii) For each element $p \in X$ and each intuitionistic open set V in Y with $f(\tilde{p}) \in V$, there is a Ib-open set U in X, such that $\tilde{p} \in U$, $f(U) \subseteq V$.

Proof. (i)⇒(ii): Assume $f: (X, \tau) \to (Y, \sigma)$ is a single valued Ib-continuous function. Let $f(\tilde{p}) \in V$ and $V \subseteq Y$ an intuitionistic open set, then $\tilde{p} \in f^{-1}(V) \in I^{(\tau)}b$-open set in X. Choose $U = f^{-1}(V)$, then $\tilde{p} \in U$ and $f(U) \subseteq V$.

(ii)⇒(i): Let V be any intuitionistic open set in Y and $\tilde{p} \in f^{-1}(V)$, then $f(\tilde{p}) \in V$, there exists a $U_{\tilde{p}}$ is an $I^{(\tau)}b$-open set in X, such that $\tilde{p} \in U_{\tilde{p}}$ and $f(U_{\tilde{p}}) \subseteq V$. Then $\tilde{p} \in U_{\tilde{p}} \subseteq f^{-1}(V)$ and $f^{-1}(V) = \cup U_{\tilde{p}}$. Therefore, $f^{-1}(V)$ is $I^{(\tau)}b$-open set in X. Therefore f is Ib-continuous function.

4. Intuitionistic b-homeomorphisms and intuitionistic$b$_irresolute functions

Definition 4.1A mapping $f:(X,\tau) \to (Y,\sigma)$ is said to be intuitionistic b irresolute, (briefly Ib irresolute), if the inverse image of every intuitionistic b open set in Y is Ib open set in X.

Example 4.1$X = \{a, b, c\} = Y$, $\tau = \{\tilde{\phi}, \tilde{X}, (X, \{a\}, \{b\}), (X, \{a\}, \phi)\}$ and $\sigma = \{\tilde{\phi}, \tilde{Y}, (Y, \{b\}, \phi), (Y, \{b\}, \{c\}), (Y, \{b\}, \{a\}), (Y, \{a, b\}, \phi), (Y, \{b, c\}, \phi), (Y, \{a, b\}, \{c\}), (Y, \{b, c\}, \{a\}), (Y, \{b\}, \{a, c\})\}$. Define, $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b,\ f(b) = c,\ f(c) = a$. Then f is Ib-irresolute.

Definition 4.2 A bijection $f:(X,\tau)\to(X,\sigma)$ is called Ib-homeomorphism if f is both Ib-continuous and Ib-open.

Example 4.2 Let $X=\{a,b,c\}=Y$, $\tau=\{\tilde{\phi},\tilde{X},(X,\{a\},\{b\})\}$, $\sigma=\{\tilde{\phi},\tilde{Y},(Y,\{b\},\phi)\}$. Define $f:(X,\tau)\to(Y,\sigma)$ by $f(a)=b$, $f(b)=c$, $f(c)=a$ then the map f is bijective, Ib-continuous and Ib-open. So f is Ib-homeomorphism.

Theorem 4.1 Let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic function. Then the following statements are equivalent.

(i) f is a Ib-irresolute.

(ii) For each $I^{(\tau)}P\tilde{p}$ of X and each $I^{(\sigma)}b$-neighborhood N of $f(\tilde{p})$, there exists an $I^{(\tau)}b$-neighborhood V of $\tilde{p}$ such that $f(V)\subseteq N$.

(iii) For each $\tilde{p}$ belongs to X and each N belongs to $I^{(\tau)}bO(\tilde{p})$, there exists V belongs to $I^{(\sigma)}bO(f(\tilde{p}))$ such that $f(V)\subseteq N$.

Proof. (i) ⇒ (ii): Assume $\tilde{p}$ belongs to X and N is an $I^{(\sigma)}b$-open set in Y containing $f(\tilde{p})$. Since f is a Ib-irresolute, there exists $A=f^{-1}(N)$ be a $I^{(\tau)}b$-open set in X containing $\tilde{p}$ so $f(A)\subseteq f(f^{-1}(N))\subseteq N$.

(ii) ⇒ (iii): Assume that $N\subseteq Y$ is a $I^{(\sigma)}b$-open set containing $f(\tilde{p})$, then by hypothesis there exists a $I^{(\tau)}b$-open set G such that $\tilde{p}$ belongs to $G\subseteq f^{-1}(N)$. Therefore, $\tilde{p}$ belongs to $f^{-1}(N)\subseteq I^{(\tau)}cl(f^{-1}(N))$. This shows that $I^{(\tau)}cl(f^{-1}(N))$ is a Ib-neighborhood of $\tilde{p}$.

(iii) ⇒ (i): Let N be a $I^{(\sigma)}b$-open set in Y, then $I^{(\tau)}cl(f^{-1}(N))$ is $I^{(\tau)}b$-neighborhood of each $\tilde{p}$ belongs to $f^{-1}(N)$. Thus for each $\tilde{p}$ is a $I^{(\tau)}b$-interior point of $I^{(\tau)}cl(f^{-1}(N))$ which implies that

$$f^{-1}(N)\subseteq I^{(\tau)}int(I^{(\tau)}cl(f^{-1}(N)))$$
$$\subseteq I^{(\tau)}int(I^{(\tau)}cl(f^{-1}(N)))\cup I^{(\tau)}cl(I^{(\tau)}int(f^{-1}(N))).$$

Therefore $f^{-1}(N)$ is a $I^{(\tau)}b$-open set in X. Hence f is a Ib-irresolute function.

Theorem 4.2 Let $f:(X,\tau)\to(Y,\sigma)$ be an intuitionistic b-continuous and intuitionistic b-open. Then f is Ib-irresolute functions.

Proof. Let $A=(X,A_1,A_2)$ be any $I^{(\sigma)}b$-open set. Then $A\subseteq I^{(\sigma)}int(I^{(\sigma)}cl(A))\cup I^{(\sigma)}cl(I^{(\sigma)}int(A))$, since f is intuitionistic b-continuous and intuitionistic b-open it follows that

$$f^{-1}(A)\subseteq f^{-1}(I^{(\tau)}int(I^{(\tau)}cl(A))\cup I^{(\tau)}cl(I^{(\tau)}int(A)))$$
$$\subseteq I^{(\tau)}int(I^{(\tau)}cl(f^{-1}(A)))\cup I^{(\tau)}cl(I^{(\tau)}int(f^{-1}(A)))$$
$$\Rightarrow f^{-1}(A)\subseteq I^{(\tau)}int(I^{(\tau)}cl(f^{-1}(A)))\cup I^{(\tau)}cl(I^{(\tau)}int(f^{-1}(A))).$$

Therefore $f^{-1}(A)$ is $I^{(\tau)}b$-open. This shows that f is Ib-irresolute functions.

Theorem 4.3 Let $f:(X,\tau)\to(Y,\sigma)$ be an Ib-irresolute $\leftrightarrow$ for all intuitionistic set A of Y, $I^{(\tau)}bcl(f^{-1}(A))\subseteq f^{-1}(I^{(\sigma)}bcl(A))$.

Proof. Let f is Ib-irresolute function, now $I^{(\sigma)}bcl(A)$ is an $I^{(\sigma)}b$-closed set. Since $f^{-1}(A)\subset f^{-1}(I^{(\sigma)}bcl(A))$, it follows from the definition of Ib-closure that

$I^{(\tau)}bcl(f^{-1}(A))\subseteq f^{-1}(I^{(\sigma)}bcl(A))$.

Conversely, suppose that A is $I^{(\sigma)}b$-closed set in Y, then $I^{(\sigma)}bcl(A)=A$. Now by hypothesis,

$$I^{(\tau)}bcl(f^{-1}(A))\subseteq f^{-1}(I^{(\sigma)}bcl(A))=f^{-1}(A)$$
$$\Rightarrow I^{(\tau)}bcl(f^{-1}(A))=f^{-1}(A).$$

Thus $f^{-1}(A)$ is an $I^{(\tau)}b$-closed set and so f is an Ib-irresolute functions.

Proposition 4.1 Suppose $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\omega)$ are both Ib-irresolute, then $gof:(X,\tau)\to(Z,\omega)$ is Ib-irresolute function.

Proof. Let $E=(X,E_1,E_2)$ be an $I^{(\omega)}b$-open in (Z,ω). Since g is an Ib-irresolute, $g^{-1}(E)$ is an $I^{(\sigma)}b$-open in (Y,σ). Since f is also an Ib-irresolute $f^{-1}(g^{-1}(E))=(gof)^{-1}(E)$ is an $I^{(\tau)}b$-open in (X,τ). Thus (gof) is an Ib-irresolute function.

Theorem 4.4 If $f:(X,\tau)\to(Y,\sigma)$ is an Ib-continuous function and $f^{-1}(I^{(\sigma)}cl(B))\subseteq(f^{-1}(B))^{-1}$ for each B belongs to $I^{(\sigma)}bO(Y)$, then f is an Ib-irresolute function.

Proof. Let B belongs to $I^{(\sigma)}bO(Y)$,

$$f^{-1}(B)\subseteq f^{-1}(I^{(\sigma)}cl(I^{(\sigma)}int(B))\cup I^{(\sigma)}int(I^{(\sigma)}cl(B)))$$
$$\subseteq (I^{(\sigma)}cl(f^{-1}(I^{(\sigma)}int(B))\cup I^{(\sigma)}int(f^{-1}(I^{(\sigma)}cl(B)))$$
$$\subseteq I^{(\sigma)}cl(I^{(\sigma)}int(f^{-1}(B))\cup I^{(\sigma)}int(I^{(\sigma)}cl(f^{-1}(B)))))$$

$\Rightarrow f^{-1}(B)\subseteq I^{(\sigma)}cl(I^{(\sigma)}int(f^{-1}(B))\cup I^{(\sigma)}int(I^{(\sigma)}cl(f^{-1}(B))))$. Thus $f^{-1}(B)$ is $IbO(X)$.

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