Mathematical Analysis for warm and cold standby system with imperfect switching

Dr.Digvijay Singh
Associate Professor

Department Of Applied Science & Humanities
J.P.I.E.T., Meerut ,India
drdigvijay2008@rediffmail.com

Abstract : The author has considered cold and warm standby redundant systems with imperfect switching. The general expression of the reliability of the system R_n is obtained for $n \le 3$. Numerical values of reliability are also tabulated for different stress-strength distributions.

Key Words : Imperfect switch, stress-strength, warm standby, cold standby.

Introduction : The main aim of this paper is to obtain the system reliability R_3 for n-cold and n-warm standby system with imperfect switching for identical strength and comparing the results for both the systems. Mathematical formulations of the models are presented and the reliability of an n-cold and n-warm standby system with imperfect switch is obtained. We have assumed some specific distributions for the stress and strength involved. viz. Exponential, Gamma and Normal. To observe the change in the values of reliabilities with parameters involved, some numerical values of reliability .

Mathematical Solutions n-cold standby model with imperfect switching for identical strength :

Let the strengths of the n-components be the same, denoted by a random variable, say X . Let $Y_1, Y_2, ..., Y_n$ be the set of independent random variables representing the stresses on the n components, when they are activated. Thus $X, Y_i = 1, 2, ..., n$ and U and V are all independent random variables. Then the reliability, R_n of the system is given by,

$$R_n = R(1) + R(2) + ... + R(n) \tag{1}$$

$$R(1) = P[X \ge Y_1] \tag{2}$$

$$R(2) = P[X < Y_1, U \ge V \text{ and } X \ge Y_2] \tag{3}$$

$$R(3) = P[X < Y_1, U \ge V \text{ and } X < Y_2, U \ge V \text{ and } X \ge Y_3] \tag{4}$$

Then we get

$$R(r) = P[X < Y_1, U \ge V \text{ and } X < Y_2, ..., U \ge V \text{ and }$$
$$X < Y_{r-1}, U \ge V \text{ and } X \ge Y_r] \tag{5}$$

We have

$$R(1)=\int_{-\infty}^{\infty}\bar{F}(y)g_1(y)dy \tag{6}$$

$$R(2)=\left\{\int_{-\infty}^{\infty}\bar{F}(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\bar{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\bar{F}(y)g_2(y)dy\right\} \tag{7}$$

$$R(3)=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\bar{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}F(y)g_2(y)dy\right\}$$

$$\left\{\int_{-\infty}^{\infty}\bar{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\bar{F}(y)g_3(y)dy\right\} \tag{8}$$

Then

$$R(r)=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\bar{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}F(y)g_2(y)dy\right\}\ldots$$

$$\left\{\int_{-\infty}^{\infty}F(y)g_r(y)dy\right\}\left\{\int_{-\infty}^{\infty}\bar{H}(v)k(v)dv\right\}\left\{\int_{-\infty}^{\infty}\bar{F}(y)g_r(y)dy\right\} \tag{9}$$

Where $F(x)$ and $H(u)$ are the cumulative distribution functions (c.d.f.) of X and U respectively i.e.,

$$F(x)=\int_{-\infty}^{x}f(x)dx \text{ and } \bar{F}(x)=1-F(x) \text{ and } \bar{H}(u)=\int_{u}^{\infty}h(u)du$$

The reliability, R_n of the system is given by 1

$$R(1)=P[X\geq Y_1] \tag{10}$$

$$R(2)=P\left[X<Y_1,\{X\geq Z_2,(U\geq V \text{ and } X\geq Y_2)\}\right] \tag{11}$$

$$R(3)=P\left[X<Y_1,\{X\geq Z_2,(U\geq V \text{ and } X<Y_2) \text{ or } X<Z_2\}\right.$$

$$\left.\{X\geq Z_3,(U\geq V \text{ and } X\geq Y_3)\}\right] \tag{12}$$

Then, we get

$$R(r)=P\left[X<Y_1,\{X\geq Z_2,(U\geq V \text{ and } X<Y_2) \text{ or } X<Z_2\}\right.$$

$$\{X\geq Z_3,(U\geq V \text{ and } X<Y_3) \text{ or } X<Z_3\},\ldots,$$

$$\{X\geq Z_{r-1},(U\geq V \text{ and } X<Y_{r-1})$$

$$\left.\text{Or } X<Z_{r-1}\}\{X\geq Z_r,(U\geq V \text{ and } X\geq Y_r)\}\right] \tag{13}$$

$$R(1)=\int_{-\infty}^{\infty}\bar{F}(y)g_1(y)dy \tag{14}$$

$$R(2)=\int_{-\infty}^{\infty}F(y)g_1(y)dy\int_{-\infty}^{\infty}\bar{F}(z)w_2(z)dz\int_{-\infty}^{\infty}\bar{H}(v)k(v)dv\int_{-\infty}^{\infty}\bar{F}(y)g_2(y)dy \tag{15}$$

$$R(3)=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\bar{F}_2(z)w_2(z)dz\int_{-\infty}^{\infty}\bar{H}(v)k(v)dv\right.$$

$$\left.\int_{-\infty}^{\infty}F(y)g_2(y)dy+\int_{-\infty}^{\infty}F_2(z)w_2(z)dz\right\}$$

$$\left\{\int_{-\infty}^{\infty}\bar{F}_3(z)w_3(z)dz\int_{-\infty}^{\infty}\bar{H}(v)k(v)dv\int_{-\infty}^{\infty}\bar{F}(y)g_3(y)dy\right\} \tag{16}$$

Then

$$R(r)=\left\{\int_{-\infty}^{\infty}F(y)g_1(y)dy\right\}\left\{\int_{-\infty}^{\infty}\bar{F}_2(z)w_2(z)dz\int_{-\infty}^{\infty}\bar{H}(v)k(v)dv\right.$$

$$\left.\int_{-\infty}^{\infty}F(y)g_2(y)dy+\int_{-\infty}^{\infty}F_2(z)w_2(z)dz\right\}\left\{\int_{-\infty}^{\infty}\bar{F}_3(z)w_3(z)dz\right.$$

$$\left.\int_{-\infty}^{\infty}\bar{H}(v)k(v)dv\int_{-\infty}^{\infty}\bar{F}(y)g_3(y)dy+\int_{-\infty}^{\infty}F_3(z)w_3(z)dz\right\}\ldots$$

$$\left\{\int_{-\infty}^{\infty}\bar{F}_r(z)w_r(z)dz\int_{-\infty}^{\infty}\bar{H}(v)k(v)dv\int_{-\infty}^{\infty}\bar{F}(y)g_r(y)dy\right\} \tag{17}$$

Where, r = 1,2,…,n

Here $F(x)$ and $H(u)$ are the c.d.f.'s of X and U respectively.

Conclusion :

$R(r), r=1,2,3$ we evaluate $R(1), R(2)$ and $R(3)$ for different distributions from their expressions. The strength parameter θ increases then reliability $R(3)$ decreases. When the stress parameter α_1 increases $R(1)$ also increases. The value of marginal reliability $R(1)$ becomes 0.7500, 0.6000, 0.5000, 0.4286 and 0.3750 respectively for $\theta = 1,.2,.3,.4$ and 5.

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