TWO DIMENSIONAL FLOW OF A VISCOUS, INCOMPRESSIBLE AND ELECTRICALLY CONDUCTING FLUID PAST A POROUS FLAT PLATE

Dr. Digvijay Singh
Associate Professor

Department Of Applied Science & Humanities
J.P.I.E.T., Meerut ,India
drdigvijay2008@rediffmail.com

Abstract : In this paper the author has considered the flow of viscous incompressible electrically conducting fluid past a porous flat plate is determined in presence of a transverse magnetic field, when there is step change in suction velocity. Laplace transform technique is employed to obtain the expression for velocity distribution, skin fraction and displacement thickness. Results have been shown graphically for different values of suction parameter λ, magnetic parameter M and time t.
Key waords : Boundary layer, Flat Plate, unsteady flow, suction velocity.

Introduction : In this paper we propose to study the problem of Purohit and Goyal for an electrically conducting fluid in presence of transverse magnetic field. For time t < 0 there is a steady laminar boundary layer flow for constant normal suction velocity and constant free stream velocity. At t = 0 there is a step change in suction velocity and the problem becomes unsteady. Using Laplace Transform technique the expressions for velocity field, skin fraction and displacement thickness are obtained which have been shown graphically for different values of suction parameter λ, magnetic parameter M and time t.

Solution of the problem : The equations of motion in the present case then reduce to the following single equation

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}} = -\frac{1}{p}\frac{\partial \bar{p}}{\partial \bar{x}} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0{}^2 \bar{u}}{\rho} \tag{1}$$

Boundary conditions :

$$\begin{aligned} &\bar{u}(0,\bar{t}) = 0; \bar{u}(\infty,\bar{t}) = U_\infty, \\ &\bar{v}(0,\bar{t}) = v_1 \quad \text{(constant)}, \ \bar{t} \le 0 \\ &\qquad\quad = v_2 \quad \text{(constant)}, \ \bar{t} > 0 \end{aligned} \tag{2}$$

Initial conditions :

$$\bar{u} = U_\infty\left[1 - \exp\left(-\frac{m}{v}|v_1|\bar{y}\right)\right] \tag{3}$$

where

$$m = \frac{1}{2}\left[1 + \left(1 + \frac{4\sigma B_0{}^2 v}{\rho|v_1{}^2|}\right)^{1/2}\right] \tag{4}$$

Equation (1) becomes

$$-\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{x}}=\frac{\sigma}{\rho}B_0^{\,2}U_\infty \tag{5}$$

Solve equation (5) then equation (1) gives

$$\frac{\partial \bar{u}}{\partial \bar{t}}+\bar{v}\frac{\partial \bar{u}}{\partial y}=v\frac{\partial^2 \bar{u}}{\partial \bar{y}^2}+\frac{\sigma_0^{\,2}}{\rho}\left(U_\infty-\bar{u}\right) \tag{6}$$

in equations (6), using (2) and (3), we get

$$\frac{\partial u}{\partial t}+\bar{v}\frac{\bar{v}}{|v_1|}\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+M\left(1-u\right) \tag{7}$$

boundary and initial conditions are

$$u(0,t)=0,u(\infty,t)=1,$$

$$\frac{1}{|v_1|}\bar{v}(0,t)=-1,t\le 0$$

$$=-\lambda,t>0 \tag{8}$$

or $$u(y,0)=1-\exp(-my),m=\frac{1}{2}\left[1+\sqrt{1}+4M\right] \tag{9}$$

Using the Laplace transform technique, equation (7) reduces to :

$$\frac{d^2u^*}{dy^2}+\lambda\frac{du^*}{dy}=(M+s)u^*=\exp(-my)-1-\frac{M}{s} \tag{10}$$

Where $\left(u^*(y,s)=\int_0^\infty e^{-st}u(y,t)dt\right)$

$$u^*(0,s)=0,u^*(\infty,s)=\frac{1}{s} \tag{11}$$

The solution of equation (10) is given by

$$u^*(y,s)=\frac{1}{s}-\frac{1}{s+\left(M+\lambda m-m^2\right)}\exp(-my)$$

$$+\left|\frac{1}{s+\left(M+\lambda m-m^2\right)}-\frac{1}{s}\right|\exp\left[-\frac{1}{2}\left(\lambda+\sqrt{\ }\left\{\lambda^2+4(M+s)\right\}y\right]\right. \tag{12}$$

Inverse Laplace transform of equation (12) becomes

$$u(y,t)=1-\exp\left[-\left(M+\lambda m-m^2\right)t-my\right]$$

$$+\frac{1}{2}\exp\left[\frac{-1}{2}y-\left(m+\lambda m-m^2\right)t\right]$$

$$\times\left[\exp.\left(\overline{\frac{1}{2}\lambda-my}\right)Erfc\left(\frac{y}{2\sqrt{t}}+\overline{\frac{\lambda}{2}-m\sqrt{t}}\right)\right]$$

$$+\exp.\left(\overline{\frac{-\lambda}{2}-my}\right)Erfc\left(\frac{y}{2\sqrt{t}}-\overline{\frac{\lambda}{2}-m\sqrt{t}}\right)$$

$$-\frac{1}{2}\exp.\left(\frac{-1}{2}\lambda y\right)\left[\exp.\left(\frac{1}{2}\sqrt{\left\{\lambda^2+4M\right\}}y\right)\right]Erfc\left[\frac{1}{2}\frac{y}{\sqrt{t}}+\sqrt{\left\{\lambda^2+4M\right\}t}\right]$$

$$+\exp\left(\frac{-1}{2}\sqrt{\left\{\lambda^2+4M\right\}}y\right)Erfc\left(\frac{1}{2}\frac{y}{\sqrt{t}}-\frac{1}{2}\sqrt{\left\{\lambda^2+4M\right\}t}\right) \quad (13)$$

The skin fraction τ_w is given by

$$\tau_w=\rho\left|v_1\right|U_\infty\left(\frac{\partial u}{\partial y}\right)_{y=0} \quad (14)$$

Using equation (13), equation (14) becomes

$$\frac{\tau_w}{\rho\left|v_1\right|U_\infty}=\exp\left[-\left(M+m\lambda-m^2\right)t\right]$$

$$\left[\left(m-\frac{1}{2}\lambda\right)\left(1+Erf.\left(\frac{\lambda}{2}-m\right)\sqrt{t}\right)-\frac{1}{\sqrt{\pi t}}\exp-\left(\frac{\lambda}{2}-m\right)^2 t\right]$$

$$+\frac{1}{2}\left[\lambda+\sqrt{\left(\lambda^2+4M\right)}Erf\left(\frac{1}{2}\left(\lambda^2+4M\right)t\right)\right.$$

$$\left.+\left(\frac{2}{\sqrt{(\pi t)}}\exp\left(-\frac{1}{4}\left(\lambda^2+4M\right)t\right)\right)\right] \quad (15)$$

The displacement thickness δ_1, as defined by Watson (9) is given by

$$\frac{\left|v_1\right|}{v}\delta_1=\int_0^\infty\left[1-u\left(y,t\right)\right]dy. \quad (16)$$

Using equation (13), equation (16) gives,

(i) When $M\neq 0,\lambda\neq m,$

$$\frac{\left|v_1\right|}{v}\delta_1=\frac{\lambda\left(\lambda-1\right)}{2M\left(\lambda-m\right)}\exp\left(-Mt\right)-\frac{\lambda}{2M}$$

$$+\frac{\lambda\left(1-\lambda\right)}{2M\left(\lambda-m\right)}\exp\left(-mt\right)Erf\left(\frac{1}{2}\lambda\sqrt{t}\right)$$

$$+\frac{1}{2M}\sqrt{\left(\lambda^2+4M\right)}Erf\left(\frac{1}{2}\sqrt{\left(\lambda^2+4M\right)t}\right)$$

$$+\frac{(\lambda-2m)}{2m(\lambda-m)}\exp\{-m(\lambda-1)t\}\left[1+Erf\left(\overline{\frac{\lambda}{2}-m\sqrt{t}}\right)\right] \quad (17)$$

(ii) When $m \neq 0, \lambda = m$

$$\frac{|v_1|}{v}\delta_1 = \frac{m}{2M} - \sqrt{\left(\frac{t}{\pi}\right)}\exp\left[-\left(M+\frac{m^2}{4}\right)t\right]$$
$$+\left(\frac{1}{m}+\frac{m}{2M}+\frac{1}{2}mt\right)\exp(-Mt)\,Erfc\left(\frac{m}{2}\sqrt{t}\right)$$
$$-\frac{1}{2M}\left(m^2+4M\right)Erf.\left(\frac{1}{2}\sqrt{\left(m^2+4M\right)t}\right) \quad (18)$$

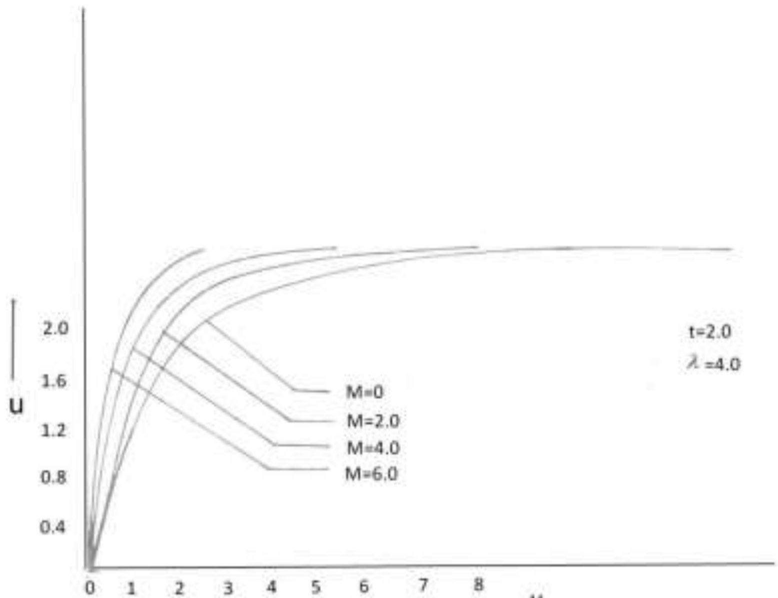
(iii) When $M = 0, \lambda \neq 1$

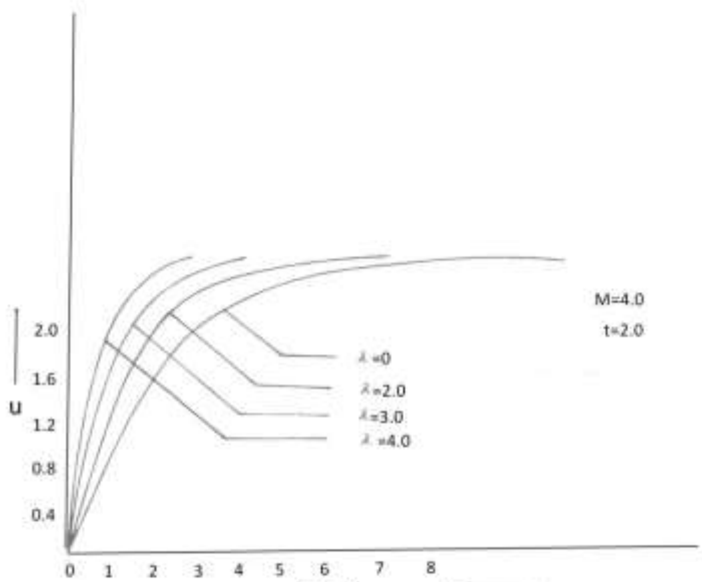
$$\frac{|v_1|}{v}\lambda_1 = \sqrt{\left(\frac{t}{\pi}\right)}\exp\left(\frac{-1}{4}\lambda^2 t\right)+\frac{1}{2}\frac{(\lambda-2)}{(\lambda-1)}\exp\{-(\lambda-1)t\}$$
$$+\frac{\lambda}{2(\lambda-1)}\frac{1}{2}\lambda t-\left[\frac{\lambda}{2(\lambda-1)}-\frac{1}{\lambda}-\frac{1}{2}\lambda t\right]Erf.\left(\frac{1}{2}\lambda\sqrt{t}\right)$$
$$+\frac{1}{2}\frac{(\lambda-2)}{(\lambda-1)}\exp\{-(\lambda-1)t\}Erf\left(\overline{\frac{\lambda}{2}-1}\sqrt{t}\right) \quad (19)$$

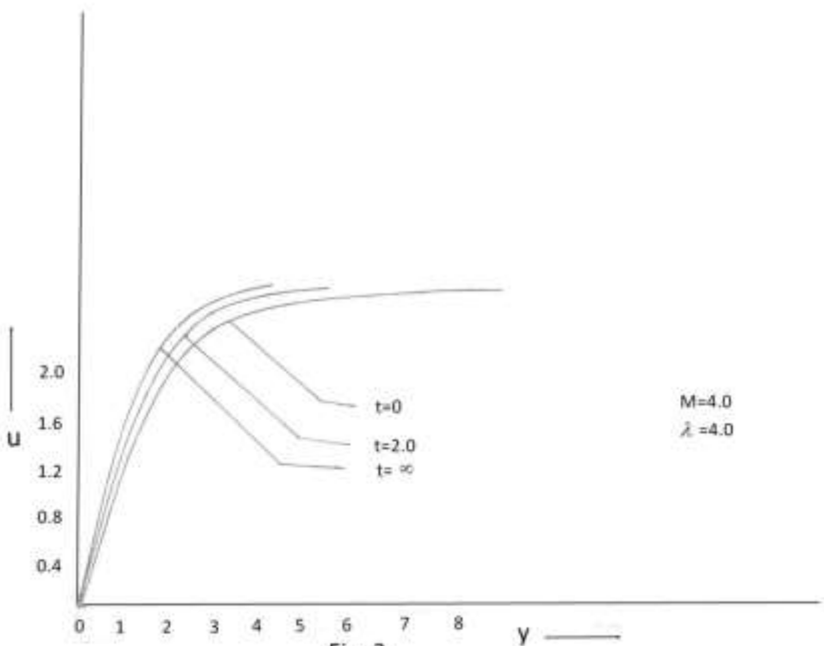
(iv) $M = 0, \lambda = 1,$

$$\frac{|v_1|}{v}\delta_1 = 1 \quad (20)$$

Numerical discussions : Figure 1, 2 and 3 for different values of t, λ and M and if is observed that velocity increases with M, λ and t. Hence it is concluded that due to increase in one of these parameters, boundary layer formation becomes sharper. Figures 4 and 5 deal with skin friction for different values of t and λ and it is seen that skin friction increases with M for fixed t or λ and also with t or λ for fixed M.

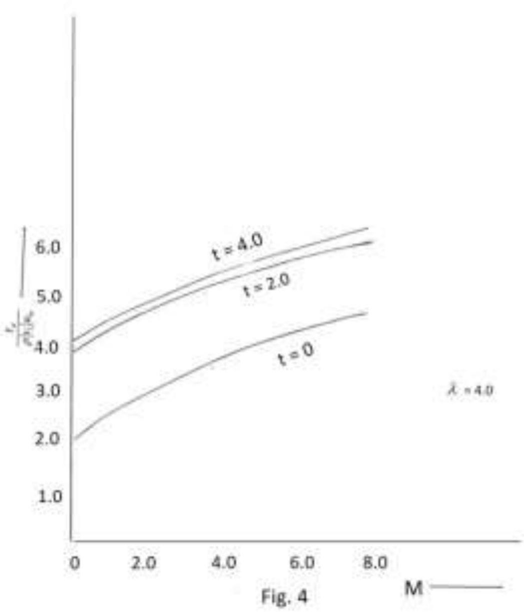

u
2.0
1.6
1.2
0.8
0.4
0 1 2 3 4 5 6 7 8
y
t=2.0
λ =4.0
M=0
M=2.0
M=4.0
M=6.0
Fig.1


u
2.0
1.6
1.2
0.8
0.4
0 1 2 3 4 5 6 7 8
y
M=4.0
t=2.0
λ =0
λ =2.0
λ =3.0
λ =4.0
Fig. 2



u
2.0
1.6
1.2
0.8
0.4
0 1 2 3 4 5 6 7 8
y
t=0
t=2.0
t= ∞
M=4.0
λ =4.0

Fig. 3



6.0
5.0
4.0
3.0
2.0
1.0
0 2.0 4.0 6.0 8.0
M
t = 4.0
t = 2.0
t = 0
λ = 4.0

Fig. 4

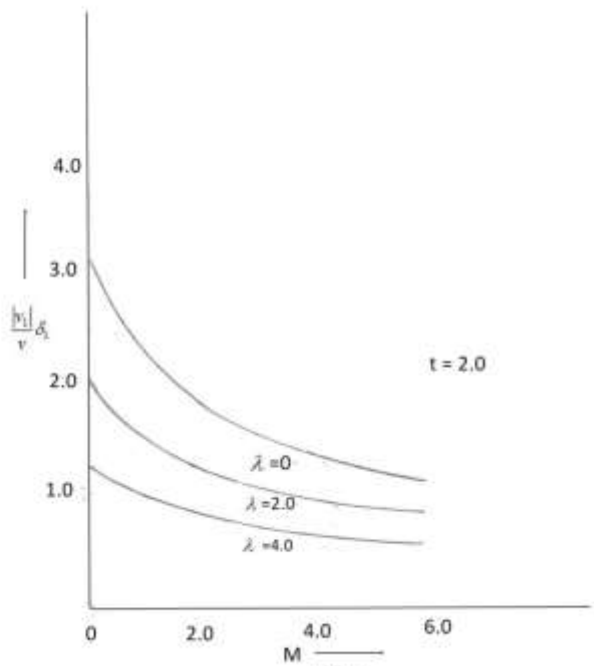


Fig. 5

References :

1. Agarwal, R.S. and Dhanapal, C., Indian J. Pure Appl. Math. 20(5), 513, (1989).
2. Choubey, K.R. and Yadav, R.R., Indian J. Theo. Physics, 34(4), 321 (1986).
3. Endelyi, A. et al., Tables of Integral Transforms, Vol. 1. McGraw Hill Book Co. Inc. New Yourk (1954).
4. Kelly, R.E., Quart J. Mech. Appl. Math. 18 (1955).
5. Purohit, G.N. and Goyal, M.C. Proc. Ind. Acad. Sci. 81, 215 (1975).
6. Schlichting, H., Boundary Layer Theory, McGraw Hill Book Co. Inc. New York (1968).
7. Soundalgekar, V.M. and Uplekar, A.G., Czech J. Phys. B29, 859 (1979).
8. Stuart, J., Proc. Roy. Soc., A 213, 116 (1955).
9. Watson, J., Quart. J. Mech. Appl. Math. 11, 302 (1955).