SOME MATHEMATICAL PROBLEMS OF A NON-HOMOGENEOUS CONE WITH SPHERICAL CAPS

Dr.Digvijay Singh
Associate Professor
Department Of Applied Science
J.P.I.E.T., Meerut ,India
drdigvijay2008@rediffmail.com

Abstracts : In this paper the author has considered problem of the forced torsional vibration of a non-homogeneous cone with spherical caps, where the density as well as the rigidity modulus varies according to some exponential law from the vertex of the cone, the applying periodic terminal couples has been solved and the effect of non-homogeneity on the stresses have been exhibited.

Keywords : Terminal couples, Homogeneous, density, stresses.

Introduction : We have considered the classical theory of linear elasticity solutions of Problems are numerous for materials whose elastic Co-efficient are same at all points within the body in question, there are materials where these vary considerable from point to point. The recent trend of research concerning non-homogeneous elasticity may by found in works of Olszak (1), Gibson (2) and Huston (3), Mukhopandhyay (4) in his paper solved a problem of the forced torsional vibration of a non-homogeneous cone with spherical caps twisted by periodic terminal couples, in which non-homogeneity assumed are in the rigidity modulus and density of the material with some power of distance from the vertex of the cone.

Solutions : The spherical polar Co-ordinates (r,θ,ϕ) with the vertex of the cone as origin and the axis of the cone coinciding with the axis of the Co-ordinate system.
In our problem we assume

$$\left.\begin{aligned} U_r &= u\theta = 0 \\ U_\phi &= f(r)\sin\theta e^{ipt} \end{aligned}\right\} \tag{1}$$

$$\tau r\phi = \mu\left[\frac{df}{dr} - \frac{f}{r}\right]\sin\theta - e^{ipt} \tag{2}$$

Considering the dimensionless Co-ordinate

$$\beta = \frac{r}{a} \tag{3}$$

$$\mu = \mu_0 e^{-k\beta}$$
$$p = p_0 e^{-k\beta} \tag{4}$$

$$\frac{\partial}{\partial r}\tau_{r\phi} + \frac{1}{r}\frac{\partial \tau_{\theta\phi}}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial \tau\phi\phi}{\partial\phi} + \frac{1}{r}\left[3\tau_{r\phi} + 2\tau_{\theta\phi}\cot\theta\right] = p\frac{\partial^2 U_\phi}{\partial t^2} \tag{5}$$

Equation (5) with the help of (3) and (4) takes the form

$$\beta^2 \frac{d^2 f}{d\beta^2} + \beta \frac{df}{d\beta}(2 - k\beta) + (q\beta + k\beta - 2) f = 0$$

Where $c^2 = \frac{\mu_0}{p_0}$ and $q = \frac{a^2 p^2}{c^2}$ (6)

Putting $f(\beta) = \beta u(\beta)$ in equation (6) becomes

$$\beta \frac{d^2 u}{d\beta^2} + (4 - k\beta)\frac{du}{d\beta} + q\beta u = 0 \tag{7}$$

Again putting $u(\beta) = e^{h\beta} v(\beta)$ in equation (7), becomes

$$\beta^2 \frac{d^2 v}{d\beta^2} + \{4 + (2h - k)\beta\}\frac{dv}{d\beta} + 4hv = 0 \tag{8}$$

Where $h = \frac{1}{2}\left[k \pm \sqrt{(k^2 - 4q)}\right]$

We get $v(\beta) = W(z); Z = -(2h - k)\beta, 2h - k > 0$

Equation (8) becomes

$$z\frac{d^2 W}{dZ^2} + (4 - Z)\frac{dW}{dZ} + \alpha W = 0 \tag{9}$$

$$W = (A + BInz)F(\alpha;4;z) + B\psi(z) \tag{10}$$

$$f(\beta) = \beta e^{h\beta}\left[(A + BInz)F(\alpha;4;z) + B\psi(z)\right] \tag{11}$$

Hence

$$\left.\begin{aligned} U_\phi &= \beta e^{h\beta}\left[(A + BInz)F(\alpha;4;z) + B\psi(z)\right]\sin\theta e^{ipt} \\ \tau_{r\phi} &= \frac{\mu}{\alpha} h\beta e^{(h-k)\beta}\left[A\Phi_1(z) + \beta\Phi_2(z)\right]\sin\theta e^{ipt} \end{aligned}\right\} \tag{12}$$

Where

$$\Phi_1(z) = F(\alpha;4;z) + k_1 F'(\alpha;4;z) \tag{13}$$

$$\Phi_2(z) = \left(\frac{k_1}{z} + Inz\right)F(\alpha;4;z) + k_1 InzF'(\alpha;4;z)$$
$$+\psi(z) + k_1\psi'(z) \tag{14}$$

$$K_1 = \frac{k - 2h}{h} \tag{15}$$

Boundary Conditions :

Let the cone with semi-vertical angle γ with two spherical caps be bounded by two radii $\beta = \beta_1$ and $\beta = \beta_2$ and it be acted by equal and opposite twisting couples. Then the boundary conditions are :

$$\int_{\theta=0}^{\gamma}\int_{\phi=0}^{2\pi} \tau_{r\phi}\alpha^3\beta^3 \sin^2\theta.d\theta.d\phi\Big|_{\beta=\beta_1} = Me^{ipt}$$

$$\int_{\theta=0}^{\gamma}\int_{\phi=0}^{2\pi} \tau_{r\phi}\alpha^3\beta^3 \sin^2\theta d\theta d\theta\Big|_{\beta=\beta_2} = Me^{ipt} \tag{16}$$

From (12) and (16) the contents A and B are given by

$$A - ML\sigma_1 \;;\quad B = -ML\sigma_2 \tag{17}$$

$$\frac{1}{f_2} = \beta_2{}^4 e^{(h-k)} \beta_2$$

and $\Phi_1(d_1), \Phi_1(d_2), \Phi_2(d_1), \Phi_2(d_2)$ are given from equation (14), (15).

Conclusion :

The effect of non-homogeneity on the stress at a given time t = T, we put $\beta_1 = 1, \beta_2 = 2, q = 2, \gamma = 45^0$, and

$$\frac{\pi\alpha^3 \tau_{p\phi}}{Me^{iPT}} = P$$

We have calculated the value of P for $\theta = 30^0$ and for different values of β in $1 \le \beta \le 2 (i) K = 3$ and $(ii) K = 0$ (homogeneous case) from equation (12) and (17).

References :

1. Olszak, W., Non-homogeneity in elasticity and plasticity (Pergamon press), London (1959).
2. Gibson, R.E., Geotechnique, 17, 58-67 (1967).
3. Huston, R.L., - ZAMM 44, 573-578 (1964).
4. Mukhopadhyay, A., Ind. Jour. Of Theo. Physics. Vol. 24, No. 3 (1976).
5. Murphy, G., Ordinary differential equations and their solutions, D.Van Nostrand Company, Inc.