Effect of Pressure Ratio on Tangential/Axial Stress in Thick Spherical Vessels

Sukhjinder Singh Sandhu[a], Tejeet Singh[b] and V.K.Gupta[c]

[a] *Assistant Professor, Department of Mechanical Engg, S. B. S. S. T. C., Ferozepur, 152004, INDIA*

[b] *Associate Professor, Department of Mechanical Engg, S. B. S. S. T. C., Ferozepur, 152004, INDIA*

[c] *Professor, Mech. Engg., UCoE, Punjabi Univ., Patiala-147002, INDIA*

Abstract

The study describes the effect of varying pressure ratio on tangential/axial stress in thick walled spherical vessels under uniform temperature field. The spherical vessel is made of functionally graded material and plain strain condition was assumed. The creep behaviour of material is governed by threshold stress based creep law. The study reveals that for assumed pressure ratio the threshold stress decreases linearly with maximum value at inner radius and minimum value at outer radius of spherical vessel. On the other hand the creep parameter *M* increases from inner radius to outer radius. The tangential/axial stress which is dependent upon creep parameters remains compressive at inner radius, however, the value become tensile for pressure ratio 3, 4 and 5.

Key words: Sphere; Tangential Stress, Creep; Pressure Ratio.

1. Introduction

The spherical pressure vessels are extensively used in many industrial applications such as gas cooled nuclear reactors, gas or liquid tanks in petroleum industries, injection pumps in automobiles *etc*. (Sultana and Mondal, 2012). It is seen that many application involving spherical pressure vessels are subjected to high temperatures for long duration and therefore material of the vessel undergoes creep stresses and thereby reducing its lifetime. Many of the structures used under high temperature are designed for minimum creep rate in the secondary stage creep during their service time. Therefore, estimation of secondary stage creep is very important from design point of view. Spherical pressure vessels are also used in power plants and in petrochemical industry, where the wall of the spherical vessel also undergoes a radial thermal gradient (Johnson and Khan, 1963; Sim, 1973; Durban and Baruch, 1974) of the order of 50°C and it requires a different analysis as compared to the vessels exposed to constant temperature conditions.

Creep analyses of thick-walled spherical pressure vessels subjected to internal and external pressures is important in engineering applications. The strains considered are assumed to be large which necessitates the use of finite strain theory for evaluating the expressions for stresses, creep strains and strain rates. The general theory developed by Bhatnagar and Arya (1973) has been applied to the solution of a specific problem using Norton's law of creep. Steady-state creep analysis of thick-walled spherical pressure vessels with varying creep properties has been presented by You and Ou (2008). Stresses in a spherical pressure vessels undergoing creep and dimensional changes has been presented by Miller (1995). Analytical and numerical analysis for the Functionally Graded thick sphere under combined pressure and temperature loading has been presented by Bayat et al (2012). In all these studies the strains are assumed to be infinitesimal and the deformation is referred with respect to original dimensions of the sphere.

The basic idea of using FGM as vessel material is to add more SiC particles at locations with high strain rates so as to reduce order of strain rates and achieve a uniform distribution of strain rates. The content of silicon carbide in Al matrix has been assumed to vary linearly, with maximum amount at the inner radius and minimum at the outer radius of spherical vessel. A mathematical model has been developed and used to estimate the tangential stress for different pressure ratios.

2. Governing Equations and Mathematical Solution for Incompressible Material

For aluminium matrix composites undergoing secondary stage creep, the relation between effective strain rate and effective stress can be described by the well-known threshold stress *(σ_o)* based creep law (Singh and Gupta, 2011) and is given by,

$$\dot{\varepsilon}_e = \{M(r)(\sigma_e - \sigma_0(r))\}^n \tag{1}$$

The symbols *M(r)* and $\sigma_o(r)$ are known as creep parameters and will vary with radius. Further the value of these parameters is dependent upon temperature, reinforcement content and reinforcement size in a composite.

In the present study, the values of M and σ_0 have been extracted from the experimental creep results reported for Al-SiC$_P$ composite under uniaxial loading, Pandey et al (1992). However, in order to determine the values of creep parameters M and σ_o for various combinations of P, V and T, not reported by Pandey et al (1992) the regression analysis has been performed. The developed regression equations are given below,

$$M(r) = 0.0287611 - \frac{0.00879}{P} - \frac{14.02666}{T} - \frac{0.032236}{V(r)} \quad (2)$$

$$\sigma_o(r) = -0.084P - 0.0232T + 1.1853\big(V(r)\big) + 22.207 \quad (3)$$

Where P, $V(r)$, T, $M(r)$ and $\sigma_o(r)$ are respectively the particle size, particle content, temperature, creep parameter and threshold stress at any radius (r). In the present work particle size is assumed as 1.7 μm while operating temperature is kept as 350°C. The reinforcement content *i.e.* silicon carbide particle (SiCp), in the sphere is assumed to vary linearly from the inner radius (a) to the outer radius (b). The variation of SiCp is described by following equation (Singh and Gupta, 2011)

Particle content $$V(r) = V_{max} - \frac{(r-a)}{(b-a)}(V_{max} - V_{min}) \quad (4)$$

Where, V_{max} and V_{min} are respectively the maximum and minimum SiCp, at the inner and outer radii. Thus for a given FG sphere containing non particle gradient both the creep parameters will be function of radius. The value of M(r) and σ_o (r) at any radius could be estimated by substituting the reinforcement content V(r) from equation (4) in into equation (2) and (3).

Let us consider a thick-walled, spherical vessel made of functionally graded Al-SiCp composite. The vessel is assumed to have inner and outer radii as a and b respectively and is subjected to both internal pressure p and external pressure q.

The steady state creep deformations in thick-walled spherical pressure vessels are spherically symmetric. The geometric relationships between radial and circumferential strain rates and radial displacement rate are

$$\dot{\varepsilon}_r = \frac{d\dot{x}_r}{dr} \quad (5)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{x}_r}{r} \quad (6)$$

Eliminating $\dot{x}_r$, from Eqns. (5) and (6) the deformation compatibility equation is obtained as,

$$r\frac{d\dot{\varepsilon}_\theta}{dr} = (\dot{\varepsilon}_r - \dot{\varepsilon}_\theta) \quad (7)$$

Considering the equilibrium of forces on an element of spherical vessel along the radial direction, we get the equilibrium equation as below,

$$\frac{r}{2}\frac{d\sigma_r}{dr} = (\sigma_\theta - \sigma_r) \quad (8)$$

where σ_θ and σ_r are respectively the circumferential and radial stresses.

Since the material of the sphere is assumed to be incompressible, therefore,

$$\dot{\varepsilon}_r + \dot{\varepsilon}_\theta + \dot{\varepsilon}_z = 0 \quad (9)$$

Where, $\dot{\varepsilon}_z$ is the axial strain rate.

The constitutive equations (Singh and Gupta, 2011) for creep along the principal direction r, θ and z can be written as below considering spherical symmetry, $\sigma_\theta = \sigma_z$,

$$\dot{\varepsilon}_r = \frac{\dot{\varepsilon}_e}{\sigma_e}(\sigma_r - \sigma_\theta) \quad (10)$$

$$\dot{\varepsilon}_\theta = \frac{\dot{\varepsilon}_e}{2\sigma_e}(\sigma_\theta - \sigma_r) \quad (11)$$

$$\dot{\varepsilon}_z = \frac{\dot{\varepsilon}_e}{2\sigma_e}(\sigma_z - \sigma_r) \quad (12)$$

Where $\dot{\varepsilon}_e$ is the effective strain rate, σ_e is the effective stress, and σ_r, σ_θ and σ_z are the stresses along r, θ and z directions respectively.

From Eqs. (10) and (11), the relationship between the radial and circumferential strain rates can be obtained as,

$$\dot{\varepsilon}_r = -2\dot{\varepsilon}_\theta \quad (13)$$

Substituting Eqn. (13), into Eqn. (9), the deformation compatibility becomes,

$$\frac{d\dot{\varepsilon}_\theta}{\dot{\varepsilon}_\theta} = -3\frac{dr}{r} \quad (14)$$

The integration of Eqn. (14), gives the circumferential strain rate as,

$$\dot{\varepsilon}_\theta = \frac{A_1}{r^3} \quad (15)$$

Where, A_1 is constant of Integration.

The effective stress in thick-walled spherical vessels subjected to internal pressure is assumed to be expressed by von-Mises equation (Singh and Gupta, 2011),

$$\sigma_e = (\sigma_\theta - \sigma_r) \tag{16}$$

Equilibrium equation (8) along with constitutive Eqs. (10) - (12) have been solved to obtain Tangential/Axial stress as given below,

$$\sigma_\theta = \frac{A_2(r)}{r^{3/n}} + \sigma_o(r) + 2\int_a^r \frac{A_2(r)}{r^{\frac{(n+3)}{n}}} dr + 2\int_a^r \frac{\sigma_o(r)}{r} dr - p \tag{17}$$

Based on the analysis presented, a computer program has been developed to calculate the steady state tangential stress of the FG spherical vessel. For the purpose of numerical computation, the inner and outer radii of the spherical vessel are taken 500 *mm* and 800 *mm* respectively, and the internal pressure is assumed to be 100 *MPa* and external pressure varies as 50 *MPa, 33.33 MPa, 25 MPa and 20 MPa* so that ratio of p/q = 2, 3, 4 and 5. The tangential stress at different radial locations of the sphere is calculated respectively from Eqn. (17).The creep parameters *M(r)* and $\sigma_0(r)$, required during the computation process, are estimated respectively from Eqs. (2) and (3).

3. Results and Discussions

On the basis of mathematical analysis, numerical calculations have been carried out to obtain the secondary stage creep behaviour of functionally graded spherical pressure vessels. The results have been obtained for different pressure ratios in FG spheres. The internal pressure is taken as 100 MPa, however external pressure is varied to obtain pressure ratio as 2, 3, 4 and 5.

3.1: Variation of Particle Content and Creep Parameters.

The distribution of SiC particles in spherical vessel plotted in Fig. 1. The maximum particle content is 30 vol% at inner radius. The content of reinforcement is assumed to decrease linearly along the radial distance. The average particle content is kept as 20 vol%.

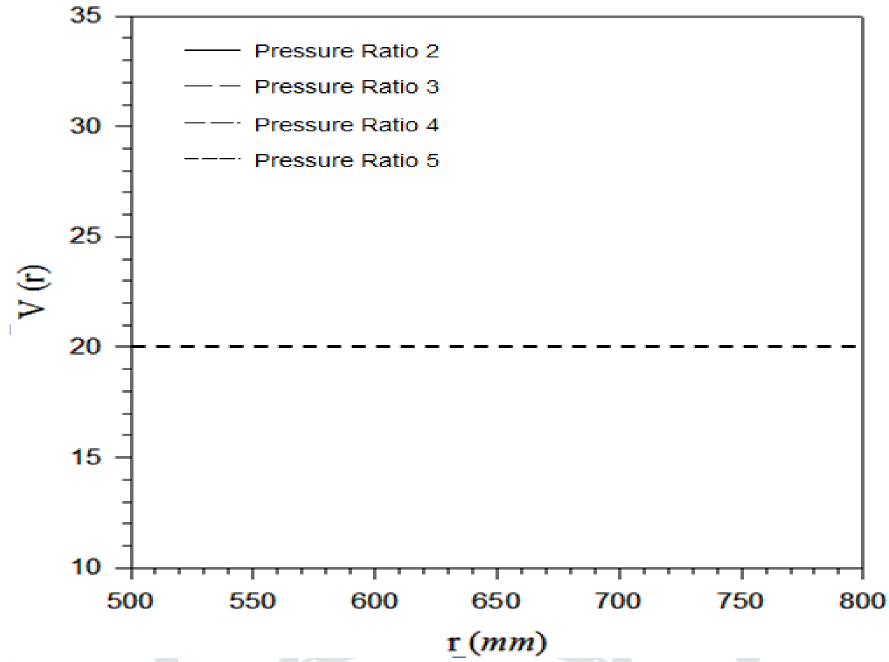

Fig. 1: Variation of SiC_p with radius.

The threshold stress will reduce linearly (Fig. 2) with maximum value (43.17 MPa) at in a radius and minimum value (21.16 MPa) at outer radius of spherical vessel.

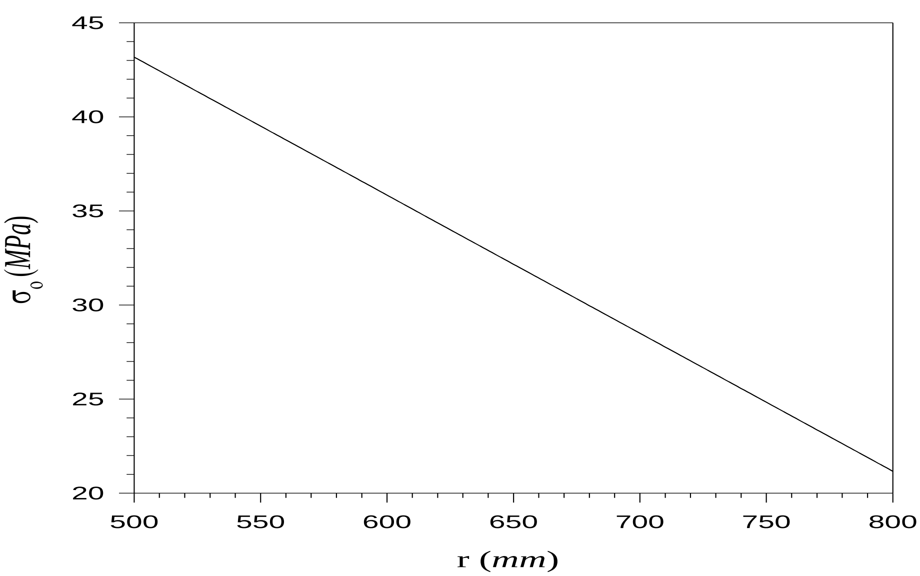

Fig. 2: Variation of Threshold stress with radius.

The threshold stress is higher at locations have greater density of SiCp. The variation of σ_0 become stepper with increase in particle gradient in FG sphere. However, the value of creep parameter 'M' will increase as one moves from inner radius to outer radius (Fig.3). The increase observed in the value of 'M' may be due to decrease in particle content at outer locations.

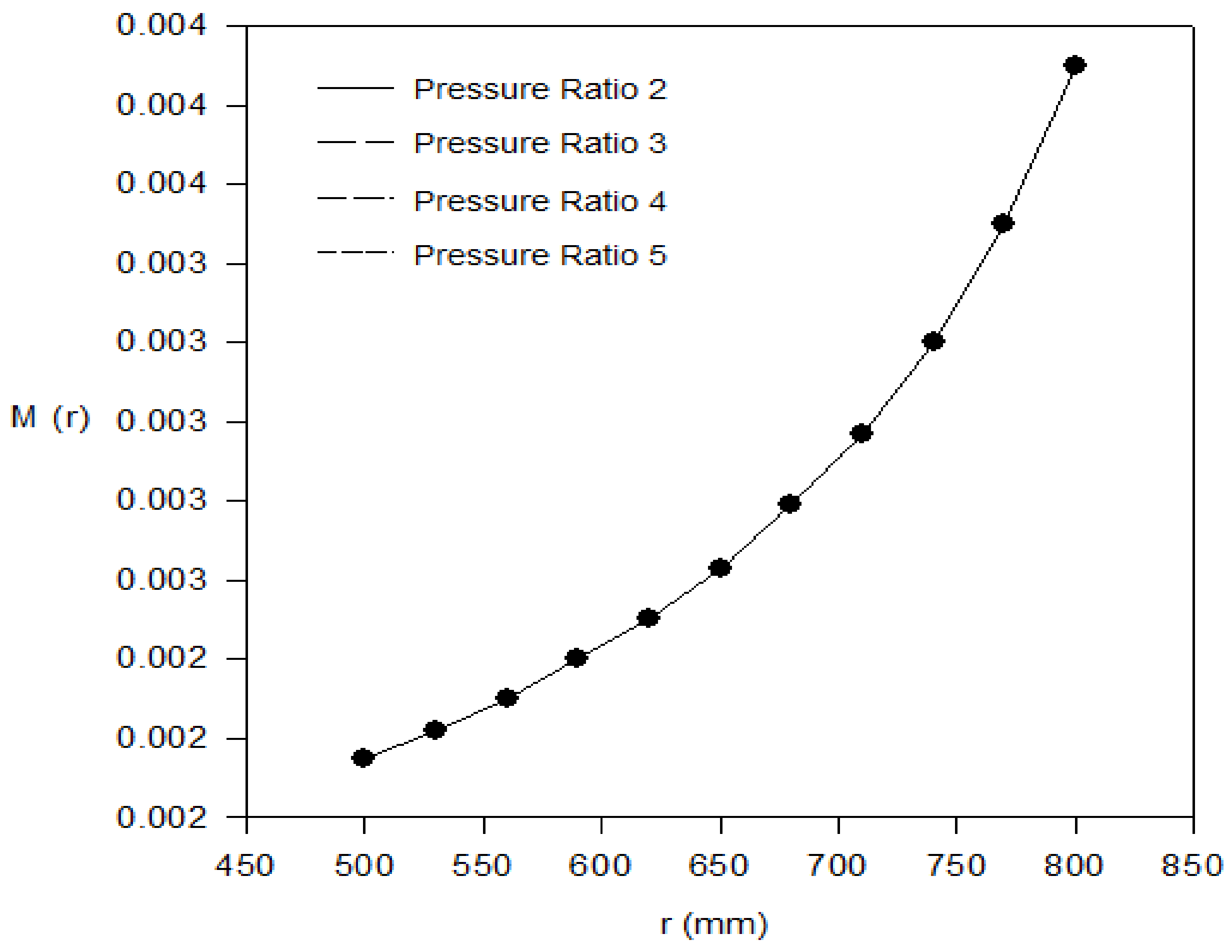

Fig. 3: Variation of Creep parameter *M* with Radius.

3.2 Variation of Tangential Stresses

To observe the effect of pressure ratio of creep stresses in a thick walled spherical vessel the pressure ratio is varied i.e. 2, 3, 4 and 5.

The tangential stress σ_θ and axial stress σ_z remains equal due to spherical symmetry and observed to increase with radius. Further value of σ_θ and σ_z remains compressive at inner and outer radius for pressure ratio 2. On the other hand it interesting to observed that the stress become tensile at middle region as pressure ratio is increased from 2 to 3. Further, increase in pressure ratio (4 and 5) leads to shift in tensile stress near inner region of sphere (i.e. at 530 mm). Further the distribution of $\sigma_\theta\ or\ \sigma_z$ remain parabolic in nature Fig.4.

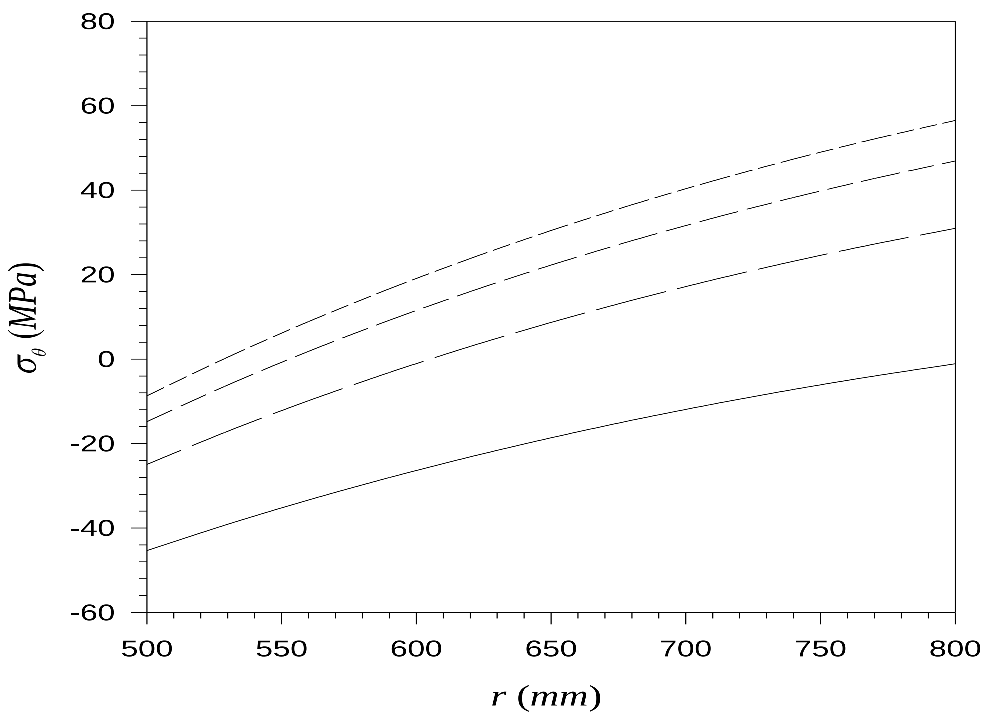

Fig. 4: Variation of Tangential/Axial Stress with radius.

4. Conclusions

The present investigation is an attempt to estimate the tangential/axial stresses in a thick-walled spherical pressure vessel made of aluminum matrix reinforced with silicon carbide particles and subjected to internal and external pressures.

The following conclusions are drawn from the present study:

1) The creep parameters are significantly affected by varying the particle content in the spherical vessel.
2) The threshold stress will decrease linearly from inner radius to outer radius of sphere. On the other hand creep parameter M has opposite trend of variation.
3) The tangential stress/axial stress is compressive near the inner and outer radius of spherical pressure vessel for pressure ratio 2. The tangential stress/axial stress become tensile with increase in pressure ratio 3, 4 and 5.

References

1. **Bayat Y., Ghannad M. and Torabi H. (2012).** "Analytical and numerical analysis for the FGM thick sphere under combined pressure and temperature loading". Journal of Archive Applied Mechanical, Vol. 82, No. 10, pp. 229-242.
2. **Bhatnagar N. S. and Arya V. K. (1974).** "Large strain creep analysis of thick-walled cylinder". International Journal of Non Linear Mech., Vol. 9, No. 2, pp. 127-140.
3. **Bhatnagar N. S. and Arya V. K. (1975).** "Creep of thick-walled spherical vessels under internal pressure considering large strain". Indian Journal of Pure and Applied Mathematics, Vol. 6, No.10, pp. 1080-1089.
4. **Bhatnagar N. S. and Gupta S.K. (1969).** "Analysis of thick-walled orthotropic cylinder in the theory of creep". Journal of Physical Soc. of Japan, Vol. 27, No. 6, pp. 1655-1662.

5. **Durban D. (1982).** "Thermo-plastic/elastic behavior of a strain hardening thick walled sphere is presented with an exact solution which is subjected to a radial temperature gradient". International Journal of Solid Structures, Vol. 19, No.7, pp. 643-652.

6. **Durban D. and Baruch P. (1974).** "Behavior of an incrementally elastic thick walled sphere under internal and external pressure". International Journal of Non-Linear Mechanics, Vol. 9, pp. 105-119.

7. **Johnson A.E. and Khan B. (1963).** "Creep of metallic thick-walled spherical vessels subject to pressure and radial thermal gradient at elevated temperatures". International Journal of Mechanical Science, Vol. 5, pp. 507-532.

8. **Johnson W. and Mellor P.B. (1961).** "Elastic plastic behaviour of thick walled spheres of non-work hardening material subjected to a steady state radial temperature gradient". International Journal of Mechanical Science, Vol. 4, pp.147-158.

9. **Miller G. K. (1995).** "Stresses in a spherical pressure vessel undergoing creep and dimensional changes". International Journal of Solids and Structures, Vol. 32, No.14, pp. 2077-2093.

10. **Singh T. and Gupta V.K. (2009).** "Effect of material parameter in steady state creep in a thick composite cylinder subjected to internal pressure". The Journal of Engineering Research, Vol. 6, No. 2, pp. 20-32.

11. **Singh T. and Gupta V.K. (2011).** "Effect of anisotropy on steady state creep in functionally graded cylinder". Composite Structures, Vol. 93, No. 2, pp. 747-758.

12. **Sultana, N. and Mondal, (2012).** "Design of spherical vessel considering barichinger effect", International Journal of Research in Engineering and Applied Sciences, Vol. 2, No. 2, pp. 696-719.

13. **You L. H. and Ou H. (2008).** "Steady-state creep analysis of thick-walled spherical pressure vessels with varying creep properties". Journal of Pressure Vessel Technology, Vol. 130, No. 1, pp.14501-14505.