TIME-DEPENDENT DOUBLE-DIFFUSIVE FLOW OVER A SEMI-INFINITE VERTICAL PLATE WITH HPM

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Abstract: This study is devoted to the analysis of a one-dimensional time-dependent double-diffusive mass & heat flow over a semi-infinite vertical plate, under a convective surface boundary condition. Using Similarity Transforms, the governing nonlinear partial differential equations have been transformed into a set of coupled nonlinear ordinary differential equations, which are solved by Homotopy Perturbation Method.

Keywords: Time-Dependent; Double-Diffusive Flow; Semi-Infinite Vertical Plate, similarity transform, Homotopy Perturbation Method

Introduction

The phenomenon of time-dependent double-diffusive flow has found wide applications in several areas of science, engineering and technological processes. This flow principle occurs in many areas such as, electronic devices cooling, aerodynamic, and cooling of nuclear reactors, drawing of copper wires, extrusion of plastic sheets and cooling of metallic plates. The problems of time-dependent double-diffusive (mass and heat) flow over a semi-infinite vertical plate have been studied extensively by many researchers. Soundalgekar, V.M. and Ganesan, P. (1980) investigated Transient Free Convective Flow Past a Semi-Infinite Vertical Plate with Mass Transfer. Elbashbeshy, E.M.A. and Ibrahim, F.N. (1993) examined Steady Free Convection Flow with Variable Viscosity and Thermal Diffusivity along a Vertical PlateTakhar, H.S., Ganesan, P., Ekambavanan, K. and Soundalgekar, V.M. (1997) presented the transient Free Convection Past a Semi- Infinite Vertical Plate with Variable Surface Temperature. They stated that, such a flow problem occurs in many industrial and technological applications. Gaur, P. and Mahanti, N.C. (2009) analyzed The Effects of Varying Viscosity and Thermal Conductivity on Steady Free Convective Flow and Heat Transfer along an Isothermal Vertical Plate in the Presence of Heat Sink. Aziz, A. (2009) studied Similarity Solution for Laminar Thermal Boundary Layer over a Flat Plate with a Convective Surface Boundary Condition. Makinde, O.D. and Olanrewaju, P.O. (2010) studied Effects on Thermal Boundary Layer over a Vertical Plate with a Convective Surface Boundary Condition. Aiyesimi, Y.M., Abah, S.O. and Okedayo, G.T. (2011) Analyzed Hydromagnetic Free Convection Heat and Mass Transfer Flow over a Stretching Vertical Plate with Suction. Abah, S.O., Eletta, B.E. and Omale, S.O. (2012) presrnted Numerical Analysis of the Effect of Free Convection Heat and Mass Transfer on the Unsteady Boundary Layer Flow Past a Vertical Plate. The Homotopy Perturbation Method is a combination of the classical perturbation technique and homotopy technique, which has eliminated the limitations of the traditional perturbation methods. He J.H. (2003) discussed Homotopy Perturbation method: a new nonlinear analytical technique. Dehghan M. and Shakeri F.(2008) use of He's Homotopy Perturbation Method for Solving a Partial Differential Equation Arising in Modeling of Flow in Porous Media. In this paper the governing equations were transformed to ordinary differential equation using similarity transformation method and then solved by Homotopy Perturbation Method.

Mathematical Formulation

A time-dependent one dimensional double-diffusive (mass and heat transfer) flow of a viscous, incompressible fluid over a semi-infinite vertical plate is considered. The *x*-axis is taken along the vertical semi-infinite plate in the upward direction and the *y*-axis is normal to the plate. The governing equation of the flow under the Boussinesq's approximation is based on the following Continuity Equation, Momentum Equation, Mass Transfer Equation, and Heat Transfer equations:

$$\frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} = \upsilon \frac{\partial^2 u}{\partial y^2} + g\beta_c(c - c_\infty) + g\beta_T(T - T_\infty) \tag{2}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial y^2} \tag{3}$$

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{4}$$

The appropriate boundary conditions for this flow are,

$$u = U, T = T_w, C = C_w, \text{ at y=0} \tag{5}$$

$$u \to 0, T \to T_\infty, C \to C_\infty, as\ y \to \infty\ at\ \ t > 0. \tag{6}$$

Nomenclature

t=Time, T=Temperature, T_∞= Free Stream Temperature, υ= Kinematic viscosity, g= Gravity, β_T= Heat transfer Coefficient, β_c= Mass transfer Coefficient, C = Concentration, C_∞= Free Stream Mass Transfer, α= Thermal Diffusion, D =Coefficient of Mass Diffusion.

Using similarity transform we introduce the dimensionless variable in order to transform the governing momentum, energy and concentration equations and the boundary conditions from partial differential equations to ordinary differential equations

$$f(\eta)=\frac{u}{u_\infty},\eta=\frac{y}{2\sqrt{vt}},\phi(\eta)=\frac{C-C_\infty}{C_w-C_\infty},\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},G_c=\frac{4tg}{u}\beta_c(C_w-C_\infty),G_T=\frac{4tg}{u}\beta_T(T_w-T_\infty),P_r=\frac{v}{\alpha},S_c=\frac{v}{D} \quad (7)$$

Substituting the above dimensionless quantities into the governing Equations (1)-(4) and the boundary conditions, we obtain the following dimensionless form, ordinary differential equation:

$$\frac{d^2f}{d\eta^2}+2\eta\frac{df}{dn}+G_T\theta(\eta)+G_c\phi(\eta)=0, \quad (8)$$

$$\frac{d^2\phi}{d\eta^2}+2\eta s_c\frac{d\phi}{dn}=0, \quad (9)$$

$$\frac{d^2\theta}{d\eta^2}+2\eta p_r\frac{d\theta}{dn}=0. \quad (10)$$

The corresponding boundary conditions are reduced to

$$f(0)=1,\ \phi(0)=1,\theta(0)=1,\ f(\infty)=0,\ ,\ \phi(\infty))=0\ \ and\ \theta(\infty)=0 \quad (11)$$

The governing boundary layer equations (8),(9) and (10) with the boundary conditions (11) are solved using Homotopy Perturbation Method. Equations (8),(9) and (10) are non-linear differential equation. To solve these equations, we introduce the following Homotopy.

$$D(f,p)=(1-p)\left[\left(\frac{d^2f}{d\eta^2}-\frac{d^2f_I}{d\eta^2}\right)\right]+p\left[\frac{d^2f}{d\eta^2}+2\eta\frac{df}{dn}+G_T\theta+G_c\phi\right]=0 \quad (12)$$

$$D(\phi,p)=(1-p)\left[\frac{d^2\phi}{d\eta^2}-\frac{d^2\phi_I}{d\eta^2}\right]+P\left[\frac{d^2\phi}{dn^2}+2\eta s_c\frac{d\phi}{dn}\right]=0 \quad (13)$$

$$D(\theta,p)=(1-p)\left[\frac{d^2\theta}{d\eta^2}-\frac{d^2\theta_I}{d\eta^2}\right]+P\left[\frac{d^2\theta}{dn^2}+2\eta p_r\frac{d\theta}{dn}\right]=0 \quad (14)$$

With the following assumption

$$\left.\begin{aligned} f&=f_0+pf_1+p^2f_2+\cdots\\ \phi&=\phi_0+p\phi_1+p^2\phi_2+\cdots\\ \theta&=\theta_0+p\theta_1+p^2\theta_2+\cdots\end{aligned}\right\} \quad (15)$$

Using equation (15) into equation (12), (13) and (14) and on comparing the like powers of p,we get the zeoth order equation,

$$\frac{d^2f_0}{d\eta^2}-\frac{d^2f_I}{d\eta^2}=0, \quad (16)$$

$$\frac{d^2\phi_0}{d\eta^2}-\frac{d^2\phi_I}{d\eta^2}=0, \quad (17)$$

$$\frac{d^2\theta_0}{d\eta^2}-\frac{d^2\theta_I}{d\eta^2}=0, \quad (18)$$

with the corresponding boundary conditions:

$$\left.\begin{aligned}&\eta=0:\ f_0=1,\phi_0=1,\theta_0=1\,;\\ &\eta=\infty:\ f_0=0,\phi_0=0,\theta_0=0\,;\end{aligned}\right\} \quad (19)$$

And first order equations are:

$$\frac{d^2f_1}{d\eta^2}+\frac{d^2f_0}{d\eta^2}+2\eta\frac{df_0}{d\eta}+G_T\theta_0+G_c\phi_0=0, \quad (20)$$

$$\frac{d^2\phi_1}{d\eta^2}+\frac{d^2\phi_0}{d\eta^2}+2\eta s_c\frac{d\phi_0}{d\eta}=0, \quad (21)$$

$$\frac{d^2\theta_1}{d\eta^2}+\frac{d^2\theta_0}{d\eta^2}+2\eta ps_r\frac{d\theta_0}{d\eta}=0, \quad (22)$$

With the corresponding boundary conditions:

$$\left.\begin{aligned}&\eta=0:\ f_1=0,\phi_1=0,\theta_1=0\,;\\ &\eta=\infty:\ f_1=0,\phi_1=0,\theta_1=0\,;\end{aligned}\right\} \quad (23)$$

And second order equations are:

$$\frac{d^2f_2}{d\eta^2}+2\eta\frac{df_1}{d\eta}+G_T\theta_1+G_c\phi_1=0, \quad (24)$$

$$\frac{d^2\phi_2}{d\eta^2}+2\eta s_c\frac{d\phi_1}{d\eta}=0, \quad (25)$$

$$\frac{d^2\theta_2}{d\eta^2}+2\eta p_r\frac{d\theta_1}{d\eta}=0, \quad (26)$$

With the corresponding boundary conditions:

$$\left.\begin{aligned}&\eta=0:\ f_1=0,\phi_1=0,\theta_1=0\,;\\ &\eta=\infty:\ f_1=0,\phi_1=0,\theta_1=0\,;\end{aligned}\right\} \quad (27)$$

Solving equations with corresponding boundary conditions, and summing up the results, letting $p\rightarrow 1$ the following functions f($\boldsymbol{\eta}$), $\boldsymbol{\theta}(\boldsymbol{\eta})$, $\boldsymbol{\phi}(\boldsymbol{\eta})$ can be obtained successively:

$$\boldsymbol{f(\eta)}=e^{-\eta}(2\eta+1)+4(\eta^2\boldsymbol{e^{-\eta}}+\boldsymbol{4\eta e^{-\eta}}+6e^{-\eta})-2(\boldsymbol{\eta e^{-\eta}}+2e^{-\eta})(1-G_T-G_C+G_TP_r+G_CS_C)-e^{-\eta}\frac{\eta^2}{2}(G_T-4P_rG_T+G_C-4S_CG_C)-24e^{-\eta}+4(G_T-4P_rG_T+G_C-4S_CG_C) \quad (28)$$

$$\boldsymbol{\theta(\eta)}=e^{-\eta}+2\eta S_ce^{-\eta}+4(S_c)^2e^{-\eta}(\eta^2+5\eta+8)-32(S_c)^2e^{-\eta}-\frac{\eta^3S_c}{3} \quad (29)$$

$$\boldsymbol{\phi(\eta)}=e^{-\eta}+2\eta P_re^{-\eta}+4(P_r)^2e^{-\eta}(\eta^2+5\eta+8)-32(P_r)^2e^{-\eta}-\frac{\eta^3P_r}{3} \quad (30)$$

SKIN-FRICTION: Skin-friction coefficient at the sheet is given by

$$C_f = xf''(0) \quad (31)$$

Where

$$\mathbf{f''(\eta)} = e^{-\eta}(4\eta^2 + 2\eta - 27) - 2(\boldsymbol{\eta e^{-\eta}})(1 - G_T - G_C + G_T P_r + G_C S_C) - \left(e^{-\eta}\frac{\eta^2}{2} + e^{-\eta} - \mathbf{2\eta e^{-\eta}}\right)(G_T - 4P_r G_T + G_C - 4S_C G_C) \quad (32)$$

NUSSELT NUMBER: The rate of heat transfer in terms of the Nusselt number at the sheet is given by

$$Nu = -\theta'(0) \quad (33)$$

where

$$\theta'(0) = -e^{-\eta} + 2S_c e^{-\eta} - 2\eta S_c e^{-\eta} - 4(S_c)^2 e^{-\eta}(\eta^2 + 3\eta - 5) - \eta^2 S_c \quad (34)$$

SHERWOOD NUMBER: Skin-friction coefficient at the sheet is given by

$$Nu = -\phi'(0) \quad (35)$$

$$\boldsymbol{\phi}'^{(\eta)} = -e^{-\eta} + 2P_r e^{-\eta} - 2\eta P_r e^{-\eta} - 4(P_r)^2 e^{-\eta}(\eta^2 + 3\eta - 5) - \eta^2 P_r \quad (36)$$

Conclusion:

The skin-friction coefficient, Nusselt number and Sherwood number are obtained by equations (32), (34) and (36) and are directly proportional to $f''(0)$ and -θ ′(0) and $-\phi'(0)$ respectively. The effects of $f''(0)$ and -θ ′(0) and $-\phi'(0)$ have been presented through Tables 1, Table 2 and Table 3 receptively. Table 1 shows the numerical calculations carried out for different values of thermal Grashoff number G_T, concentration Grashoff number G_c, Prandtl number Pr, Schmidt number Sc for the Skin friction coefficient. Table 2 represent that the numerical values of temperature distribution $\theta'(0)$ decrease as Schmidt number S_C increases. Table 3 represent that the numerical values of Concentration distribution $\theta'(0)$ decrease as Prandtl number Pr increases. It is observed from Figure 1 and 2 that the fluid velocity f (η) increases due to increase in thermal Grashoff Number. G_T and concentration Grashoff number G_c. From Figure 3 it is observed that the value of temperature distribution θ decreases due to increase in Prandtl number . It is observed from Figure 4 that concentration distribution ϕ decreases due to increase in Schmidt number Sc. Figure 5 shows Velocity profiles for various values of thermal Grashoff numberG_T, concentration Grashoff number G_c, Prandtl number Pr, Schmidt number Sc . An increase in the thermal Grasshof and the Concentration Grashof number leads to increase in the velocity profile. And that, rise in the Prandtl number leads to a decrease in the temperature profile. And also a rise in the Schmidt number produces a decrease in the concentration profile.

Table 1 : The coefficient of skin friction at the plate for various values of Grashoff number

G_T	G_c	P_r	S_c	$f''(0)$
0.1	0.1	0.72	0.24	-1.73230
0.5	0.1	0.72	0.24	-1. 77537
1.0	0.1	0.72	0.24	-1.77154
0.1	0.5	0.72	0.24	-1.75323
0.1	1.0	0.72	0.24	-1.80125
0.1	0.1	3.00	0.24	-1.53748
0.1	0.1	7.10	0.24	-1.98574
0.1	0.1	0.72	0.62	-1.95467
0.1	0.1	0.72	2.62	-2. 21047

Table 2 : Nusslet number at the plate for various values of Schmidt number

S_c	$-\theta'(0)$	S_c	$-\theta'(0)$
0.0	1.284514	1	0.208451
0.1	0.913433	1.5	-0.987469
0.2	0.9604478	2.8	-0.957469
0.5	0.8021276	2.0	-0.975746

Table 3 : Sherwood number at the plate for various values of Prandtl number

P_r	$-\phi'(0)$	P_r	$-\phi'(0)$
0.0	- 0.793624	3.00	-0.929184
0.3	-0.774384	3.72	-1.971584
0.5	-0.814324	7.01	-1.918863
0.7	-0.899363	7.10	-2.896388

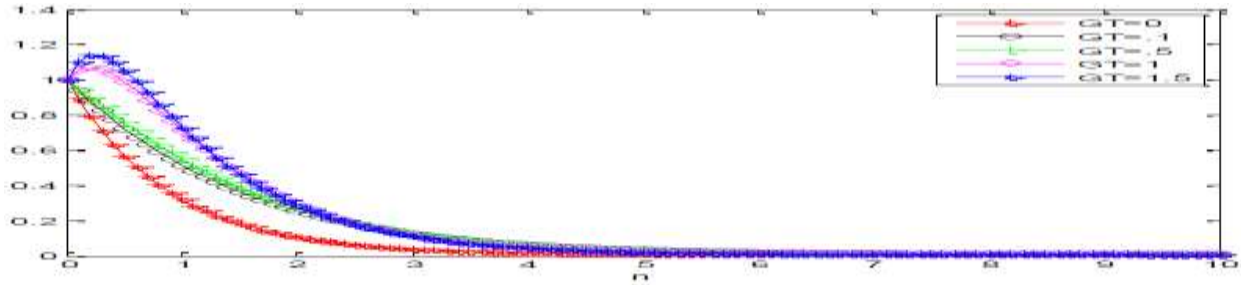

Figure 1 : Velocity distribution f(η) versus η when Gc = 0.1, Pr = 0.72, Sc = 0.24

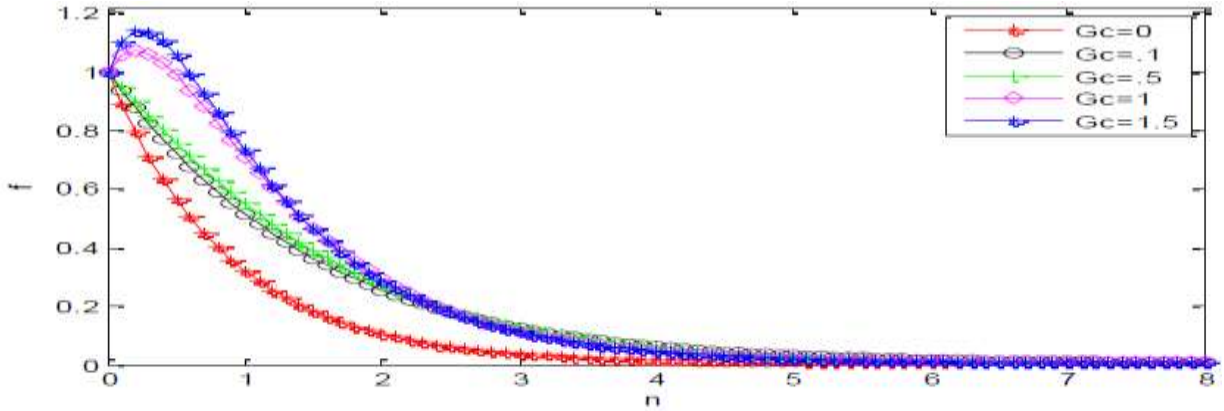

Figure 2 : Velocity distribution f (η) versus η

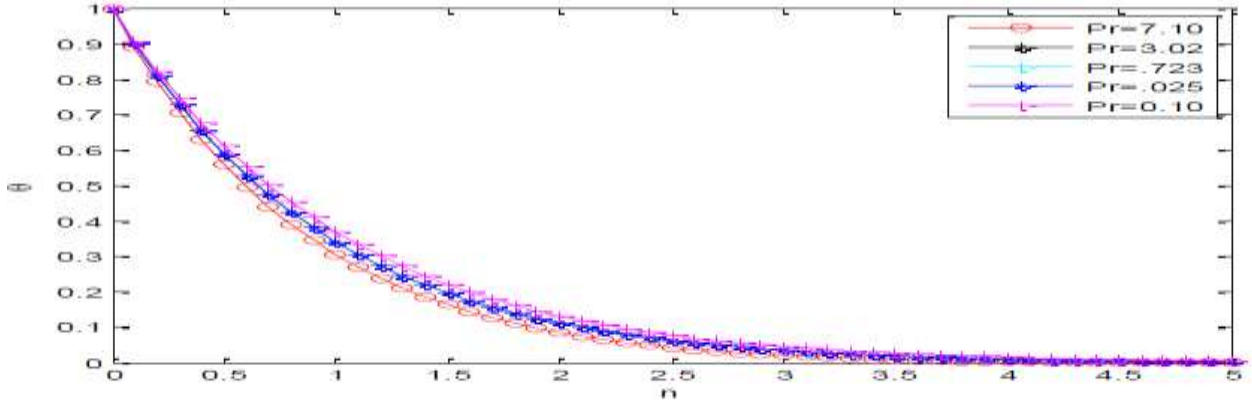

Figure 3 : Temperature distribution θ versus η when GT = GC = 0.1, Sc = 0.24

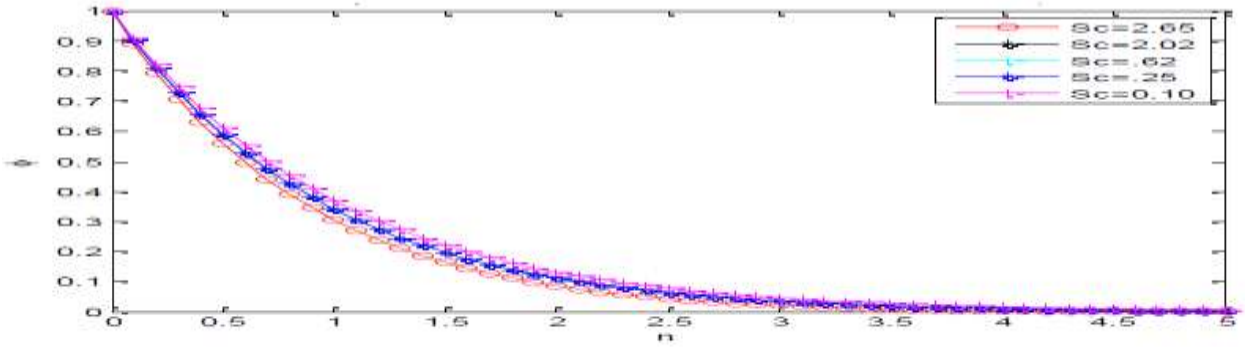

Figure 4 : Concentration distribution ϕ versus η when GT = GC = 0.1, Pr = 0.723

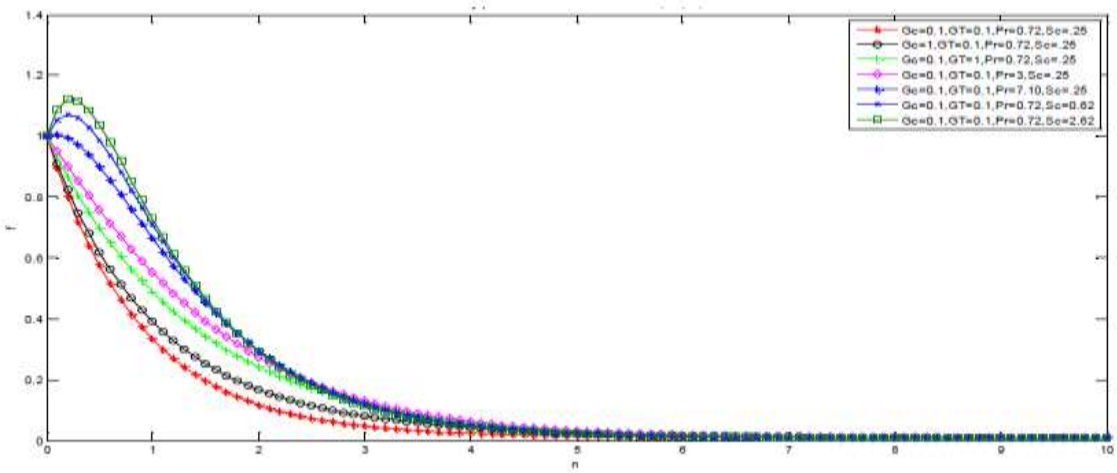

Figure 5 : Velocity distribution f (η) versus η

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