E-CORDIAL LABELING OF SOME PATH UNION GRAPHS

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Abstract: Path union of graph G i.e. P m(G) is obtained by fusing a copy of graph G at each vertex of a path P m. Vertex of fusion is same and fixed for all graphs. We discuss e-cordial labeling of P m(G) for G = claw, paw, kite and show that P m(G) is e-cordial.

Key words: E-cordial, path union, fusion, edge, vertex.

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Introduction:
In 1997 Yilmaz and Cahit [4] introduced a weaker version of edge graceful labeling called E-cordial. The word cordial was used first time in this paper. Let G be a graph with vertex set V and edge set E. Let f be a function that maps E into {0, 1}. Define f on V by f(v) = Σ(f(uv))/ (uv)∈ E (mod 2). The function f is called as E-cordial labeling if |e f(0)−e f(1)|≤1 and |v f(0)−v f(1)|≤1. Where e f(i) is the number of edges labeled with i = 0, 1 and v f (i) is the number of vertices labeled with i = 0, 1. We also use v f (0, 1) = (a, b) to denote the number of vertices labeled with 0 are a in number and that with 1 are b in number. Similarly e f (0, 1) = (x, y) to denote number of edges labeled with 0 are x in number and that labeled with 1 are y in number. A lot of work has been done in this type of labeling and the above mentioned paper gave rise to number of papers on cordial labeling. A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit has shown that Trees T n with n vertices and Complete graphs K n are E-cordial. Yilmaz and Cahit [3] showed that Friendship graph C n for all n and fans F n for n not congruent to 1 (mod 4). They observe that a graph with n vertices is not E-cordial if n = 2 (mod 4). One may refer A Dynamic survey of graph labeling for more details on completed work.

Preliminaries:

Fusion of vertex. Let G be a (p, q) graph. Let u and v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1 vertices and at least q-1 edges. If u ∈ G 1 and v ∈ G 2, where G 1 is (p 1, q 1) and G 2 is (p 2, q 2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p 1+p 2-1 vertices and q 1 + q 2 edges. Sometimes this is referred as u is identified with v.

Path union of G i.e. P m(G) is obtained by taking a path P m and m copies of graph G. Fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges, where G is a (p, q) graph. If we change the vertex on G that is fused with vertex of Pm then we generally get a path union non isomorphic to earlier structure. In this paper we define a e-cordial function f that does not depends on which vertex of given graph G is used to obtain path union. This allows us to obtain path union in which the same graph G is fused with vertices of Pm at different vertices of G, as our choice and the same function f is applicable to all such structures that are possible on P m(G).

Main Results:
Theorem 4.1 Path union of claw is e-cordial.
Proof: There are two possible structures of P m(G). From figure 4.1 it follows that one can take path union at vertex ‘u’ giving structure 1 and the one point union at point v will give structure 2. The two structures are non-isomorphic. Take a path P m=(v 1, v 2, ..v m). Each edge on Pm has label ‘0’. At each vertex fuse a copy of Type A labeling at vertex u. The resultant graph has label distribution given by v f (0, 1) = (2m, 2m), e f (0, 1) = (2m-1, 2m). If we form path union at vertex v of claw then also the above type A label will work and label distribution also same.
Theorem 4.2 Path union of paw is e-cordial for all m.
Proof: Define a function \( f: E(G) \rightarrow \{0,1\} \). It gives us following two types of labeled copies of paw:

There are three possible structures of \( P_m(G) \). From figure 4.3 it follows that one can take path union at vertex ‘u’ giving structure 1 and the path union at point v will give structure 2 and path union at w giving structure 3. All structures are pairwise non-isomorphic. Take a path \( P_m = (v_1, v_2, \ldots, v_m) \). Each edge on \( P_m \) has label ‘0’. At each vertex \( v_i \), fuse a copy of Type A labeling (at vertex u) for all \( i \equiv 1 \pmod{2} \) and copy of Type B if \( i \equiv 0 \pmod{4} \). The resultant graph has label distribution given by \( v_1(0,1) = (2m, 2m) \), \( e_0(0,1) = (2x+1, 2x+1) \) for odd number \( m = 2x-1 \); \( x = 0, 1, 2 \). \( v_1(0,1) = (2m, 2m) \), \( e_0(0,1) = (4+5(x-1), 5x) \) for even number \( m = 2x \), \( x = 1, 2 \). If we form path union at vertex ‘v’ or ‘w’ of claw then also the above type A label will work and label distribution is also same. Thus the function \( f \) is independent of the vertex of paw used to form pathunion.

Theorem 4.3 Path union (\( P_m(\text{house}) \)) of house is e-cordial for \( m \) is not congruent to 2(mod 4)
Proof: Define a function \( f: E(G) \rightarrow \{0,1\} \). It gives us following three types of labeled copies of house graph. Take a path \( P_m = (v_1, v_2, \ldots, v_m) \). Each edge on \( P_m \) has label ‘0’. At \( v_1 \) fuse a copy of Type A label, at \( v_2 \) fuse a copy of Type B label, at \( v_3 \) fuse type C label. For \( i > 5 \) at each vertex \( v_i \), fuse type A label. For \( i > 3 \) at each vertex \( v_i \), fuse type C label. For \( i > 2 \) at each vertex \( v_i \), fuse type B label. At \( v_1 \) fuse a copy of type C label, at \( v_2 \) fuse a copy of type A label, at \( v_3 \) fuse type C label. For \( i > 5 \) at each vertex \( v_i \), fuse type C label. For \( i > 3 \) at each vertex \( v_i \), fuse type C label. For \( i > 2 \) at each vertex \( v_i \), fuse type C label. For \( i > 1 \) at each vertex \( v_i \), fuse type B label. At \( v_1 \) fuse type B label. At \( v_2 \) fuse type A label. At \( v_3 \) fuse a copy of type B label, at \( v_4 \) fuse a copy of type B label, at \( v_5 \) fuse a copy of type B label, at \( v_6 \) fuse a copy of type B label, at \( v_7 \) fuse a copy of type B label.
type C label, at vertex $v_i$ fuse type A label. For $i > 5$ at each vertex $v_i$ fuse a copy of Type C labeling for all $i \equiv 1 \pmod{4}$, Type B labeling for all $i \equiv 2 \pmod{4}$ and copy of Type A if $i \equiv 0, 3 \pmod{4}$.

The resultant graph has label distribution given by $v_i(0,1) = (3,2)$, $e_i(0,1) = (3,3)$ for $m = 1$, for $m = 2$ is given by $v_i(0,1) = (6,4)$, $e_i(0,1) = (7,6)$.

For all $m$ of type $4x$, $x = 2, 3, \ldots$ We have label distribution given by $v_i(0,1) = (10x,10x_x)$, $e_i(0,1) = (14x,14x-1)$.

For $m = 4$ we have $v_i(0,1) = (10,10)$, $e_i(0,1) = (13,14)$.

For all $m$ of type $4x+1$, $x = 1, 2, \ldots$ we have label distribution given by $v_i(0,1) = (13+10x(1),12+10x(1))$, $e_i(0,1) = (17+14x(1),17+14x(1))$.

For all $m$ of type $4x+2$, $x = 1, 2, \ldots$ we have label distribution given by $v_i(0,1) = (14+10x(1),16+10x(1))$, $e_i(0,1) = (20+14x(1),21+14x(1))$.

For all $m$ of type $4x+3$, $x = 0, 1, 2, \ldots$ We have label distribution given by $v_i(0,1) = (7+10x,8+10x_x)$, $e_i(0,1) = (14x+10,14x+10)$.

The function $f$ is independent of the vertex of dart bowtie used to form path union. The graph is $e$-cordial. 

Theorem 4.6 Path union $(G = P_m(dart))$ of dart is $e$-cordial for $m$ is not congruent to $2 \pmod{4}$.

Proof: Define a function $f:E(G)\to\{0,1\}$. It gives us following three types of labeled copies of dart graph.

Take a path $P_m = (v_1, v_2, \ldots v_m)$. Each edge on $P_m$ has label ‘0’. At vertex $v_1$ fuse a copy of type A label, at vertex $v_2$ fuse a copy of type B label, at vertex $v_3$ fuse type C label. For $i > 3$ at each vertex $v_i$ fuse a copy of Type A labeling for all $i \equiv 0 \pmod{4}$ and copy of Type B if $i \equiv 1 \pmod{4}$, copy of Type C if $i \equiv 2 \pmod{4}$.

The resultant graph has label distribution given by $v_i(0,1) = (3,2)$, $e_i(0,1) = (3,3)$ for $m = 1$, for $m = 2$ is given by $v_i(0,1) = (6,4)$, $e_i(0,1) = (6,7)$. For all $m$ of type $4x$, $x = 1, 2, \ldots$ we have label distribution given by $v_i(0,1) = (10x,10x_x)$, $e_i(0,1) = (14x,14x-1)$.

For all $m$ of type $4x+1$, $x = 0, 1, 2, \ldots$ we have label distribution given by $v_i(0,1) = (13+10x(1),12+10x(1))$, $e_i(0,1) = (17+14x(1),17+14x(1))$.

For all $m$ of type $4x+2$, $x = 0, 1, 2, \ldots$ we have label distribution given by $v_i(0,1) = (14+10x(1),16+10x(1))$, $e_i(0,1) = (20+14x(1),21+14x(1))$.

For all $m$ of type $4x+3$, $x = 0, 1, 2, \ldots$ we have label distribution given by $v_i(0,1) = (7+10x,8+10x_x)$, $e_i(0,1) = (14x+10,14x+10)$.

The function $f$ is independent of the vertex of dart used to form pathunion.

Conclusions:

In this paper we have discussed and shown that path union $P_m(G)$ is $e$-cordial for certain choice of $G$. The $e$-cordial function $f$ we defined is such that irrespective of the vertex on $G$ used to fuse with path vertex to obtain $P_m(G)$, the graph is $e$-cordial.

We have shown that 1) Path union of claw is $e$-cordial for all $m$. 2) Path union of paw is $e$-cordial for all $m$. 3) Path union ( $P_m($ house $)$ ) of house is $e$-cordial for $m$ is not congruent to $2 \pmod{4}$. 4) Path union ( $G = P_m(\text{bull})$ ) of bull is $e$-cordial for $m$ is not congruent to $2 \pmod{4}$. 5) Path union ( $G = P_m(\text{bowtie})$ ) of is $e$-cordial for $m$ is not congruent to $2 \pmod{4}$. 6) Path union ( $G = P_m(\text{dart})$ ) of dart is $e$-cordial for $m$ is not congruent to $2 \pmod{4}$.

Our work confirms one observation by Cahit that a graph with $n$ vertices is not $e$-cordial if $n = 2 \pmod{4}$.

References: