A STUDY ON GENERALIZED FUZZY BI-IDEAL IN SEMIGROUP

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Abstract: In this paper, we introduced the cocept of a fuzzy set and the concept was applied to define fuzzy subgroups and ideals and also defined a fuzzy semigroup and various kinds of fuzzy ideals in semigroups and characterized them.

Keywords: Fuzzy ideals, Fuzzy subgroups, Fuzzy semigroup, Fuzzy bi- ideals, Intuitionistic Fuzzy set.

Introduction

Zadeh introduced the concept of a fuzzy set for the first time and this concept was applied by Rosenfeld to define fuzzy subgroups and fuzzy ideals. Based on this crucial work, N.Kuroki defined a fuzzy semigroup and various kinds of fuzzy ideals in semigroups and characterized them. On the other hand, Mo and Wang defined some kinds of fuzzy ideals generated by fuzzy subsets in a semigroup with an identity element and Xie reproved the results of Mo and Wang using the level subsets. However the fuzzy bi-ideal generated by a fuzzy subset in a semigroup has not yet been defined and studied. In this note we are able to define the fuzzy bi-ideal generated by a fuzzy subset in a semigroup and obtain the same results, as special cases of our main results, that Mo and Wang ([8]) found in a semigroup with an identity element or a regular semigroup. In section 2 we give some definitions and propositions which will be used in the next section . In section 3 we define the fuzzy bi-ideal generated by a fuzzy subset in a semigroup, define the fuzzy bi-ideal generated by a fuzzy subset A such that $A \subseteq A^2$ in a semigroup with an identity element, and find, as special cases, the fuzzy bi-ideal generated

The concept of intuitionistic fuzzy set was introduced by K. T. Atanassov [1, 2], as a generalization of the notion of fuzzy set. In N. Kuroki gave some properties of fuzzy ideals and fuzzy bi-ideals in semigroups. The concept of (1,2)-ideals in semigroups was introduced by S. My topic, we consider the intuitionistic fuzzification of the concept of several ideals in a semigroup S, On fuzzy Bi-ideals in semigroup

In 1980, Grosek and Satko defined the notion of a left almost ideal of a semigroup a non-empty subset GL of a semigroup S is said to be a left almost ideal of S if s$G_L \cap G_L \neq \emptyset$ for all s ∈ S. A right A-ideal of S is similarly defined. If G is both left and right almost ideal of S, then G is called an almost ideal of S

Fuzzy bi-ideals generated by Fuzzy subset semigroup.

Definition

A function B from a set X to the closed unit interval [0, 1] in R is called a fuzzy subset in X.
For every $x \in$ X, B(x) is called the membership grade of x in B.
A fuzzy subset in X is called a fuzzy point if and only if it takes the value 0 for all y ∈ X except one, say, $x \in$ X.
If its value at x is α (0 < α ≤ 1), we denote this fuzzy point by $x\alpha$, where the point x is called its support.
The fuzzy point $x\alpha$ is said to be contained in a fuzzy subset A, denoted by $x\alpha \in$ A, if and only if $\alpha \leq A(x)$

Definition

A triangular norm (briefly t-norm) is a function T : [0,1]×[0,1] ⟶ [0,1] satisfying, for each p,q,r,s in [0,1],
(i) T(p,1) = p
(ii) T (p, q) ≤ T (r , s) if p ≤ r and q ≤ s
(iii) T (p, q) = T (q ,p)
(iv) T (p, T(q,r)) = T (T(p,q),r.

Definition

A t-norm T : [0,1]×[0,1] ⟶ [0,1] is continuous if T is continuous with respect to the usual topologies. It is well known that the function T_m : [0,1]×[0,1] ⟶ [0,1] defined by

$$T_m(a,b) = \min(a,b),$$

The function T_p : [0,1]×[0,1] ⟶ [0,1] defined by

$$T_p(a,b) = ab$$

and the function T_M: [0,1] × [0,1] ⟶ [0,1] defined by

$$T_M(a,b) = \max(a+b-1,0)$$ are continuous t-norms.

For fuzzy sets U,V in a set X, Liu ([7]) defined U ∘V by

$$(U\circ V)(x) = \begin{cases} \sup_{ab=x} \min\big(U(a),V(b)\big) & if\ ab = x \\ 0 & if\ ab \neq x \end{cases}$$

Definition

Let X be a set and let U,V be two fuzzy sets in X. U∘V is defined by

$$(U\circ V)(x) = \begin{cases} \sup_{ab=x} T(U(a),V(b) & if\ ab = x \\ 0 & if\ ab \neq x \end{cases}$$

Theorem

Let $A_1, A_2, \ldots, A_n$ be fuzzy subsets of a set S.
Then

(i) $(A_1 \cup A_2 \cup \ldots\ldots\cup A_n) S \subseteq A_1 S \cup A_2 S \cup \ldots\ldots\cup A_n S$.

(ii) $S (A_1 \cup A_2 \cup \ldots\ldots\cup A_n) \subseteq SA_1 \cup SA_2 \cup \ldots\ldots\cup SA_n$

Proof

Since S (b) = 1,

$[A_1 \cup A_2 \cup \ldots\ldots\cup A_n) S](x) = \sup_{ab=x} T((A_1 \cup A_2 \cup \ldots\ldots\cup A_n)(a), S(b))$

$= \sup_{ab=x} \max[A_1 (a), A_2(a), \ldots\ldots A_n(a)$

$[A_1 \cup A_2 \cup \ldots\ldots\cup A_n) S](x) = \sup_{ab=x} \max[A_1 (a), A_2(a), \ldots\ldots A_n(a)$

Since S(b)=1,

$(A_1 S \cup A_2 S \cup \ldots\ldots\cup A_n S)(x)$

$$= \max[\sup_{ab=x} T(A_1 (a), S(b)), \ldots\ldots T(A_n(a), S(b))$$

$$= \max[\sup_{ab=x} A_1 (a), \sup_{ab=x} A_2(a), \ldots.., \sup_{ab=x} A_n(a)]$$

Thus

$(A_1 \cup A_2 \cup \ldots\ldots\cup A_n) S \subseteq A_1 S \cup A_2 S \cup \ldots\ldots\cup A_n S$.

Similarly

$S (A_1 \cup A_2 \cup \ldots\ldots\cup A_n) \subseteq SA_1 \cup SA_2 \cup \ldots\ldots\cup SA_n$

Definition

Let S be a semi group .The fuzzy subset H in S is a fuzzy subsemigroup of S if

$$H(xy) \geq T(H(x), H(y) \text{ for all } x, y \in S.$$

A fuzzy subsemigroup B of S is called a fuzzy bi-ideal of S if

$$B(xyz) \geq T\big(B(x), B(y)\big)$$

Definition

Let S be a semigroup.S is called a regular semi-group if for every $x \in S$,
there exists $a \in S$ such that , $xax = x$

Theorem

Let A be a fuzzy subset in a semigroup S. Then the fuzzy bi-ideal F generated by A is $A \cup A^2 \cup ASA$.
That is, F(x) = max $[A(x) \sup_{ab=x} T\big(A(a), A(b)\big), \sup_{ab=x} T(A(c), A(b))]$

Proof

Let $\{J_i : i \in I\}$ be the collection of all fuzzy bi-ideal of S containing A.
Since T is a continuous and increasing function

$$(J_i S J_i)(x) = \sup_{ab=x} T(J_i S(a), J_i(b))$$
$$= \sup_{ab=x} T(\sup_{cd=a} T(J_i(c), S(d)), J_i(b))$$
$$= \sup_{ab=x} T(\sup_{cd=a} J_i(c), J_i(b)$$
$$= \sup_{ab=x} \sup_{cd=a} T(J_i(c), J_i)(b)$$
$$= \sup_{cdb=x} T(J_i(c), J_i)(b))$$
$$\leq \sup_{cdb=x} J_i(cdb)$$
$$(J_i S J_i)(x) = J_i(x)$$

For each $i \in I$.
Thus $ASA \subseteq (J_i S J_i) \subseteq J_i$ for each $i \in I$.
Since each J_i is a fuzzy semigroup ,

$$(J_i S J_i)(x) = \sup_{ab=x} T(J_i(a), J_i(b))$$
$$\leq \sup_{ab=x} (J_i(ab)$$
$$(J_i S J_i)(x) = (J_i(x)$$

That is,

$$A^2 \subseteq J_i J_i \subseteq J_i \text{ for each } i \in I.$$

Hence

$$A \cup A^2 \cup ASA \subseteq \cap_{i \in I} J_i$$

By a theorem(3.5)

$(A \cup A^2 \cup ASA)S(A \cup A^2 \cup ASA) \subseteq (AS \cup A^2 S \cup ASAS)(A \cup A^2 \cup ASA)$

Since $A \subseteq S$ and S is a semigroup,

$$(AS \cup A^2 S \cup ASAS)(A \cup A^2 \cup ASA) \subseteq (AA \cup AS^2 \cup AS^3)(A \cup A^2 \cup ASA)$$
$$\subseteq AS(A \cup A^2 \cup ASA)$$
$$\subseteq ASA \cup AA^2 S \cup ASASA)$$
$$\subseteq ASA \cup AS^2 A \cup AS^3 A$$
$$(AS \cup A^2 S \cup ASAS)(A \cup A^2 \cup ASA) = ASA$$

Thus $(A \cup A^2 \cup ASA)S(A \cup A^2 \cup ASA) \subseteq A \cup A^2 \cup ASA$.
Let H= $A \cup A^2 \cup ASA$.
Then $HSH \subseteq H$.
Since T is a continuous and increasing function .

$$H(xyz) \geq (HSH)(xyz) = \sup_{ab=xyz} T[HS(a), H(b)]$$
$$= \sup_{ab=xyz} T\Big[\sup_{cd=a} T(H(c), S(d), H(b)\Big]$$
$$= \sup_{ab=xyz} T\Big[\sup_{cd=a} H(c), H(b)\Big]$$
$$= \sup_{ab=xyz} \sup_{cd=a} T\big(H(c), H(b)\big)$$

$$= \sup_{cdb=xyz} T\big(H(c), H(b)\big)$$

$$H(xyz) \geq T\big(H(x), H(y)\big).$$

Since $A \subseteq S$ and S is a semigroup,

$$(A \cup A^2 \cup ASA)(A \cup A^2 \cup ASA) \subseteq (A \cup AS \cup ASS)(A \cup SA \cup SSA)$$
$$\subseteq (A \cup AS)(A \cup SA)$$
$$\subseteq A^2 \cup ASA \cup ASA \cup ASSA$$
$$(A \cup A^2 \cup ASA))(A \cup A^2 \cup ASA\ ) \subseteq A \cup A^2 \cup ASA$$

By a proposition $A \cup A^2 \cup ASA$ is a fuzzy subsemigroup.

Thus H=$A \cup A^2 \cup ASA$ is s fuzzy bi-ideal of S containing A.

Hence $A \cup A^2 \cup ASA = \bigcap_{i \in I} J_i$.

Since T is continuous and increasing,

$$(ASA)(\ x) = \sup_{ab=x} T(AS(a), A(b))$$
$$= \sup_{ab=x} \mathrm{T}\left(\sup_{cd=a} \mathrm{T}\big(\mathrm{A(c), S(d)}\big), \mathrm{A(b)}\right)$$
$$= \sup_{ab=x} T\left(\sup_{cd=a} A(c), A(b)\right)$$
$$= \sup_{ab=x} \operatorname{sub}_{cd=a} \mathrm{T}\big(\mathrm{A(c), A(b)}\big)$$
$$(ASA)(\ x) = \sup_{cdb=x} T\big(A(c), A(b)\big).$$

Thus

$$\mathrm{F}(x) = \max\ [A(x) \sup_{ab=x}\ T\big(A(a), A(b)\big), \sup_{ab=x} T(A(c), A(b))]$$

Conclusion

In this paper, we consider some operation on fuzzy bi-ideal in semigroup in order to Study the structure of fuzzy semigroup .After that we study intuitionistic fuzzy bi-ideal in semigroup and fuzzy almost bi-ideal in semigroup. Let A be a fuzzy subset in a semigroup S. Then the fuzzy bi-ideal generated by A is $A \cup A^2ASA$ .And let A be a fuzzy subset in a semigroup S with identity element e such that $A \subseteq A^2$ and T be a minimum operation .Then the fuzzy bi-ideal generated by ASA. Here using definition of intuitionistic fuzzy bi-ideal and almost bi-ideal solving some theorem .The obtained result can be used to solve some social network problem and to decide whether the corresponding graph is balanced or dusterable.

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