HYDROKINETICS FLOW NEAR TIME-VARYING ACCELERATED POROUS PLATE IN MOVING SYSTEM

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ABSTRACT: ***In this paper an initial value investigation has been made of the motion of an incompressible, homogeneous, viscous fluid over a porous plate with uniform suction. Both the plate and the fluid are in the state of a solid body rotation with constant angular velocity about z-axis normal to the plate and plate is assumed to be accelerated with given velocity. The solution in general feature of the unsteady hydrodynamic boundary layer flow in a rotating system with suction has been obtained. The results obtained have been compared with previous investigation.***

Keywords: An incompressible fluid; porous plate, Angular velocity suction, unsteady motion, Navier-Stoke's equation, Continuity Laplace Transformation.

Introduction

The flow of a viscous incompressible and electrically conducting fluid past moving plate in the presence of external magnetic field has been investigated. In case of infinite plate with and without the induced magnetic field by the current. Gupta (1972) has studied an exact solution for the flow past a plate with uniform suction in a rotating reference frame. In the present paper under different condition has been reviewed.

Mathematical Formulation of Basic Equations

The Navier-Stoke's equations and the equation continuity for the unsteady motion of a viscous fluid in a rotating reference frame are :

$$\frac{\partial \vec{u}}{\partial t}+(\vec{u}.\nabla)\vec{u}+2\Omega\vec{k}\times\vec{u}=\frac{1}{\rho}\nabla p+\nu\nabla^2\vec{u} \qquad (1)$$

$$div\,\vec{u}=0 \qquad (2)$$

Where Ω is constant angular velocity about *z*-axis.

Where $\vec{u}=(u,v,\omega)$ is the velocity vector, $\vec{k}$ the unit vector along *z*-axis, *p* the pressure, ρ be the density of fluid, ν the kinematic viscosity.

The velocity is assumed to be dependent on *z* and *t*, so that

$$\vec{u}(z.t)=\left[(z,t),v(\tau,t),\omega(\tau,t)\right] \qquad (3)$$

In absence of pressure gradient, the equation of motion can be written as :

$$\frac{\partial u}{\partial t}-\omega_0\frac{\partial u}{\partial z}-2\Omega v=\nu\frac{\partial^2 u}{\partial z^2} \qquad (4)$$

$$\frac{\partial v}{\partial t}-\omega_0\frac{\partial v}{\partial z}+2\Omega u=\nu\frac{\partial^2 v}{\partial z^2} \qquad (5)$$

Here $\omega=\omega_0$ is constant and $\omega_0>0$ for suction and $\omega_0<0$ for blowing.

Now we take $q=u+iv$ and multiplying equation (5) and adding with equation (4) where $i=\sqrt{-1}$

$$\frac{\partial}{\partial t}(u+iv)-\omega_0\frac{\partial}{\partial z}(u+iv)+2\Omega i(u+iv)=\nu\frac{\partial^2}{\partial z^2}(u+iv)$$

$$\frac{\partial q}{\partial t}-\omega_0\frac{\partial q}{\partial z}+2\Omega iq=\nu\frac{\partial^2 q}{\partial z^2} \qquad (6)$$

The boundary conditions for the present problem are :

$$q(z,t)=0,\ \text{for all}\ z=0\ \text{and}\ t\le 0 \qquad (7)$$

$$q(z,t)=\lambda e^{t\omega t}t^{n},\ \omega=\omega_0 \text{ for } z=0 \text{ and } t>0 \quad (8)$$

and $$q\to 0 \; as\; z\to\infty \text{ for } t>0$$

where λ in constant with the dimensions of velocity (9)

Solution of the Problem

Now we introduce non-dimensional variables and non-dimensional parameters in the form

$$\eta=\frac{z\lambda}{\nu},\ t=\Omega t',\ U=\frac{q}{\lambda} \quad (10)$$

and $$R=\frac{W_0}{\lambda},\ N=\frac{2\Omega\nu}{\lambda^2},\sigma=\frac{\omega}{\Omega} \quad (11)$$

Then equation (6) and initial condition (7) – (9) become

$$\frac{\partial^2 U}{\partial\eta^2}+R\frac{\partial U}{\partial\eta}-iNU=\frac{N}{2}\frac{\partial U}{\partial t} \quad (12)$$

U = 0 every where for $t\le 0$ (13)

$$U=\lambda e^{i\sigma t}.t^{m} \text{ at } \eta=0,\ t>0 \quad (14)$$

and U = 0 or infinite as $\eta=\infty,\ t>0$ (15)

Now we solve the initial value problem we apply the Laplace transform $\overline{U}(\eta,p)$ of $U(\eta,t)$ defined by the integral

$$\overline{U}(\eta,p)=\int_0^\infty e^{-pt}U(\eta,t)\,dt \quad (16)$$

The Laplace transform of equation (12) and the boundary conditions (13) – (14) are given by

$$\frac{d^2\overline{U}}{d\eta^2}+\frac{d\overline{U}}{d\eta}-\left(iN+\frac{p}{2}N\right)\overline{U}=0 \quad (17)$$

$$\overline{U}(\eta,p)=\frac{\lambda\Gamma(m+1)}{(p-i\sigma)^{m+1}}\ at\,\eta=0 \quad (18)$$

and $\overline{U}(\eta,p)=0$ or infinite as $\eta\to\infty$ (19)

Taking $\frac{d}{d\eta}\equiv D,$ equation (17) can be written as

$$\left[D^2+RD-\left(iN+\frac{PN}{2}\right)\right]\overline{U}=0$$

The auxiliary equation corresponds to

$$D^2+RD-\left(iN+\frac{PN}{2}\right)=0$$

$$\therefore D=\frac{-R\pm R^2+4\left(iN+\frac{PN}{2}\right)}{2}$$

$$=\frac{-R}{2}\pm\sqrt{\frac{n}{2}}\sqrt{P+\frac{R^2+4iN}{2N}}$$

Thus equation (17) is written as

$$\overline{U}(\eta,P)=C_1\exp\left[\frac{-R\eta}{2}+\eta\sqrt{\frac{N}{2}}\sqrt{P+\frac{R^2+4iN}{2N}}\right]+C_2\exp\left[\frac{-R\eta}{2}+\eta\sqrt{\frac{N}{2}}\sqrt{P+\frac{R^2+4iN}{2N}}\right] \quad (20)$$

Using boundary conditions (18)-(19) to equation (20) we obtain

$$\overline{U}(\eta,P)=\frac{\lambda\Gamma(m+1)}{(P-iq)^{m+1}}\exp\left[\frac{R\eta}{2}-\eta\sqrt{\frac{N}{2}}.\sqrt{P+\frac{R^2+4iN}{2N}}\right] \quad (21)$$

Inverting equation (21) by Fourier Mellin inversion integral, we get (22)

$$U(\eta,t)=\frac{\lambda\Gamma(m+1)}{2\pi i}\exp\left(\frac{-\eta R}{2}\right)\int_{c-i\infty}^{c+\infty}\exp\left[Pt-\eta\sqrt{\frac{N}{2}}.\left(P+\frac{R^2+4iN}{2N}\right).\frac{dp}{(p-i\Gamma)^{m+1}}\right] \quad 22$$

Let us put

$$Q=\eta\sqrt{\frac{N}{2}},x^2=\left(P+\frac{R^2+4in}{2N}\right),\alpha=\left(i\Gamma+\frac{R^2+4iN}{2N}\right)$$

Than we have

$$U(\eta,t)=\lambda\Gamma(\mu+1)\exp\left(-\frac{\eta R}{2}\right)I(\eta,t,\alpha,m) \quad (23)$$

Where $I(\eta,t,\alpha,m)$

$$=\quad \frac{1}{2\pi i}\int_{Br_2}\exp\left[\left\{x^2-\frac{R^2+4iN}{2N}\right\}t-Qx\right]\frac{2x}{(x^2-\alpha)}dx \quad (24)$$

The path Br_2 is Brom which path defined. Now we have to find the values of $I(\eta,t,\alpha,m)$ for different values of *m*.

Let $F(\alpha)=I(\eta,t,\alpha,0)$

$$=\frac{1}{2\pi i}\int_{Br_2}\exp\left[\left\{x^2-\frac{R^2+4iN}{2N}\right\}t-Qx\right]\frac{2x}{(x^2-\alpha)}dx \quad (25)$$

Differentiating (25) w.r.t. to $'\alpha'$ we get

$$I(\eta,t,\alpha,1)=\frac{dF}{d\alpha}+tF \quad (26)$$

Again differentiating (26) successively (*m* – 1) times w.r.t. to $'\alpha'$ we get $I(\eta,t,\alpha,m)$ for different values of *m*.

Also, $$I(\eta,t,\alpha,0)=\frac{1}{2}\exp(i\sigma t)\left[\exp\left(Q\sqrt{\alpha}\right)\right]erf\,c\left(\frac{Q+2t\sqrt{\alpha}}{2\sqrt{t}}\right)+\exp\left(-Q\sqrt{\alpha}\right)erf\,c\left(\frac{Q-2t\sqrt{\alpha}}{2\sqrt{t}}\right) \quad (27)$$

With the help of equation (26) and (27) we get

$$I(\eta,t,\alpha,1)=\frac{1}{2}\exp(i\sigma t)\left[\exp\left(Q\sqrt{\alpha}\right)\right]erf\,c\left(\frac{Q+2t\sqrt{\alpha}}{2\sqrt{t}}\right)\left(\frac{Q}{2\sqrt{\alpha}}+1\right)-\exp\left(Q\sqrt{\alpha}\right)erf\,c\left(\frac{Q-2t\sqrt{\alpha}}{2\sqrt{t}}\right)\left(\frac{C}{2\sqrt{\alpha}}-t\right) \quad (28)$$

Similarly, we can get $I(\eta,t,\alpha,2)$ and in general $I(\eta,t,\alpha,m)$ for different integral values of m.

Some Special Cases

When $i\sigma t\neq 0$ and *m* = 0 the velocity field is obtained with the help of equations (23) and (27) as :

$$U\frac{(\eta+t)}{\lambda}=\frac{1}{2}\exp\left(i\sigma t-\frac{R\eta}{2}\right)\left[\exp\left\{\eta\left(\frac{N}{2}\right)^{\frac{1}{2}}\left(i\sigma+\frac{R^2+i4N}{2N}\right)^{\frac{1}{2}}\right\}\right.$$

$$+erf\ C\left\{\eta\left(\frac{N}{2}\right)^{\frac{1}{2}}+\left(i\sigma+\frac{R^2+4iN}{2N}\right)^{\frac{1}{2}}\sqrt{t}\right\}$$

$$+\exp\left\{-\eta\left(\frac{N}{2}\right)^{\frac{1}{2}}\left(i\sigma+\frac{R^2+4iN}{2N}\right)^{\frac{1}{2}}\right\}$$

$$\times\ erf\ C\left\{\frac{\eta}{2}\left(\frac{N}{2t}\right)^{\frac{1}{2}}-\left(i\sigma+\frac{R^2+4iN}{2N}\right)^{\frac{1}{2}}\sqrt{t}\right\} \qquad (29)$$

Where erf $C(x)$ is the complementary error function

$$erf\ C(x)=1-erf\ (x)=\frac{2}{\sqrt{\pi}}\int_x^\infty e^{-z^2}dz \qquad (30)$$

The solution (29) describe the general feature of unsteady hydrodynamic boundary layer flow in rotating fluid including the effect of uniform suction according as R > 0 or R < 0.

This result agrees with the known result. By taking R = 0 the velocity distribution is exactly identical with known result.

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