STATE SPACE MODELING AND SIMULATION OF DC-DC CONVERTERS USING SIMULINK

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Abstract : Switched mode DC-DC converters are the simplest power electronic circuits which convert one level of electrical voltage into another level by the action of switching. These converters have received an increasing deal of interest in many research areas. This is due to their wide applications like power supplies for personal computers, office equipment, appliance control, telecommunication equipment, DC motor drives, automotive, aircraft, etc. The analysis, design, control and stabilization of switching converters are the main factors that need to be considered. The design of power electronic converter circuit by using closed loop scheme needs modeling and then simulating the converter using the mathematical equations. This can easily be done with the help of state equations using MATLAB/SIMULINK as a tool for simulation of those state equations. This paper describes an efficient method to learn, analyze and simulation of power electronic converters, using system level nonlinear, and switched state-space models.

IndexTerms - Switching converters, MATLAB/SIMULINK, system modeling, cascade control, subsystem, Simulation

I. INTRODUCTION:

The emergence of integrated circuit around 1964 due to transistor’s miniaturization facilitated a great revolution of electronics which allowed the emergence of various portable applications. Consequently, mobile equipment industry has known strong growth and mobiles integrate more and more functionalities in smaller volumes. In other words, battery-powered mobile equipment has become an important pillar of the electronic consumer market [1]. However, all these mobile equipment have the same major weakness: their battery provides a limited operating time, which can only be increased in two ways. First, the energy density of the battery can be increased by developing new battery chemistries. Second, the battery energy can be used more efficiently by improving energy management [2]. We will focus on the latter, and especially on voltage conversion, which is used in mobile equipment. A way used to optimize the battery run-time consists in inserting a DC-DC converter between the battery and supplied load. In recent years, these converters have received an increasing deal of interest in many areas of applications due to maintain the voltage supplied to the load constant from no load to full load with high conversion efficiency. The controller design for any system needs knowledge about system behavior [3]. Usually this involves a mathematical description of the relation among inputs to the process, state variables, and output. This description in the form of mathematical equations which describe behavior of the system (process) is called model of the system. This paper describes an efficient method to learn, analyze and simulation of power electronic converters, using system level nonlinear, and switched state-space models. The Boost converter is one of the simplest but most useful power converters [4]. It is a step-up converter that converts an unregulated DC input voltage to a regulated DC output at a lower voltage. Figure 2 shows the basic circuit configuration used in the buck converter. As can be seen, it consists of a power MOSFET switch Q, flywheel diode D, inductor L, output capacitor C, and load resistance R. The inductor L acts as energy storage element that keeps the current flowing while the diode facilitates inductor current wheeling during the OFF time of the MOSFET. Filter made of capacitor (C) is normally added to the output of the converter to reduce output voltage ripple [5-7]. The MATLAB/SIMULINK software package can be advantageously used to simulate power converters. This study aims at development of the models for all basic converters and studying its open loop response, so these models can be used in case of design of any close loop scheme [8]. Also as a complete exercise a closed scheme case has been studied using cascaded control for a boost converter [9-10]. Controller design for any system needs knowledge about system behavior. Usually this involves a mathematical description of the relation among inputs to the process, state variables, and output [11-14]. This description in the form of mathematical equations which describe behavior of the system is called model of the system. This paper describes an efficient method to learn, analyze and simulation of power electronic converters, using system level nonlinear, and switched state-space models. The MATLAB/SIMULINK software package can be advantageously used to simulate power converters. This study aims at developing of the models for all basic converters and studying its open loop response, so these models can be used in case of design of any close loop scheme [15]. Also as a complete exercise a closed scheme case has been studied using cascaded control for a boost converter.

II. Simulink Model Construction of DC-DC Switching Converter:

The System modeling is probably the most important phase in any form of system control design work. The choice of a circuit model depends upon the objectives of the simulation. If the goal is to predict the behavior of a circuit before it is built, the model is built using a designer with valuable information about the system dynamics. Due to the difficulty involved in solving general nonlinear equations, all the governing equations will be put together in block diagram form and then simulated using Matlab’s Simulink program. Simulink will solve these nonlinear equations numerically, and provide a simulated response of the system dynamics. To obtain a nonlinear model for power electronic circuits, one needs to apply Kirchhoff’s circuit laws. To avoid the use of complex mathematics, the electrical and semiconductor devices must be represented as ideal components (zero ON voltages, zero OFF currents, zero switching times). Therefore, auxiliary binary variables can be used to determine the state of the switches. It must be ensured that the equations obtained by the use of Kirchhoff’s laws should include all the permissible states due to power semiconductor devices being ON or OFF.

III. Modeling of Open-Loop DC-DC Converters:

A. Modeling of Buck Converter:

The buck converter with ideal switching devices will be considered here which is operating with the switching period of T and duty cycle D Fig. 1. [1]. The state equations corresponding to the converter in continuous conduction mode (CCM) can be easily understood by
applying Kirchhoff's voltage law on the loop containing the inductor and Kirchhoff's current law on the node with the capacitor branch connected to it. When the ideal switch is ON, the dynamics of the inductor current $i_L(t)$ and the capacitor voltage $v_C(t)$ are given by,

$$\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L} (V_{in} - v_o), \quad 0 < t < dT, \quad Q: ON \\
\frac{dv_c}{dt} &= \frac{1}{C} (i_L - v_o) \\
\end{align*}$$

(1)

and when the switch is OFF are presented by,

$$\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L} (-v_o), \quad dT < t < T, \quad Q: OFF \\
\frac{dv_c}{dt} &= \frac{1}{C} (i_L - v_o) \\
\end{align*}$$

(2)

These equations are implemented in Simulink as shown in Fig. 2 using VARIOUS SIMULINK blocks such as multipliers, summing blocks, and gain blocks, and subsequently fed into two integrators to obtain the states $i_L(t)$ and $v_C(t)$ [2-4].

**B. Modeling of Boost Converter:**

The boost converter of Fig. 3 with a switching period of $T$ and a duty cycle of $D$ is given. Again, assuming continuous conduction mode of operation, the state space equations when the main switch is ON are shown by, [1].

$$\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L} (V_{in}) \\
\frac{dv_c}{dt} &= \frac{1}{C} (v_o) \\
\end{align*}$$

(3)

and when the switch is OFF
Fig. 4 shows these equations in Simulink using multipliers, summing blocks, and gain blocks, and subsequently fed into two integrators to obtain the states $i_L(t)$ and $v_C(t)$.[2][3][4].

\[
\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L} (V_o - v_o), \quad dT < t < T, \quad Q: \text{OFF} \\
\frac{dv_C}{dt} &= \frac{1}{C} (i_L - v_o) / R
\end{align*}
\]  

Fig. 3 DC-DC Boost Converter

C. Modeling of Buck-Boost Converter:

In Fig. 5 a DC-DC buck-boost converter is shown. The switching period is $T$ and the duty cycle is $D$. Assuming continuous conduction mode of operation, when the switch is ON, the state space equations are given by, [1]

\[
\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L} (V_o) \\
\frac{dv_C}{dt} &= \frac{1}{C} (-v_o) / R
\end{align*}
\]  

and when the switch is OFF

\[
\begin{align*}
\frac{di_L}{dt} &= \frac{1}{L} (v_o) \\
\frac{dv_C}{dt} &= \frac{1}{C} (-i_L - v_o) / R
\end{align*}
\]
These equations are implemented in Simulink as shown in Fig. 6 using multipliers, summing blocks, and gain blocks, and subsequently fed into two integrators to obtain the states $i_L(t)$ and $v_C(t)$.

**D. Modeling of Cuk Converter:**

The Cuk converter of Fig. 7 with switching period of $T$ and duty cycle of $D$ is considered. During the continuous conduction mode of operation, the state space equations are as follows,

$$
\begin{align*}
\frac{di_{L1}}{dt} &= \frac{1}{L_1} (v_{in}) \\
\frac{dv}{dt} &= \frac{1}{C_1} (-v_C) \\
\frac{dt}{dt} &= \frac{1}{C_2} (i_{L2} - v_C) \\
\frac{dv}{dt} &= \frac{1}{L_2} (-v_C + v_o) \\
\end{align*}
$$

When the switch is OFF the state space equations are represented by

$$
\begin{align*}
\frac{di_{L1}}{dt} &= \frac{1}{L_1} (v_{in} - v_o) \\
\frac{dv}{dt} &= \frac{1}{C_1} (i_{L2}) \\
\frac{dt}{dt} &= \frac{1}{C_2} (i_{L2} - v_o) \\
\frac{dv}{dt} &= \frac{1}{L_2} (-v_C) \\
\end{align*}
$$
IV. Simulation Closed-Loop of DC-DC Converters Using Cascaded Control:

The simulation model for cascaded control of DC-DC switching converters is build using the above-mentioned steps is as shown in Fig. 9. The DC-DC buck, boost, buck-boost, and Cuk converters was previously designed, and simulated on digital computer using MATLAB package.

The complexity of device models and switching nature of switching converters make simulation difficult due to converge in Pspice. Simulink is a windows oriented dynamic modeling package that is an extension to MATLAB. The advantage is that models are entered as block diagrams after corresponding mathematical equations are developed for the target system. MATLAB uses ordinary differential equation solver (ode45) to solve sets of linear and non-linear differential equations which in this case are emulated by block diagrams. Thus to simulate an electrical system such as DC-DC converter, one has to write equations for various blocks in the system and construct an equivalent block diagram using icons in SIMULINK. The parameters for individual icons can be set for the process. Finally, a choice of equation solver and simulation time is made. The output of system could be observed or recorded into file. SIMULINK also provides the
feature of writing S-functions which implements the equations of a block. The disadvantage in using s-functions is that no bode plots can be observed if an s-function is in the block diagram.

V. Results and Conclusions:
The inductor current and capacitor voltage for open loop simulation of all converters are as shown in Fig.10.

![Fig. 10 Output voltage and inductor current Open-loop for Buck Converter](image)

![Fig. 10 Output voltage and inductor current Open-loop for Boost Converter](image)

![Fig. 10 Output voltage and inductor current Open-loop for Buck-Boost Converter](image)
The intuitive signal flow interface in SIMULINK makes this mathematical model and its corresponding masked subsystem very easy to create. The results of Closed loop using a cascaded control scheme for a boost converter is shown in Fig. 10(a). Here the output voltage rises up to 21.3V (6.5%) for the step variation of load from 10Ω to 13Ω (30%). The output voltage resumes its reference value (of 20 V) within 15ms after the transient variation of load. As per fig 10(b), for a step change at the input voltage from 10V to 18 V (80%) (at 0.5 Sec instant), a satisfactory performance is obtained in the output voltage which has a rise up to 22.8V (14%), but it is quickly dropped to its set value (20 V) within 16 ms. Simulation results verify that the control scheme in this section gives stable operation of the power supply. The output voltage and inductor current can return to the steady state even when it is affected by line and load variation.

VI. Conclusions:
This paper analysis nonlinear, switched, state-space models for buck, boost, buck-boost, and Cuk converters. The simulation environment is quite suitable to design the modeling circuit, and to learn the dynamic behavior of different converter structures in open loop. A key challenge to design switching regulators is to maintain almost constant output voltage within acceptable regulation. The simulation results obtained, show that the output voltage and inductor current can return to steady state even when it is affected by input voltage and load variation, with a very small over shoot and settling time.

References