Abstract- In today’s era, power generation and devices working low power output are the main point of attraction all over the world. In this context, thermoelectric devices (TEs) possess large number of advantages like low power operation, no mobile parts, no hazardous operating media, ruggedness and minimum operating maintenance requirements. That is why these devices are highly applicable in number of emerging fields like biomedical, commercial, defense, aeronautics etc. They work on the principle of direct conversion of heat energy into electrical power output and vice-versa following Seebeck and Peltier effects. They can also be integrated/cascaded with other energy generation routes like Solar/PV, Solar/Thermal or high temperature exhaust from various industrial processes. In present work, single- and 2-stage thermoelectric coolers are thermodynamically reviewed in view of 1st/2nd law of thermodynamics. It is found that the thermoelectric coolers made up of semiconductor p-type and n-type modules are highly efficient at low power output in comparison to the conventional cooling options. The variations in the coefficient of performance (COP) of these devices is investigated and presented with respect to different number of thermoelectric modules for various working electrical current values. It is also found that effective balance between various performing variables is very much required for improving the operating parameters of these devices.

Keywords- Thermoelectric Coolers, 1st/2nd law of thermodynamics, Thermodynamic optimization, finite time thermodynamics, Coefficient of performance

I. INTRODUCTION

The major problems with thermoelectric coolers (TECs) are their low COP and cooling load that is because of their electrical, thermal and thermoelectric material characteristics. The upgradation in the operating characteristics of these devices can be obtained with two possible options as to employ new/improved thermoelectric material with high figure of material or by configuring these devices in two-stage mode [1-2]. To maintain maximum temperature gradient between the cold/hot junction, multi-stage configuration of thermoelectric coolers can be utilized as each stage can contribute towards the small temperature difference and hence, two-stage system possesses much better cooling capacity [3-4]. Linder et al. [5] utilized multi-stage cascaded thermoelectric heat pump to be utilized for various practical applications. They investigated the proposed configurations to improve upon its performance and operating variable conditions. Ma and Riffat [6] reviewed the existing and possible potential of thermoelectric devices in numerous applications and presented an overall view of their possible improvement in all the fields. Additionally, Riffat and Ma [7] proposed a comprehensive review on the improvement of COP of TECs in context with its performance improvement. In addition to this, they simulated and experimentally tested the reverse thermoelectric cooler while considering all performance and design variables [8]. Meng et al. [9] carried out optimization in the performance characteristics of thermoelectric cooler which was powered by the thermoelectric generator and presented the various design variables for its further upgradation in practical applications. Chen et al. [10] studied the impact of heat transfers on TEC driven by the thermoelectric converter. The thermoelectric cooler powered by the generator system is highly compatible, efficient and useful device for the remote areas where power generation is still a big issue of concern. Kaushik et al. [11] accomplished energy/exergy investigations on reverse thermoelectric cooler to observed the effects of external/internal irreversibilities on the concert evaluation of these devices. Kaushik et al. [12] investigated and modelled a thermoelectric generator system with internal/external irreversibilities in order to optimize its performance and operating conditions for various applications in power generation/cascading with cooling devices. Hans et al. [13] studied 2-stage thermoelectric converter in two proposed modes electrically parallel/series in order to analyze the improvement in performance characteristics in different modes. Similarly, they carried out the thermodynamic investigations on reverse thermoelectric cooler i.e., heat pump in 03 different configurations, separated/parallel and series modes [14]. Kaushik et al. [15] theoretically and experimentally investigated Solar PV operated thermoelectric cooler device for the applications in storage of food and medicine in rural and remote areas. This system is very much useful as it overcomes all the environmental hazards and power issues of conventional refrigeration system and offers an environmental friendly output at low cost. Later, they carried out an experiment to expand the throughput of the system with respect to the environmental conditions and operating parameters [16]. Kaushik et al. [17] carried out exergy/energy investigations on annular mode thermoelectric heat pump device with various operating parameters in order to propose the scope of improvement in their performance characteristics. Tan et al. [18] evaluated and obtained an optimal study on the cooling temperature of single-stage thermoelectric cooler with the use of second law of thermodynamics and found that the performance evaluation of these devices is incomplete without the use of entropy generation law. These devices lead to high entropy generation while performing and should be optimized for various operating parameters. It is observed that thermoelectric coolers need lot of improvement in coefficient of performance and cooling capacity areas for which soft computing based non-sorted genetic algorithm is a possible option to optimize the performance parameters. The operation of genetic algorithm is based on the natural selection of genes and it chooses the best ones to yield the optimum output in minimum computational time. The similar principle can be applied in area of thermoelectric devices in order to improve their performance characteristics by multi-objective optimization. This technique proposes the best possible balance amongst the operating parameters and chooses their optimum values in minimum possible time. Many researchers have carried out multi-objective optimization study on several thermal/heat energy converting devices viz. Brayton heat engine, Stirling heat engine, thermoelectric devices etc. [19-31]. Hans et al. [32] investigated thermoelectric generators in electrical series mode with the use of multi-objective based genetic algorithm optimizing approach with the aim of obtaining the performance and design variables. They carried out simultaneous optimization of electrical output power/efficiency with respect to working electric current, nos. of thermoelectric couple in two stages and operating range of cold/hot junction temperatures. Later, they applied genetic algorithm tool to the thermoelectric heat pump in order to get best compromise between the COP and heating load [33]. They utilized a decision-making...
technique i.e., Fuzzy Bellman Zadeh to excerpt the optimal solution from the Pareto front. Arora et al. [34] investigated thermoelectric generators in electrical series/parallel modes with the use of NSGA-II based multi-objective optimization technique with the aim of obtaining the performance and design variables. They carried out simultaneous optimization of electrical output power, 1st law efficiency and entropy generation with respect to working electric current, nos. of thermoelectric couple in two stages and operating range of cold/hot junction temperatures. This technique offers number of advantages as to get maximum possible power output/efficiency with smallest possible entropy generation which is the requirement of today’s era. Similarly, they choses different sets of objective function to optimize the performance characteristics of thermoelectric generators in electrically parallel/series configurations [35]. Later, Arora et al. [36] carried out the multi-objective optimization investigation on reverse thermoelectric cooler and found the effect of Thomson irreversibility on the device operation. They carried out the comprehensive comparison of the devices in view of optimizing its performance with an appropriate balance amongst the design variables.

In this view, a single- and 2-stage TEC have been extensively reviewed on the basis of 1st/2nd laws of thermodynamics. The systems are investigated with the internal irreversibilities as Fourier and Joule effects. The external irreversibility owing to finite heat transfer between the source and the sink are also reviewed. An attempt has also been made to formulate the coefficient of performance with respect to different electric current values. The variations in the coefficient of performance of these devices is investigated and presented with respect to different number of thermoelectric modules for various working electrical current values. The present study proposes a unique optimization idea in view of designing best performing thermoelectric coolers with minimum cost in single and two-stage configuration. This study is highly applicable in practical designing of cooling devices in all areas of applications.

II. SYSTEM DESCRIPTION

Single-stage and 2-stage thermoelectric coolers

Fig. 1(a-b) illustrates the 1-stage thermoelectric cooler model and its generalized form along the outside heat transfers. The multi-stage thermoelectric cooler with electrical series connection of subsequent stages is shown in Fig. 2. The thermoelectric cooler contains N/P-types semiconductor modules assembled together in series electrical but in parallel thermally. The subsequent figures reveal the thermal heat flow from cold junction to hot junction due to the external electrical supply by a voltage source. The temperatures of cold/hot junctions can be represented as Tc/Th respectively. It works on the principle of Peltier effect which states that a temperature difference can be maintained between two junctions due to an electrical current supplied by an external source voltage. The hot junction and the cold junction are associated with a heat source and a sink with temperatures TH and TC. The semiconductor modules possess electrical resistance of Rp/Rn as shown in Fig. 1(b). The cooler takes out the heat from the object kept at the cold junction and rejects it to the hot junction as shown in Figs 1-2. The amount of heat flowing in the circuit can be quantified as NαITc and get rejected towards the sink of the cooler.
II. GENERAL THERMODYNAMIC MODELING EQUATIONS

Apply 1st law of thermodynamics, the energy conservation equation for semi-conductor thermoelectric couple is given as

\[ Q_{\text{storage}} = Q_{\text{in}} - Q_{\text{out}} + Q_{\text{gen}} - Q_{\text{loss}} \]  

Here, \( Q_{\text{in}}/Q_{\text{out}} \) are the input/output heats of the couple and \( Q_{\text{gen}} \) denotes heat generated owing to Joule effect. Now, \( Q_{\text{loss}}/Q_{\text{storage}} \) are ignored, and Eqn. (1) is reframed as

\[ \frac{d}{dx}(T_p + dT_p)k_pA_p - \frac{dT_p}{dx}k_pA_p + (J_p) \frac{A_pL_p}{\sigma_pL_p} - \tau I dT = 0 \]  

Likewise, the eqn. for n-type thermoelectric couple is written as

\[ \frac{d}{dx}(T_N + dT_N)k_NA_N - \frac{dT_N}{dx}k_NA_N + (J_N) \frac{A_NL_N}{\sigma_NL_N} - \tau I dT = 0 \]  

Boundary condition for N/P type semi-conductor legs are specified as

\[ T_p(0)=T_N(0)=T_c \]  

\[ T_p(L_p)=T_N(L_p)=T_l \]  

The temperature dependent properties are taken into account. Additional, \( k_p/\sigma_p \) is the function of \( T_p \) while \( k_N/\sigma_N \) is the function of \( T_N \). Put the boundary conditions to Eqns (2) and (3), differential equations are reframed as

\[ \tilde{k} p A_p \frac{dT_p^2}{dx^2} + \frac{I^2}{\sigma_p A_p} - \tau I \frac{dT_p}{dx} = 0 \]  

\[ -k_N A_N \frac{dT_N^2}{dx^2} + \frac{I^2}{\sigma_N A_N} - \tau I \frac{dT_N}{dx} = 0 \]  

Several heat transfer equations are obtained as:
\[ Q_{h1} = \left( \alpha_1 T_n I + \frac{1}{2} I^2 R_1 - K_1 (T_h - T_m) \right)n \] (8)

\[ Q_{c1} = \left( \alpha_1 T_m I - \frac{1}{2} I^2 R_1 - K_1 (T_h - T_m) \right)n \] (9)

\[ Q_{h2} = \left( \alpha_2 T_m I + \frac{1}{2} I^2 R_2 - K_2 (T_m - T_c) \right)m \] (10)

\[ Q_{c2} = \left( \alpha_2 T_c I - \frac{1}{2} I^2 R_2 - K_2 (T_m - T_c) \right)m \] (11)

Combining Eqs. (9) and (10) gives

\[ Q_{c1} = Q_{h2} \] (12)

\[ \alpha_2 T_m I - \frac{1}{2} I^2 R_2 - K_2 (T_m - T_c) = \alpha_2 T_m I + \frac{1}{2} I^2 R_1 - K_2 (T_m - T_c) \] (13)

Eqn. (14) is applicable for series configuration whereas Eqn. (15) is well obeyed in case parallel configuration of TEC,

\[ I_1 = I_2 = I \] (14)

\[ I_1 + I_2 = I; V_1 = V_2 \] (15)

The coefficient of performance of the TEC is

\[ COP = \frac{Q_{c2}}{Q_{h1} - Q_{c2}} \] (16)

The COP of TEC is reliant on the ensuing stated constraints, laterally through the material properties.

\[ COP = f(I, T_h, T_c, m, n) \]

### III. Results and Discussions

Fig. 3 reveals the schematic pattern of coefficient of performance of a single state cooler with the electric current for several number of thermoelectric legs. It signifies that for an optimum value of number of thermoelectric elements, the system possesses highest cooling capacity at same magnitude of current. Moreover, the optimal magnitude of working electric current can also be found from the variation pattern as shown below. It also verifies the fact that the designer should be careful about the cost analysis of the system as number of element is directly related to the cost of it.

![Fig. 3. COP versus ‘I’ for various values of ‘N’ in single-stage operation](image)

Fig. 4 shows a parabolic variation of COP with thermoelectric legs number in top stage ‘n’, for several electric current values. The maximum COP is found for I=5 with several of thermoelectric legs in top stage as 15, which is just half the value of thermoelectric legs in the upper stage to the sum of thermoelectric legs in two stages. It clearly signifies that for an optimum value of total of thermoelectric legs at the upper stage i.e. n=15 and M=30, the COP of the TEC comes out as 2.48. For an ideal outcome of working electrical current COP first increases and after attaining a maximum value, it declines with ‘n’. This relationship is very helpful in for optimizing COP with number of thermoelectric element pairs.
IV. The characteristics of COP with the total of thermoelectric legs of the upper stage \( n \) for different values of current is revealed in the figure given above. This figure verifies the results shown in Fig. 5 as for any chosen value \( I \), COP is a monotonically increasing function of \( n \). As we increase the total of thermoelectric legs in upper stage, the system yields better coefficient of performance.

From Fig. 6 one can make out that, COP of the TEC in electrical parallel configuration/mode holds inverse relationship with the total of thermoelectric legs in bottommost stage. For a preset value of working electrical current, maximum value of COP comes at \( m=5 \) and eventually decreases afterwards.

Fig. 5 and Fig. 6 gives very important graphical relationships to optimize COP of the cooler in electrical parallel configuration/mode with total of thermoelectric legs in upper and lower stages.

Fig. 7 explains the correlation of the COP with total of thermoelectric legs in electrically parallel TEC. One should go for higher values of \( M \) and \( n \) but lower values of \( m \) to obtain considerable value of COP for the system. The comparative performance evaluation of single- and two-stage cooler illustrates the statistics that as we cascade more number of stages the maximum temperature difference can also be achieved in the system. Although the cost goes up with the multi-stage system configuration but one has to trade-off between the economic and the performance criteria while its designing. The work can be further enhanced to obtain an appropriate balance between the coefficient of performance and the cooling capacity as the one parameter attains its maximum value on the verge of other. This balance can be achieved by optimizing the two objectives simultaneously with the help of multi-objective optimization using soft computing techniques.
IV. CONCLUSIONS
In present work, single-stage and two-stage thermoelectric coolers are thermodynamically reviewed in view of thermodynamic principles. It is found that the thermoelectric coolers made up of semiconductor p-type and n-type modules are highly efficient at low power output in comparison to the conventional cooling options. The devices are investigated with the consideration of internal/external irreversibilities as Fourier, Joule effects and finite heat transfer between the source and the sink. An attempt has also been made to formulate the coefficient of performance with respect to different electrical current values. The variations in the coefficient of performance of these devices is investigated and presented with respect to different number of thermoelectric modules for various working electrical current values. The study also verifies the fact that the designer should be careful about the cost analysis of the system as number of element is directly related to the cost of it. With the rise in the total of thermoelectric legs in the upper stage, the system yields better coefficient of performance as the upper stage is directly linked to the performance of the device. The comparative performance evaluation of single- and two-stage cooler illustrates the statistics that as we cascade more number of stages the maximum temperature difference can also be achieved in the system. Although the cost goes up with the multi-stage system configuration but one has to trade off between the economic and the performance criteria while its designing. It is also found that effective balance between various performing variables is very much required for improving the operating parameters of devices. The balance can be achieved by employing soft computing based multi-objective optimization techniques viz. NSGA-II, TLBO, PSO etc. The present study proposes a unique optimization idea in view of designing best performing thermoelectric coolers with minimum cost in single and two-stage configuration. This study is highly applicable in practical designing of cooling devices in all areas of applications.
REFERENCES


