Variance of Time to Recruitment for a Two Graded Manpower System with Inter-Decision Times Having Independent and Non-Identically Distributed Random Variables under Geometric Process Wastage

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Abstract :In this paper, an organization subjected to a random exit of personnel due to policy decisions taken by the organization is considered; there is an associated loss of manpower if a person quits the organization. As the exit of personnel is unpredictable, a recruitment policy involving two thresholds, optional and mandatory is suggested to enable the organization to plan its decision on appropriate univariate policy of recruitment. Based on shock model approach, two mathematical models are constructed using an appropriate univariate policy of recruitment. The analytical expressions for mean and variance of time to recruitment is obtained wheni) loss of manpower forms a geometric process ii) inter-decision times forms a sequence of independent and non-identically distributed exponential random variables iii) optional and mandatory thresholds having exponential distribution.

Keywords: Manpower planning, shock models, univariate recruitment policy, hypo-exponential distribution, geometric process.

I. INTRODUCTION

Many models have been discussed using different kinds of wastage and different types of distributions for the threshold. Such models could be seen in [1],[2],[3]. The problem of time to recruitment is studied by several authors both for a single and multigraded system for different types of thresholds, according as the inter-decision times are independent and identically distributed random variables (or) correlated random variables. For a multi-graded system ,in [7] the author has obtained the performance measures namely mean and variance of the time to recruitment for a two graded system, when (i) loss of manpower and the threshold for the loss of manpower in each grades are exponential random variables (ii) inter-decision times are independent and identically distributed exponential random variables forming the same renewal process for both grades and (iii) thresholds for the organization is the max(min) of the thresholds for the two grades max(min) model using the above cited univariate cumulative policy of recruitment.

In [8] the author has extended the results for geometric thresholds when the inter-decision times for the two grades form two different renewal processes. In [9] the authors have studied the results in [8] when the threshold for the organization is the sum of the thresholds for the grades .In [8] the work in [9] is studied when the loss of manpower and thresholds are geometric random variables according as the inter-decision times for the two grades are correlated random variables or forming two different renewal processes. This author has also obtained the mean time to recruitment for constant combined thresholds using a univariate max policy of recruitment. In all the earlier research works the monotonicity of inter-decision times which do exists in reality is not taken into account. In [9] the above limitation is removed and the authors have obtained the mean time to recruitment for a single grade manpower system by assuming that (i) inter-decision times form a geometric process in which the monotonicity is inbuilt in the process itself (ii) loss of manpower is a sequence of independent and identically distributed exponential random variables and (iii) distribution of the thresholds for the loss of manpower in the organization is exponential. In [11] the authors have obtained the mean time to recruitment for a two graded manpower system with a univariate policy of recruitment involving combined thresholds using geometric process for inter-decision times. In [12] the authors have obtained mean and variance of time to recruitment for a two graded manpower system with a univariate policy of recruitment involving (i) loss of manpower and inter-decisions times are independent and non-identically distributed exponential random variables (ii) thresholds optional and mandatory follows exponential random variables. Recently in [13], the authors have obtained mean and variance of time to recruitment for a two graded manpower system with a univariate policy of recruitment when i) loss of manpower forms a geometric process ii) inter-decision times forms a sequence of independent and non-identically distributed exponential random variables and iii) optional and mandatory thresholds having exponential distribution.

The objective of the present paper is to study the problem of time to recruitment for a two graded manpower systems and to obtain the mean and variance of time to recruitment using CUM univariate recruitment policy for exponential thresholds with loss of manpower forms a geometric process, inter-decision times forms a sequence of independent and non-identically distributed exponential random variables and optional and mandatory thresholds having exponential distribution. The analytical results obtained and are numerically illustrated and the influence of nodal parameters on the mean and variable of time to recruitment is studied.

II. Notations

X_i	:	Loss of man hours due to the i^{th} policy decision epoch i=1,2,3,…forming a geometric process with rateα.
X_1	:	Exponential distribution with parameter θ.
S_k	:	Cumulative loss of man hours in the first k^{th} decisions k=1,2,…$S_k=\sum_{i=1}^{k} X_i$.
$J_i(.)$	:	Probability distribution function of X_i, the i^{th} term of geometric process.

$j_i(.)$: probability density function of S_k.
$g_k^*(.)$: Laplace transform of convolution of density function of S_k.
U_k : Inter-decision times are independent and non-identically distributed exponential random variables between (k-1)th and kth decisions with parameters β_i (β_i>0).
$F_k(.)$: Distribution function of U_k.
$f_k(.)$: Probability density function of $U_k$with mean $\frac{1}{\beta_i}$ (β_i>0).
Y_1,Y_2: Continuous random variables denoting the optional threshold for grade1 and 2 respectively following an exponential distribution with parameters $\lambda_1$and $\lambda_2$respectively.
Z_1,Z_2 : Continuous random variables denoting the mandatory threshold for grade 1 and 2 respectively following an exponential distribution with parameters μ_1 and $\mu_2$respectively.
W : Continuous random variable denoting the time to recruitment in the organization.
p : Probability that the organization is not going for recruitment whenever the total loss of manpower crosses Y.
$V_k(t)$: Probability that exactly k decisions are taken in [0,t).
L(.) : Distribution function of W, l(.) : The probability density function of W.
$l^*(.)$: Laplace transform of l(.).
E(W) : Expected time to recruitment.
V(W) : Variance of time to recruitment.

CUM policy: **Recruitment is done whenever the cumulative loss of manpower crosses the mandatory threshold. The organization may or may not go for recruitment if the cumulative loss of manpower crosses optional threshold.**

III. Main Result

Analytical results for the above cited measures related to time to recruitment, we are derived for the present model.
The survival function of W is given by

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)\, P(S_k<Y) + \sum_{k=0}^{\infty} V_k(t)\, P(S_k\geq Y)\, P(S_k<Z)\, p \quad (1)$$

3.1 Model I : Maximum Model Y= Max (Y_1, Y_2) , Z= Max (Z_1, Z_2).
Conditioning upon S_k and using the law of total probability in equation (3), it can be shown that

$$P(S_k<Y) = \int_0^{\infty} P(S_k < Y \mid S_k = x)\, g_k(x)\,dx$$

$$= g_k^*(\lambda_1) + g_k^*(\lambda_2) - g_k^*(\lambda_1+\lambda_2)$$

ie. $P(S_k<Y) = D_1 + D_2 - D_3$ (2)

$P(S_k<Z) = g_k^*(\mu_1) + g_k^*(\mu_2) - g_k^*(\mu_1+\mu_2)$

$P(S_k<Y) = D_4 + D_5 - D_6$ (3)

Using (2) and (3) in (1) and on simplification, the tail distribution of time to recruitment is given by

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t) A_k \quad (4)$$

Where $A_k = [(D_1 + D_2 - D_3)(1 - p((D_4 + D_5 - D_6)) + p(D_4 + D_5 - D_6)]$
and
$D_1 = g_k^*(\lambda_1)$, $D_2 = g_k^*(\lambda_2)$, $D_3 = g_k^*(\lambda_1+\lambda_2)$, $D_4 = g_k^*(\mu_1)$, $D_5 = g_k^*(\mu_1)$, $D_6 = g_k^*(\mu_1+\mu_2)$. (5)

From renewal theory Medhi (2009),it is known that
$V_k(t) = F_k(t) - F_{k+1}(t)$ with $F_0(t) = 1$.
Since l(t) =-$\frac{d}{dt}P$(W>t), from (7) it can be shown that

$$l^*(s) = \sum_{k=0}^{\infty} [f_{k+1}^*(s) - f_k^*(s)]A_k \quad (6)$$

Since

$$E(W) = -\left[\frac{d}{ds} l^*(s)\right]_{s=0} \quad (7)$$

$$E(W^2) = \left[\frac{d^2}{ds^2} l^*(s)\right]_{s=0} \quad (8)$$

$$V(W) = E(W^2) - (E(W))^2 \quad (9)$$

Since $\{X_i\}$is a geometric process then distribution and density function of $\{X_i\}$are given below
$J_i(x)=P(X_i\leq x)=G(\alpha^{k-1}x)$, k=1,2,3,…, $J_i(x)= \alpha^{k-1}g(\alpha^{k-1}x)$

$$J_i^*(s)=\alpha^{k-1}g^*(\alpha^{k-1}s) = g^*\left(\frac{s}{\alpha^{k-1}}\right) \quad (10)$$

$$\text{Since } g_k^*(s) = \prod_{r=1}^{k} g^*\left(\frac{s}{\alpha^{r-1}}\right), k=1,2,3\ldots \quad (11)$$

From (10), let us consider $g_k^*(\lambda_1) = \prod_{r=1}^{k} g^*\left(\frac{\lambda_1}{\alpha^{r-1}}\right)$

X_1 forms an exponential distribution with parameter θ, g(x) = $\theta e^{-\theta x}$

$$g^*(\lambda_1) = \prod_{r=1}^{k} \int_0^\infty e^{-(\frac{\lambda_1}{\alpha^{r-1}})x}\theta e^{-\theta x}\,dx$$

$$g^*(\lambda_1) = \prod_{r=1}^{k} \left[\frac{\theta\alpha^{r-1}}{(\alpha^{r-1}+\lambda_1)}\right] = G_1$$

Similarly

$$g^*(\lambda_2) = \prod_{r=1}^{k} \left[\frac{\theta\alpha^{r-1}}{(\alpha^{r-1}+\lambda_2)}\right] = G_2 \text{ , } g^*(\lambda_1+\lambda_2) = \prod_{r=1}^{k} \left[\frac{\theta\alpha^{r-1}}{\alpha^{r-1}+(\lambda_1+\lambda_2)}\right] = G_3$$

$$g^*(\mu_1) = \prod_{r=1}^{k} \left[\frac{\theta\alpha^{r-1}}{(\alpha^{r-1}+\mu_1)}\right] = G_4 \text{ , } g^*(\mu_2) = \prod_{r=1}^{k} \left[\frac{\theta\alpha^{r-1}}{(\alpha^{r-1}+\mu_2)}\right] = G_5 \text{ , } g^*(\mu_1+\mu_2) = \prod_{r=1}^{k} \left[\frac{\theta\alpha^{r-1}}{\alpha^{r-1}+(\mu_1+\mu_2)}\right] = G_6$$

Using the above results in (5), it can be shown that

$$l^*(s) = \sum_{k=0}^{\infty} [f_{k+1}^*(s) - f_k^*(s)]\,[(G_1+G_2-G_3)(1-p((G_4+G_5-G_6))+p(G_4+G_5-G_6)] \tag{12}$$

Inter-decision times form independent and non-identically distributed exponential random variables, so it follows hypo-exponential distribution with parameters β_i

The probability density function is $f_k(t) = \sum_{i=1}^{k} b_i\,\beta_i e^{-\beta_i t}$ and the Laplace transform is

$$f_k^*(s) = \sum_{i=1}^{k} b_i \frac{\beta_i}{\beta_i+s}\text{, where } b_i = \prod_{\substack{j=1\\ j\neq i}}^{k} \frac{\beta_j}{\beta_j-\beta_i} \tag{13}$$

Using (13) in (12), it can be shown that

$$l^*(s) = \sum_{k=0}^{\infty}\sum_{i=1}^{k+1} \left(b_i \frac{\beta_i}{\beta_i+s}\right) [(G_1+G_2-G_3)(1-p((G_4+G_5-G_6))+p(G_4+G_5-G_6)] \tag{14}$$

From (7) and (8), we get

$$E(W) = \sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}}[(G_1+G_2-G_3)(1-p((G_4+G_5-G_6))+p(G_4+G_5-G_6)] \tag{15}$$

Equation (15) gives meantime to recruitment for maximum model.

$$E(W^2) = \left[\frac{d^2}{ds^2} l^*(s)\right]_{s=0} = 2\sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}{}^2}[(G_1+G_2-G_3)(1-p((G_4+G_5-G_6))+p(G_4+G_5-G_6)] \tag{16}$$

Using (15) and (16) in (9), we get variance of time to recruitment for maximum model.

3.1.1Numerical Illustrations

The analytical expressions for the performance measures namely mean and variance of time to recruitment are analyzed numerically by varying one parameter and keeping other parameters fixed. The effect of nodal parameter 'α' on the performance measures are shown in the following tables.

Effect of α on performance measures

β_2=0.006, β_3=0.007, β_4=0.008, β_5=0.009, β_6=0.010, λ_1=0.01, λ_2=0.02, μ_1=0.03, μ_2=0.04, p=0.06, θ=0.05, For any value of β_1, A_0 = 0.

α>1	**1.5**	**1.8**	**2.5**	**3.5**	**4.5**
E(W)	16.6751	16.6762	16.6779	16.6793	16.6804
V(W)	$3.7752\text{x}10^3$	$3.7753\text{x}10^3$	$3.7755\text{x}10^3$	$3.7756\text{x}10^3$	$3.7757\text{x}10^3$
α<1	**0.957**	**0.825**	**0.735**	**0.655**	**0.563**
E(W)	16.6706	16.6683	16.6661	16.6634	16.6588
V(W)	$3.7747\text{x}10^3$	$3.7745\text{x}10^3$	$3.7742\text{x}10^3$	$3.7740\text{x}10^3$	$3.7735\text{x}10^3$

Findings:

1. As α>1, the X_k's, k=1,2,3……..n form a increasing sequence and hence the loss of man hours between decision epochs will increase. Consequently, the expected time to recruitment and the variance of the time to recruitment increase.
2. As α<1, the X_k's, k=1,2,3……..n form a decreasing sequence and hence the loss of man hours between decision epochs will decrease. Consequently, the expected time to recruitment and the variance of the time to recruitment decrease.

3.2Model II : Minimum ModelY= Min (Y_1, Y_2) , Z= Min (Z_1, Z_2).

Proceeding as in model I it can be shown for the present model that

$$P(S_k<Y) = g_k^*(\lambda_1+\lambda_2) = D_3 \tag{17}$$

$$P(S_k<Z) = g_k^*(\mu_1+\mu_2) = D_6 \tag{18}$$

$$P(W>t) = \sum_{k=0}^{\infty} V_k(t)[D_3(1-pD_6)+pD_6] \tag{19}$$

$$E(W)=\sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}}[G_3)(1-pG_6)+pG_6)] \tag{20}$$

Equation (20) gives meantime to recruitment for minimum model.

$$E(W^2)=2\sum_{k=0}^{\infty} \frac{1}{\beta_{k+1}{}^2}[G_3(1-pG_6)+pG_6] \tag{21}$$

Using (20) and (21) in (9), we get variance of time to recruitment for minimum model.

3.2.1Numerical Illustrations

The analytical expressions for the performance measures namely mean and variance of time to recruitment are analyzed numerically by varying one parameter and keeping other parameters fixed. The effect of nodal parameter 'α' on the performance measures are shown in the following tables.

Effect of α on performance measures

β_2=0.006, β_3=0.007, β_4=0.008, β_5=0.009, β_6=0.010,λ_1=0.01, λ_2=0.02, μ_1=0.03, μ_2=0.04, p=0.06, θ=0.05

α>1	**1.5**	**1.8**	**2.5**	**3.5**	**4.5**
E(W)	16.0062	16.0337	16.0724	16.1012	16.1174
V(W)	$3.6506x10^3$	$3.6537x10^3$	$3.6579x10^3$	$3.6610x10^3$	$3.6628x10^3$
α<1	**0.957**	**0.825**	**0.735**	**0.655**	**0.563**
E(W)	15.9125	15.8712	15.8346	15.7937	15.7325
V(W)	$3.6404x10^3$	$3.6359x10^3$	$3.6320x10^3$	$3.6275x10^3$	$3.6209x10^3$

Findings:

1. As α>1, the X_k's, k=1,2,3……..n form a increasing sequence and hence the loss of man hours between decision epochs will increase. Consequently, the expected time to recruitment and the variance of the time to recruitment increase.
2. As α<1, the X_k's, k=1,2,3……..n form a decreasing sequence and hence the loss of man hours between decision epochs will decrease. Consequently, the expected time to recruitment and the variance of the time to recruitment decrease.

REFERENCES

[1] Barthlomew.D.J. andMorris.B.R., (1971), Aspects of Manpower Planning, Elsevier Publishing Company, New York.
[2] Bartholomew.D.J, (1973), StochasticMoels for Social Processes,2ndEd.,John Wiley & Sons, Chichester.
[3] Barthlomew.D.J and Forbes.A.F,(1979), Statistical Technique for Manpower Planning, John Wiley and Sons.
[4] Dhivya.S, Srinivasan.A and Vasudevan.V,(2011), Stochastic Models for the Time to Recruitment in a Two Graded Manpower System Using Same Geometric Process for Inter-decision Times, Proceedings of the Internaional Conference on Computational and Mathematical Modeling, Narosa Publishing House Pvt.Ltd, pp 276-283.
[5] Esther Clara.S. (2011), Contributions to the Study on Some Stochastic Models in Manpower Planning, Ph.DThesis, Bharathidasan University.
[6] Muthaiyan.A and Sathiyamoorthi.R,(2010), A Stochastic Model Using Geometric Process for Inter-arrival Time Between Wastages, ActaCienciaIndica, 46M(4): 479-486.
[7] Parthasarathy.S, (2003), On Some Stochastic Models for Manpower Planning Using SCBZ Property, Ph.D, Thesisat Department of Statistics, Annamalai University.
[8] Sendhamizhselvi.S, (2009), A Study on Expected Time to Recruitment in Manpower Model for a Multigraded System Associated with an Univariate Policy of Recruitment, Ph.D Thesis, Bharathidasan University.
[9] Sureshkumar.R, Gopal.G and Sathiyamoorthi.P, (2006), Stochastic Models for the Expected Time to Recruitment in an Organization with Two Grades, International Journal of Management and Systems, 22(2), pp 147-164.
[10] Srinivasan.A and Vasudevan.V,(2012),A Stochastic Models for Time to Recruitment in a Two Graded Manpower System with Two Types of Thresholds Using Same Geometric Process for Inter-decision Times, Proceeding of Heber International Conference on Applications of Mathematics and Statistics, pp 667-677.
[11] Srinivasan.A and Vasudevan.V, (2012), A Stochastic Model on the Mean Time to Recruitment for a Two Graded Manpower System Associated with a Univariate Policy of Recruitment Involving Combined Thresholds UsingSame Geometric Process for Inter-decision Times,Recent Research in Science and Technology, 4(12), pp:18-23.
[12] SendhamizhSelvi.S and Jenita.S, (2016), Estimation of Mean and Variance of time to Recruitment in a Two Graded Manpower System with Two Continuous Thresholds for Depletion having Independent and Non-identically Distributed Random Variables, Proceedings of Heber International Conference on Applications of Actuarial Science, Mathematics, Management and Computer Science, pp 116-127.
[13] SendhamizhSelvi.S and Jenita.S, (2017), Mean and Variance of time to Recruitment in a Two Graded Manpower System with Inter-Decision Times having Geometric Process and Depletion having Independent and Non-identically Distributed Random Variables, International Journal of Pure and Applied Mathematics (IJPAM), 117(5), pp: 109-117.