Cordial Labeling Of One Point Union Of Tail- C_5 Garphs

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Abstract: We discuss graphs of type $G^{(k)}$ i.e. one point union of k-copies of G for cordial labeling. We take G as tail graph. A tail graph (or antenna graph) is obtained by attaching a path P_m to a vertex of given graph. It is denoted by tail(G,P_m) where G is given graph. We take G as C_5 and restrict our attention to m = 2, 3 in P_m. Further we consider all possible structures of $G^{(k)}$ by changing the common point and obtain non-isomorphic structures. We show all these structures as cordial graphs. This is called as invariance of different structures of $G^{(k)}$ under cordial labeling.

Key words: cordial, one point union, tail graph, cycle, labeling, path.

Subject Classification: 05C78

1. Introduction

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Holton [6] Graph Theory by Harary [7], A dynamic survey of graph labeling by J.Gallian [9] and Douglas West.[10].I.Cahit introduced the concept of cordial labeling[6]. f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f(u)-f(v)|. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one .Similarly number of edges labeled with 0 i.e.$e_f(0)$ and number of edges labeled with 1 i.e.$e_f(1)$ differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that : every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [9].

Our focus of attention is on one point unions on different graphs. For a given graph there are different one point unions (upto isomorphism) structures possible. It depends on which point on G is used to fuse to obtain one point union. We have shown that for G = bull on C_3,bull on C_4,C_3^+,C_4^+-e the different path union $P_m(G)$ are cordial [4].It is called as invariance under cordial labeling. We use the convention that $v_f(0,1) = (a,b)$ to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b. Further $e_f(0,1) = (x,y)$ we mean the number of edges labeled with o are x and number of edges labeled with 1 are. The graph whose cordial labeling is available is called as cordial graph. In this paper we define tail graph and obtain one point union graphs on it.

3. Preliminaries

3.1 Tail Graph: A (p,q) graph G to which a path P_m is fused at some vertex. This also can be explained as take a copy of graph G and at any vertex of it fuse a path P_m with it's one of the pendent vertex. It's number of vertices are P+m-1 and edges are by q + m-1. It is denoted by tail(G, P_m). In this paper we fix G as C_3 and take P_m for m =2, 3, 4, 5.

3.2 Fusion of vertices. Let $u \neq v$ be any two vertices of G. We replace these two vertices by a single vertex say x and all edges incident to u and v are now incident to x. If loop is formed then it is deleted.[6] 3.3 $G^{(K)}$ it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p, q) graph then $|V(G_{(k)}| = k(p-1)+1$and $|E(G)|= k.q$

Results Proved:

Theorem4.1All non- isomorphic one point union on k-copies of graph obtained on G = tail(C_5,P_2) also called as flag(C_5) given by $G^{(k)}$ are cordial graphs. Proof:

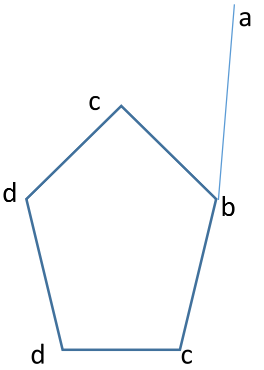


Fig 4.1 Four non-isomorphic one point union are possible at points 'a', 'b','c' and 'd'

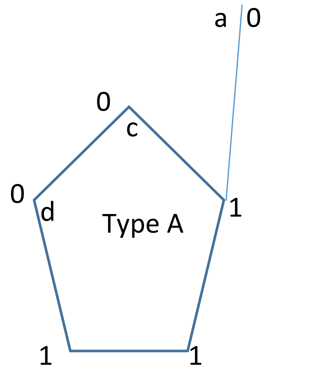


Fig 4.2 $v_f(0,1) = (3,3)$ $e_f(0,1)= (3,3)$

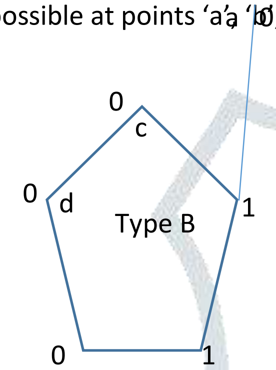


Fig 4.3 $v_f(0,1) = (4,2)$ $e_f(0,1)= (3,3)$

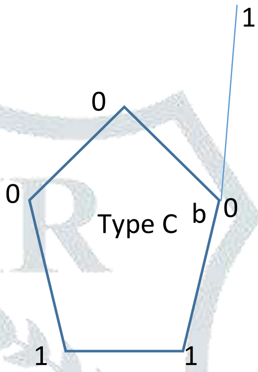


Fig 4.4 $v_f(0,1) = (3,3)$ $e_f(0,1)= (3,3)$

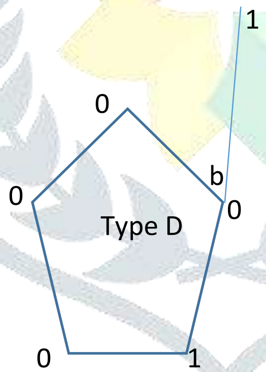


Fig 4.5 $v_f(0,1) = (4,2)$ $e_f(0,1)= (3,3)$

From Fig 4.1 it follows that we can take one point union at four vertices 'a', 'b', 'c'and 'd'. For the one point union at vertices a, c or d we fuse the type A and Type B label at vertices a, c or d respectively. For the one point union at vertex 'b' we use type C and type D label and fuse it at vertex b. For given k , if k = 2x then x copies of type A (type C) and x copies of type B (type D) are fused at desired point .If K = 2x+1 then one more copy of Type A (type C) is used than the copies of type B (type D) used.

In both case the label number distribution is given by $v_f(0,1)$ =(3+5x, 3+5x), $e_f(0,1)$= (3k,3k) where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1)$ = (6+5(x-1),5+5(x-1)), $e_f(0,1)$ = (3k,3k). Thus the graph is cordial. #

Theorem 4.2 All non- isomorphic one point union on k-copies of graph obtained on G = tail(C_5,$2P_2$) given by $G^{(k)}$ are cordial graphs. Proof:

From Fig 4.6 it follows that we can take one point union at four vertices 'a', 'b', 'c'and 'd'. For the one point union at vertices a, c or d we fuse the type A and Type B label at vertices a, b or c respectively. For the one point union at vertex 'b' we use type C and type D label and fuse it at vertex b. For given k , if k = 2x then x copies of type A (type

C) and x copies of type B (type D) are fused at desired point .If K = 2x+1 then one more copy of Type A (type C) is used than the copies of type B (type D) used.

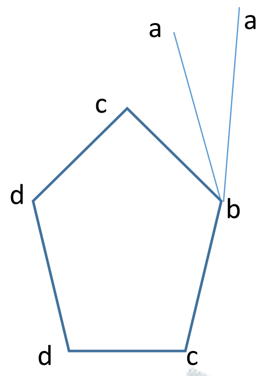


Fig 4.6 Four non-isomorphic one point union are possible at points 'a', 'b','c' and 'd'

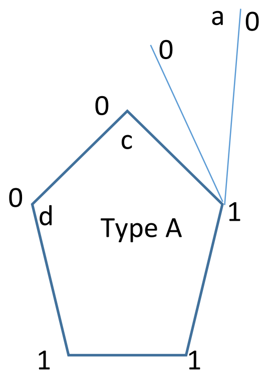


Fig 4.7 $v_f(0,1) = (4,3)$ $e_f(0,1)= (3,4)$

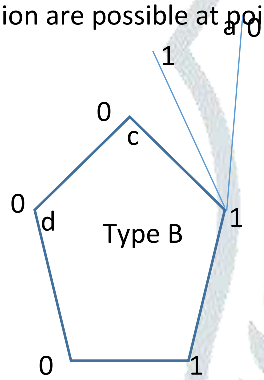


Fig 4.8 $v_f(0,1) = (4,3)$ $e_f(0,1)= (4,3)$

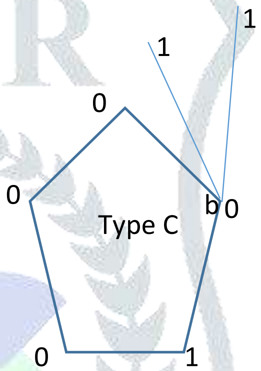


Fig 4.9 $v_f(0,1) = (4,3)$ $e_f(0,1)= (3,4)$

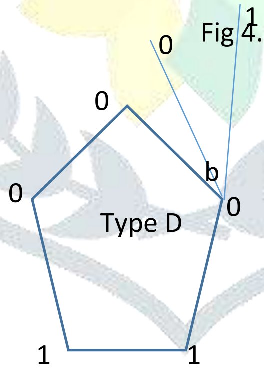


Fig 4.10 $v_f(0,1) = (4,3)$ $e_f(0,1)= (3,4)$

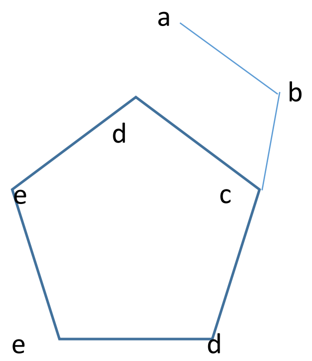


Fig 4.11 Four non-isomorphic one point union are possible at points 'a', 'b','c', 'd','e'.

In both cases the label number distribution is given by $v_f(0,1)$ =(4+6x, 3+6x), $e_f(0,1)$= (3+7x,4+7x) where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1)$ = (7+6(x-1),6+6(x-1)), $e_f(0,1)$ = (7(k-1),7(k-1)). Thus the graph is cordial.

Theorem 4.3 All non- isomorphic one point union on k-copies of graph obtained on G = tail(C_5,P_3) given by $G^{(k)}$ are cordial graphs.

Proof:

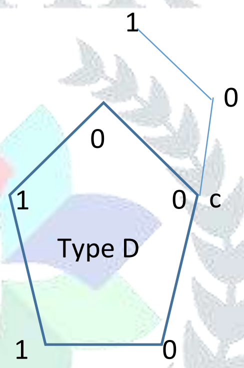


Fig 4.12 $v_f(0,1)$ = (4,3) $e_f(0,1)$= (4,3)

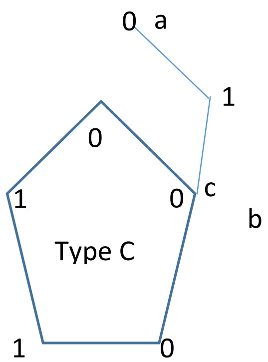


Fig 4.12 $v_f(0,1)$ = (4,3) $e_f(0,1)$= (3,4)

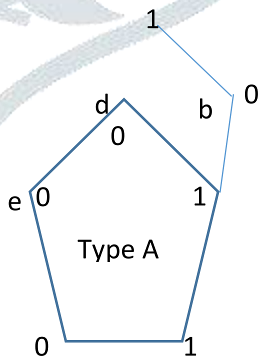
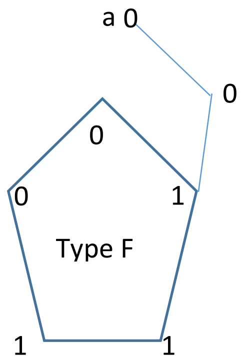


Fig 4.12 $v_f(0,1)$ = (4,3) $e_f(0,1)$= (4,3)



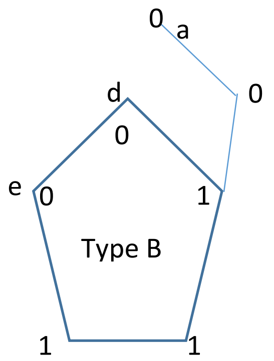


Fig 4.12 $v_f(0,1) = (4,3)$ $e_f(0,1)= (4,3)$ Fig 4.12 $v_f(0,1) = (4,3)$ $e_f(0,1)= (4,3)$

From Fig 4.11 it follows that we can take one point union at five vertices ‘a’, ‘b’, ‘c’, ‘d’or ‘e’. For the one point union at vertices b,d,or e we fuse the type A and Type B label at vertices b,d or vertex e respectively. For the one point union at vertex ‘a’ we use type C and type F label and fuse it at vertex a. For one point union at point c we use type D and type C labeling and fuse it at vertex c. For given k , if k = 2x then x copies of type A (type C) (Type D)and x copies of type B (type F) (Type C) are fused at desired point .If K = 2x+1 then one more copy of Type A (type C) (type D) is used than the copies of type B (type F)(Type C) used. In all cases the label number distribution is given by $v_f(0,1) =(4+6x, 3+6x)$, $e_f(0,1)= (3+7x,4+7x)$ where k = 2x+1, x=0,1,2… If k = 2x; x= 1,2,.. then we have, $v_f(0,1) = (7+6(x-1),6+6(x-1))$, $e_f(0,1) = (7(k-1),7(k-1))$. Thus the graph is cordial.

Conclusions: In this paper wedefine some new families obtained from C5 and fusing to one of it’s vertex pendent edges upto two or a path oflength 2. We show that 1) 1All non- isomorphic one point union on k-copies of graph obtained on G = tail(C_5,P_2) also called as flag(C_5) given by $G^{(k)}$ are cordial graphs. 2) All non- isomorphic one point union on k-copies of graph obtained on G = tail(C_5,2P_2) given by $G^{(k)}$ are cordial graphs. 3) All non-isomorphic one point union on k-copies of graph obtained on G = tail(C_5,P_3) given by $G^{(k)}$ are cordial graphs.

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