Economic Ordering Policy for Deteriorating and Breakable Item with Non-zero Level of Inventory

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Abstract: The developed model represents the required EOQ verses demand rate, required stocked EOQ in the store to satisfy the inventory level is non-zero for fluctuation in demand, optimal time run. The assumptions include : demand is constant and deterioration , break-ability rates are constant and they occur simultaneously with same or different values and the finite horizon planning, with shortage cost for only the required stocked quantity, Sensitivity analysis for the proposed data was represented the many values lies in range of the deterioration and break-ability rates and various ratio between the shortage cost and purchasing cost to achieved the optimal total cost ,optimal run time.

***Index Terms* - Inventory level -Deterioration-Break-ability -Optimal time run -Total Cost Function (TC).**

I. INTRODUCTION

II. The inventory models with deterioration rate or break-ability rate represented by many researchers as Nahmias (1982) proposed perishable inventory model theory. Benkherouf (1997) developed model for a deterministic order level inventory with two shortage facilities. Liang, Zhou (2011) represented developed inventory model with two period times. Chung, Huang (2007) developed the model with optimal retailers ordering policies for deteriorating items with limited shortage capacity under trade cridte financing. Aggarwal and Jaggi (1995) porposed ordering policies of deteriorating items under conditons of permissible delay in payments, Benkherouf (1997) developed deteriminstic order level inventory model for dtereiorating items with two shortage facilites. Chang, Ouyang and Teng (2003) investigated model for EOQ for deteriorating items under supplier cridets linked to oedering quantity. Das, Maity and Maiti (2007), Goal (1985) Hsieh, Huang (2007) porposed model to determine economic order quantity under conditionally permissible delay in payments, Huang (2006), Huang (2007) developed model to determine optimal retailers replenishment decisions in the EPQ model under two levels of trade credit policy. Hwang, Shinn(1997) developed model with retailers pricing and lot sizing policy for exponentionally deteriorating products under the condition of permissible delay in payments, Lee, Hsu (2009) developed the model under assumptions as two warehouse production , deteriorating inventory items and time dependent demands. Liang, Zhou (2011) developed two-warehouse inventory model for deteriorating items under conditionally permissible delay in payment. *Liao (2008)*, Maiti (2008*), Rong and Mahapatra* (2006). Abdullah and Muley (2018) represnted model with non-zeo inventory to handel the demand and offering rates. Rong, Mahapatra and Maiti (2008) developed model with two-warehouse inventory model for a deteriorating item with partially fully backlogged shortage and fuzzy lead time, *Sana*,Chaudhuri (2008). Zhou (2003). there are many items having deterioration and break-ability characteristics at same time as the blub and each items packed in can made from glass as medicines and milk which are packed in glass as olive, olive oil, these types of items require advanced model to manage the items with two warehouse inventory close validity with minimum risk of break-ability characteristic at first and after that deterioration for industry to determine the optimal economic ordering quantity ,optimal total cost of per unit in each those items across different class interval. The proposed model concerned among all of items especially items which packed in glass. , the deterioration rate may equivalent the break-ability rate or variant. This is paper include this consideration furthermore that the inventory policy may required that the inventory should not be zero under any circumstances for many consideration as fluctuation in demand emergency demand also variation in replenishment run to stock, with shortage cost for the shortage EOQ only, the constant as the ratio between the shortage cost and purchasing cost was proposed and represented.

2. Material and methods

2.2. Assumptions and notions

2.2.1. Assumptions

In this paper the mathematical model is developed with the following assumptions

1) Planning horizon is finite.
2) Replenishment rate is infinite.
3) Single item inventory control.

4) Demand and deterioration and break-ability rates are constant.
5) There is no replacement or repair of breakable items during the period under consideration.
6) Shortage cost is allowed.
7) The lead time is zero.
8) The inventory level at the end of planning horizon will be zero.
9) The cost factors are deterministic.
10) The total relevant cost consists of fixed ordering, purchasing and holding cost.

2.3. Notations

D= The demand rate quantity in period $[0, t_1]$.
C = The present value of purchasing cost.
C_S = The shortage cost of Q_0.
I_h =The holding cost.
Q = The total order quantity in period [0,t_1].
Q_1= The required order quantity in period [0,t_1].
Q_0= The enduring quantity in period [0,t_1]
TC_A = The total fixed ordering cost during [0, t_1].
TC_P = The total purchasing cost during [0,t_1].
TC_h = The total holding cost during [0, t_1].
TC_T= The total relevant cost during [0, t_1].

2.4. Parameters

T = The length of the finite planning horizon.
$I_1(t)$ = The inventory level at time [0,t_1].
$I_0(t)$ = The fixed inventory level at time $[0, t_1]$ when there is no demand.
t_1 = The length of replenishment
ϴ = The constant deterioration rate
ϕ = The constant break-ability rate

3. Mathematical model

Let $I(t)$ is the inventory level at any time t, $0 \le t \le t_1$, Depletion due to demand and deterioration rate. The first order differential equation that describes the instantaneous state of $I_1(t)$ over the open interval [0, t_1] is given by.

$$\frac{dI(t)}{dt} + (\theta + \phi)I(t) = -D, 0 \le t \le t_1 , 0 \le \theta \le 1 , 0 \le \phi \le 1 \quad (1)$$

Let $I_0(t)$ is the inventory level at any time t, $0 \le t \le t_1$, Depletion due to demand and deterioration, break-ability rates. The first order differential equation that describes the instantaneous state of $I_0(t)$ over the open interval [$0, t_1$] is given by.

$$\frac{dI_0(t)}{dt} + (\theta + \phi)I_0(t) = 0 \quad , 0 \le t \le t_1, I(0) = Q \quad (2)$$

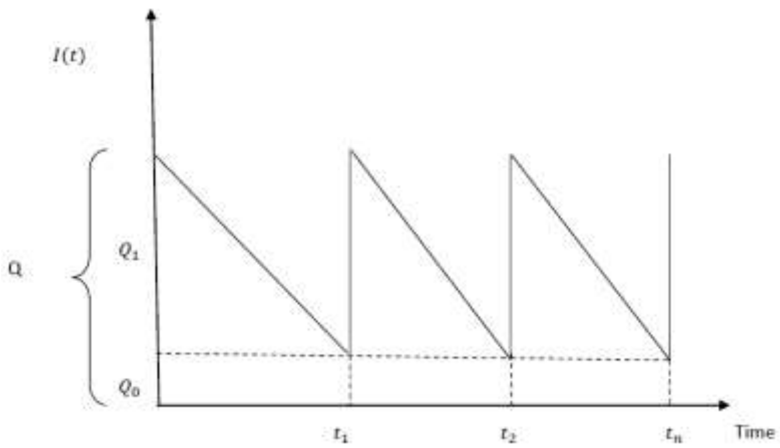

Fig.1 Graphical representation of two warehouse inventory control diagram

$$I(t) = I_{01}(t)\int_t^{t_1} De^{(\theta+\phi)u} = \frac{D}{(\theta+\phi)}\left(e^{(\theta+\phi)(t_1-t)} - 1\right), I_{01}(t) = e^{-(\theta+\phi)t} \quad (3)$$

$$I_0(t_1) = I(t)e^{-(\theta+\phi)(t_1-t)}, I(0) = Q \quad (4)$$

$$Q_1 = \frac{D}{(\theta+\phi)}\left(e^{(\theta+\phi)t_1} - 1\right)\left(1 - e^{-(\theta+\phi)t_1}\right) \quad (5)$$

3.1. Fixed ordering cost

We assumed the number of replenishment is N so that the fixed ordering cost over the planning horizon under the inflation consideration is:

$$TC_A = A \quad (6)$$

3.2. Purchasing cost

According to fig.1 of inventory level the purchasing cost of

$$TC_P = CI_1(0) = \frac{CD}{(\theta+\phi)}\left(e^{-(\theta+\phi)t_1} - 1\right)\left(1 - t_1e^{-(\theta+\phi)t_1}\right) \quad (7)$$

3.3. Holding cost excluding the cost of interest

We find the average inventory quantity to obtain holding cost

$$TC_h = I_h\int_0^{t_1} I_r(t)dt = \int_0^{t_1}\frac{I_hD}{(\theta+\phi)}\left(e^{e^{-(\theta+\phi)t_1}} - 1\right)\left(1 - t_1e^{-(\theta+\phi)t_1}\right)dt = \frac{DI_h}{(\theta+\phi)}\left(\frac{e^{(\theta+\phi)t_1}}{(\theta+\phi)} - \frac{t_1^2}{2} - t_1 - \frac{t_1e^{-(\theta+\phi)t_1}}{(\theta+\phi)} - \frac{e^{-(\theta+\phi)t_1}}{(\theta+\phi)^2} - \frac{1}{(\theta+\phi)} + \frac{1}{(\theta+\phi)^2}\right) \quad (8)$$

3.3. Shortage cost for the remaining quantity in stock with in [0, t_1]

The total shortage cost per-first run is as

$$TC_{C_S} = \frac{C_SD}{(\theta+\phi)}\left(e^{e^{-(\theta+\phi)t_1}} - 1\right)\left(1 - t_1e^{-(\theta+\phi)t_1}\right) \quad (9)$$

3.5. 1.Economic order quantity during [0, t_1]

To find EOQ by minimizing the total cost function fewer than two constrains by langrage's multiplier to minimize the total cost.

$$TC = TC_A + TC_P + TC_h + TC_{CS}$$

The total cost of first run is defined as

$$TC = \frac{1}{t_1}[TC_A + TC_P + TC_h + TC_{CS}] \quad (10)$$

By substituting the Eq. (6, 7, 8, and 9) in the Eq. (10) then it can be rewritten as

$$TC = \frac{1}{t_1}\left(A + \frac{CD}{(\theta+\phi)}\left(e^{-(\theta+\phi)t_1} - 1\right)\left(1 - t_1e^{-(\theta+\phi)t_1}\right) + \frac{DI_h}{(\theta+\phi)}\left(\frac{e^{(\theta+\phi)t_1}}{(\theta+\phi)} - \frac{t_1^2}{2} - t_1 - \frac{t_1e^{-(\theta+\phi)t_1}}{(\theta+\phi)} - \frac{e^{-(\theta+\phi)t_1}}{(\theta+\phi)^2} - \frac{1}{(\theta+\phi)} + \frac{1}{(\theta+\phi)^2}\right) + \frac{C_SD}{(\theta+\phi)}\left(e^{e^{-(\theta+\phi)t_1}} - 1\right)\left(1 - t_1e^{-(\theta+\phi)t_1}\right)\right) \quad (11)$$

By using Taylor's series about origin point

$$e^{(\theta+\phi)t_1} = 1 + (\theta+\phi)t_1$$

$$e^{-(\theta+\phi)t_1} = 1 - (\theta+\phi)t_1$$

$$TC = \frac{1}{t_1}\left(A + CD(t_1 - t_1^2) + \frac{DI_ht_1^2}{2(\theta+\phi)} + C_SDt_1\right) \quad (12)$$

$$\frac{dTC}{dt_1} = \frac{-A}{t_1^2} - CD + \frac{DI_ht_1}{2(\theta+\phi)} + C_SD = 0$$

$$t_1{}^* = \sqrt{\frac{2(\theta+\phi)A}{D(I_h+2(\theta+\phi)Ck(1-k))}} \tag{13}$$

Where

$0 < k < 1,\ k = \frac{C_S}{C}$

$\frac{d^2TC}{dt_1{}^2} = \frac{2A}{t_1{}^3}\ ,\ t_1{}^* > 0$

Then
$\frac{d^2TC}{dt_1{}^2} = \frac{2A}{t_1{}^3} > 0$

Then
The total cost of first run has minimum value at $t_1{}^*$

$$TC^* = \ D(C + C_S) + (\frac{AD(I_h+2(\theta+\phi)Ck(1-k)}{2(\theta+\phi)})^{\frac{1}{2}} + D\big(I_h - C(\theta+\phi)\big)\Big(\big(\frac{2A}{D(\theta+\phi)(I_h+2(\theta+\phi)Ck(1-k)}\big)^{\frac{1}{2}}\Big)$$

$$Q_1{}^* = \frac{D}{(\theta+\phi)}\big(e^{(\theta+\phi)t_1{}^*} - 1\big)(1 - t_1{}^* e^{-(\theta+\phi)t_1{}^*}) \tag{14}$$

4. Sensitivity analysis

The assumption of the parameters of total cost in for the two inventory models as follows:

Example1

$$D = 300, I_h = 0.05\$\ , C =, 20\$, A = 15\$$$

Table1. The sensitivity analysis

θ	ϕ	D	k	$t_1{}^*$	Q^*	$Q_1{}^*$	$Q_0{}^*$
0.0000225	0.0000448	300	0.05	0.011587	3.476077	3.4358	0.040277
0.005	0.005	300	0.05	0.120386	36.1375	31.79229	4.34521
0.05	0.05	300	0.05	0.204124	61.86652	49.49323	12.37329
0.06	0.06	300	0.05	0.207763	63.11238	50.32283	12.78955
0.07	0.07	300	0.05	0.210485	64.08499	50.98777	13.09722
0.08	0.08	300	0.05	0.212598	64.87643	51.54513	13.3313
0.1	0.1	300	0.05	0.215666	66.11529	52.45845	13.65684
0.5	0.5	300	0.05	0.226455	76.244	62.47698	13.76702
0.55	0.55	300	0.05	0.22672	77.24855	63.60051	13.64804
0.6	0.6	300	0.05	0.226941	78.25473	64.72926	13.52547
0.7	0.7	300	0.05	0.227289	80.28395	67.0096	13.27435
0.75	0.75	300	0.05	0.227429	81.31119	68.16379	13.1474
0.8	0.8	300	0.05	0.227552	82.34889	69.32864	13.02024
0.85	0.85	300	0.05	0.22766	83.39823	70.50497	12.89325
0.9	0.9	300	0.05	0.227757	84.46019	71.69347	12.76671
0.95	0.95	300	0.05	0.227843	85.53561	72.89476	12.64085
0.0000225	0.0000448	400	0.25	0.009997	3.998823	3.958847	0.039976
0.005	0.005	400	0.25	0.07746	30.99587	28.5968	2.399071
0.05	0.05	400	0.25	0.096825	38.91794	35.18604	3.731904
0.06	0.06	400	0.25	0.097333	39.1614	35.39397	3.767429
0.07	0.07	400	0.25	0.097701	39.34883	35.55664	3.792188
0.08	0.08	400	0.25	0.09798	39.50065	35.69059	3.810057
0.1	0.1	400	0.25	0.098374	39.7392	35.90606	3.833136
0.5	0.5	400	0.25	0.099668	41.92177	38.13986	3.781911
0.55	0.55	400	0.25	0.099698	42.14826	38.38262	3.765645
0.6	0.6	400	0.25	0.099723	42.37421	38.62511	3.749104
0.7	0.7	400	0.25	0.099763	42.82623	39.11069	3.715534
0.75	0.75	400	0.25	0.099779	43.05288	39.35427	3.698617
0.8	0.8	400	0.25	0.099792	43.28025	39.59858	3.681663

0.85	0.85	400	0.25	0.099804	43.50846	39.84377	3.664698
0.9	0.9	400	0.25	0.099815	43.73766	40.08992	3.647742
0.95	0.95	400	0.25	0.099825	43.96792	40.33711	3.630812

Table.2 The regression equation is as

$Q_1^* = 11.6 + 198\, t_1^* + 451339\theta - 451325\phi$
Predictor Coef SE Coef T P
Constant 11.628 1.462 7.95 0.000
t_1^* 197.661 8.330 23.73 0.000
θ 451339 107759 4.19 0.000
ϕ -451325 107759 -4.19 0.000
S = 2.77242 R-Sq = 97.7% R-Sq(adj) = 97.5%
Analysis of Variance
Source DF SS MS F P
Regression 3 9346.3 3115.4 405.32 0.000
Residual Error 28 215.2 7.7
Total 31 9561.5
Durbin-Watson statistic = 0.233735

5. Conclusion

In this paper the deterioration, break-ability represented in the proposed model simultaneously with constant demand rate , developed the mathematical model by the above assumptions and apply it in real life where the break-ability and deterioration for items have these properties output of the mathematical model is minimizing the total cost of items and obtaining the economic order quantity as required quantity and the required EOQ which required to stay in stocked to satisfy the inventory level was non-zero . The output of proposed model of required EOQ as dependent variable in dependent variables as optimal run time, deterioration and break-ability rates is represented in linear equation with strong indexes as significance of these variables to determine the required EOQ with adjusted square of determinant as 97.5 and high index for acceptance model as zero probability value as table2, the break-ability rate cannot treat it or replace it by negative coefficient for it whereas the deterioration rate can by sale it with lower price in as possible time. The obtained figures for the output model are explained the behavior for the main parameters in this model as required EOQ and shortage EOQ in stock per-run time versus optimal run time as Fig. 2 Illustrates the behavior of optimal run time versus deterioration and break-ability rates as the optimal run time increased when the risk rate increase. Fig.3 The required EOQ increased when optimal run time increased, also the required stocked EOQ was increased when optimal run time increased till 50% for each deterioration and break-ability rates and it was decreased for more than 50%. Fig. 4 showed the gap between the required EOQ, required stocked EOQ is increased when demand rate was increasing similarly that in fig. 5But with the ratio between the shortage cost and purchasing cost.

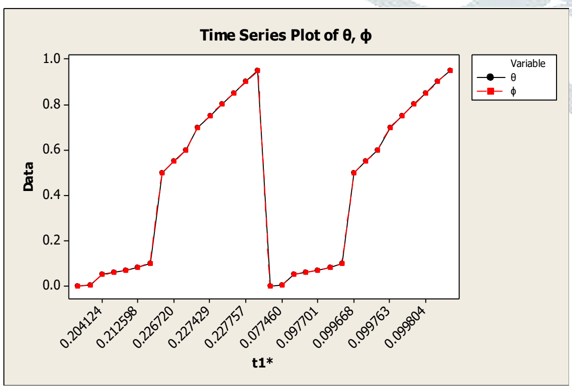

Fig.2. Graphical representation optimal run time VS deterioration ,and break-ability rates

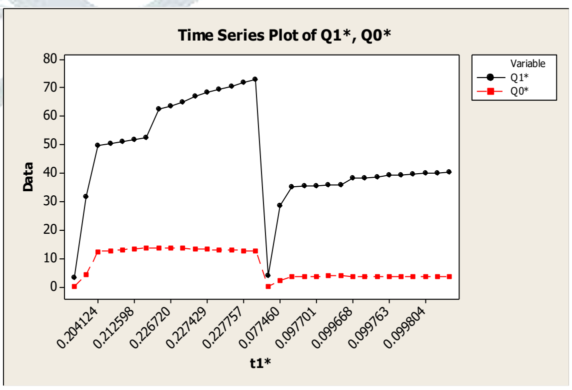

Fig.3. Graphical representation the required EOQ and stocked EOQ VS optimal run time

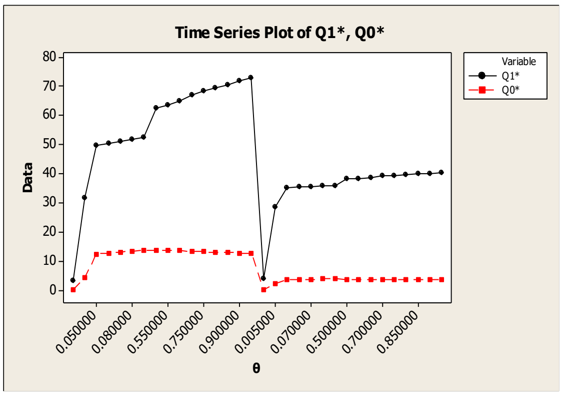

Fig.4. Graphical representation EOQ VS deterioration rate

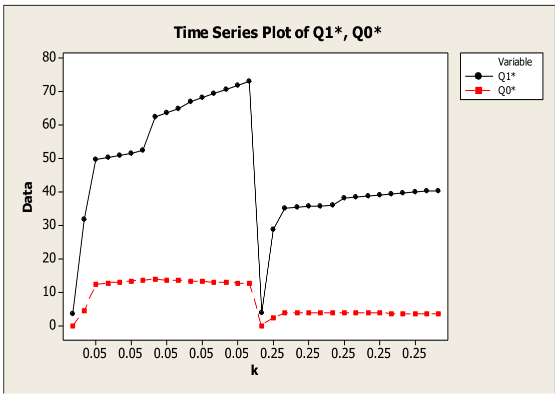

Fig.5. Graphical representation the required EOQ and remaining EOQ in stock VS optimal time run

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