CONSTRUCTION OF CONTROL CHART FOR MEAN UNDER INVERSE RAYLEIGH DISTRIBUTION WITH PROCESS CAPABILITY

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Abstract: If the population is not normal there is a need to develop a separate procedure for the construction of control limits. In this research paper we assume that the quality variate follows inverse Rayleigh model and develop control limits with process capability for such a data on balance with the presently available control limits.

Keywords: Control chart, Inverse Rayleigh distribution and Process capability.

I. INTRODUCTION

A control chart is a graphical device that detects variations in any variable quality characteristic of a product. Given a specified target value of the quality characteristic, production of the concerned product has to be designed so that the associated quality characteristic for the products should be ideal i.e, if the products are showing variations in the desirable quality, the variations must be within control in some admissible sense. There should be two limits within which the allowable variations are supposed to fall. Whenever this happens, the production process is defined to be in control (Amitava Mitra, 2001). Otherwise, it is out of control. Based on this principle it is necessary to think of the control limits on either side of the target value in such a way that under normal conditions the limits should include most of the observations. With this backdrop, the well known Shewart (1931) control charts are developed under the assumption that the quality characteristic follows a normal distribution.

Edgeman (1989) has developed control chart constants for Mean and Range charts under Inverse Gaussian distribution in a unified way for a skewed distribution where the constants are dependent on the coefficient of skewness of the distribution. Inverse Rayleigh distribution (IRD) is another situation of a skewed distribution that was not paid much attention with respect to development of control charts. At the same time it is one of the probability models applicable for life testing and reliability studies. In this research article proposed mean control chart under IRD with process capability with an example.

II. CONCEPTS AND TERMINOLOGIES

a. Upper specification limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.

b. Lower specification limit (LSL)

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

c. Tolerance level (TL)

It is a statistical interval within which, with some confidence level, a specified proportion of a sampled population falls. It is the difference between USL and LSL, TL = USL-LSL

d. Process capability (CP)

Process capability compares the output of an in-control process to the specification limits by using capability indices. The comparison is made by forming the ratio of the spread between the process specifications to the spread of the process values, as measured by 6 process standard deviation units.$\text{i. e. } C_p = \frac{TL}{6\sigma} = \frac{USL-LSL}{6\sigma}$.

III. METHODS AND MATERIALS

Let X_1, X_2, X_3,...,X_n be a random sample of size n supposed to have been drawn from an inverse Rayleigh distribution with scale parameter σ and location parameter zero. If this is considered as a subgroup of an industrial process data with a targeted population average, under repeated sampling the statistic mean gives whether the process average is around the targeted mean or not. Statistically speaking, we have to find the 'MOST PROBABLE' limits within which mean falls. It is well known that 3σ limits of normal distribution include 99.73% of probability. Hence, we have to search two limits of the sampling distribution of sample mean in inverse Rayleigh distribution such that the probability content of these limits is 0.9973. Symbolically, we have to find L, U such that $P(L \leq \bar{\bar{X}} \leq U)$ =0.9973 and the mean of the inverse Rayleigh distribution is 1.7728. If $\bar{X}$ is the mean of a data following an inverse Rayleigh distribution with scale parameter σ, $\bar{X}$=σ z. Then we get

$$P(A_{2p}^{*}\bar{\bar{X}} \leq \bar{X}_i \leq A_{2p}^{**}\bar{\bar{X}})=0.9973$$

where

$\bar{\bar{X}}$ is the grand mean
$\bar{X}_i$ is the ith subgroup mean
$A_{2p}^{*}=Z_{0.00135}/1.7728$
$A_{2p}^{**}= Z_{0.99865}/1.7728$.

These constants are named as percentile constants of mean chart.

IV. CONSTRUCTION OF CONTROL CHART FOR MEAN

The following data provided by S.C.Gupta and V.K.Kapoor (2001, Page No. 1.17) is considered here and this data on the basis of fuses, sample of five being taken every hour.

Table 1: Sample data on the basis of fuses

Sample No.	Observations					Mean	Mean under IRD
	1	2	3	4	5		
1	42	65	75	78	87	69.4	39.15
2	42	45	68	72	90	63.4	35.76
3	19	24	80	81	81	57.0	32.15
4	36	54	69	77	84	64.0	36.10
5	42	51	57	59	78	57.4	32.38
6	51	74	75	78	132	82.0	46.25
7	60	60	72	95	138	85.0	47.95
8	18	20	27	42	60	33.4	18.84
9	15	30	39	62	84	46.0	25.95
10	69	109	113	118	153	112.4	63.40
11	64	90	93	109	112	93.6	52.80
12	61	78	94	109	136	95.6	53.93
					Mean	71.6	40.39

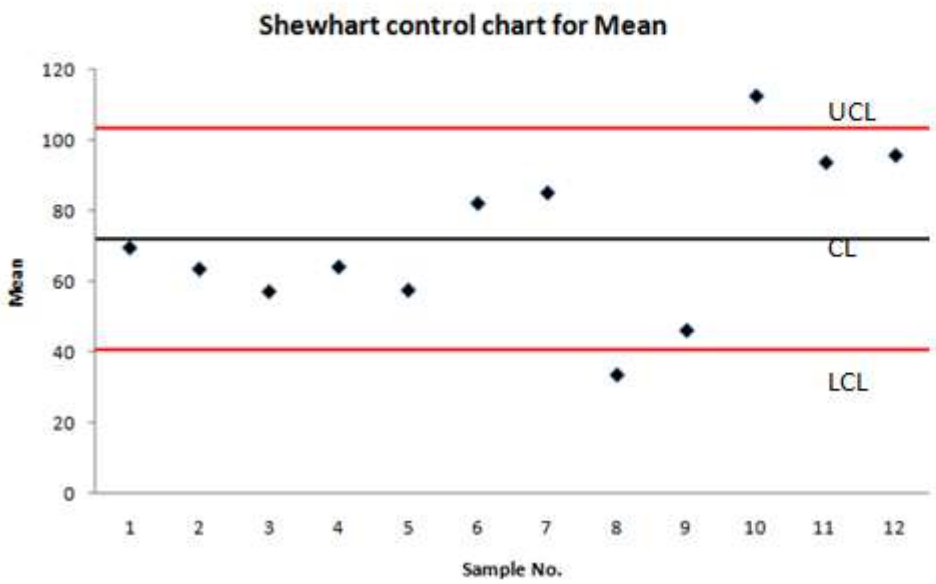
a. Construction of Shewhart control chart for Mean

The 3σ control limits suggested by Shewhart (1931) are $\overline{\overline{X}} \pm 3\left({}^{\sigma}/_{\sqrt{n}}\right)$

$$UCL_{\overline{\overline{X}}} = \overline{\overline{X}} + 3\left({}^{\sigma}/_{\sqrt{n}}\right) = 71.6 + \left(\frac{3 \times 24}{\sqrt{5}}\right) = 103.8$$

$$CL_{\overline{\overline{X}}} = \overline{\overline{X}} = 71.6$$

$$LCL_{\overline{\overline{X}}} = \overline{\overline{X}} - 3\left({}^{\sigma}/_{\sqrt{n}}\right) = 71.6 - \left(\frac{3 \times 24}{\sqrt{5}}\right) = 39.4$$



However the control limit interval hereafter refers to as CLI, is the difference between the control limits value. Therefore, for the Shewhart control chart for mean, the control limit interval will be determined using the expression:

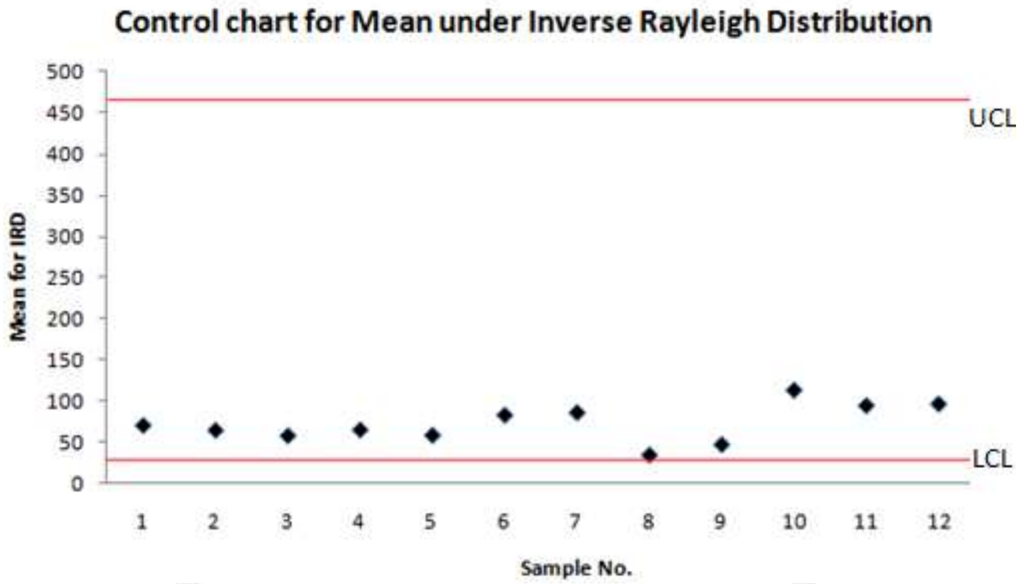
$$CLI_{\overline{\overline{X}}\text{ using R}} = \left({}^{6\sigma}/_{\sqrt{n}}\right) = \left(\frac{6 \times 24}{\sqrt{5}}\right) = 64.3$$

From the result, it is clear that the process is out of control, since the sample number 8 goes below the lower control limit and the sample number 10 goes above the upper control limit with the control limit interval is 64.3 for n=5.

b. Construction of control chart for Mean under Inverse Rayleigh Distribution

The control limits for $P(A^{*}_{2p}\tilde{\overline{X}} \le \tilde{\overline{X}}_i \le A^{**}_{2p}\tilde{\overline{X}})$=0.9973 under Inverse Rayleigh Distribution (IRD) are $A^{*}_{2p}\tilde{\overline{X}}$ and $A^{**}_{2p}\tilde{\overline{X}}$

$$\mathrm{LCL}_{\tilde{\tilde{X}}} = A_{2p}^{*}\tilde{\tilde{X}} = 0.693882 \times 40.39 = 28$$
$$\mathrm{UCL}_{\tilde{\tilde{X}}} = A_{2p}^{**}\tilde{\tilde{X}} = 11.491603 \times 40.39 = 464.1$$



However the control limit interval hereafter refers to as CLI, is the difference between the control limits value. Therefore, for the control chart for mean under IRD, the control limit interval will be determined using the expression:

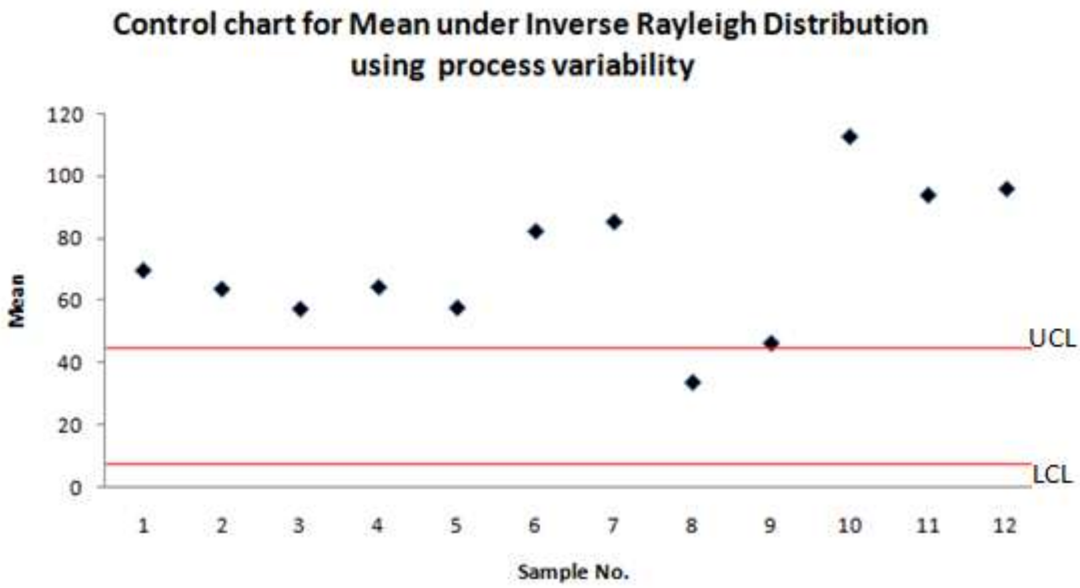
$$\mathrm{CLI}_{\tilde{\tilde{X}}} = \left(A_{2p}^{*}\tilde{\tilde{X}} \text{ and } A_{2p}^{**}\tilde{\tilde{X}}\right) = 436.1$$

From the result, it is clear that the process is out of control, since the sample numbers 8 and 9 go below the lower control limit and the control limit interval is 436.1 for n=5.

c. Construction of control chart for Mean under Inverse Rayleigh Distribution using process capability

Difference between upper specification and lower specification limits is 44.56, which termed as tolerance level (TL) and choose the process capability (Radhakrishnan and Balamurugan, 2012) is 2.0, it is found that the value of σ_{IRD} is 3.71. The control limits of using process capability for mean, a specified tolerance level with the control limits $(\sigma_{\mathrm{IRD}} Z_{0.00135}$ and $\sigma_{\mathrm{IRD}} Z_{0.99865}) = 0.9973$ under Inverse Rayleigh Distribution (IRD).

$$\mathrm{LCL}_{\tilde{\tilde{X}}} = \sigma_{\mathrm{IRD}} Z_{0.00135} = 3.71 \times 0.693882 = 2.6$$
$$\mathrm{UCL}_{\tilde{\tilde{X}}} = \sigma_{\mathrm{IRD}} Z_{0.99865} = 3.71 \times 11.491603 = 42.7$$



However the control limit interval hereafter refers to as CLI, is the difference between the control limits value. Therefore, for the control chart for mean under IRD with process variability (Nanthakumar, 2015), the control limit interval will be determined using the expression:

$$\mathrm{CLI}_{\tilde{\tilde{X}}} = (\sigma_{\mathrm{IRD}} Z_{0.00135} \text{ and } \sigma_{\mathrm{IRD}} Z_{0.99865}) = 40.1$$

From the result, it is clear that the process is out of control, since the sample numbers 6, 7, 10, 11 and 12 go above the upper control limit and the control limit interval is 40.1 for n=5.

Table 2: Assessment of Shewhart, IRD and IRD using process capability control charts

Control limits	Shewhart control chart	IRD	IRD using process capability
LCL	39.4	28	2.6
UCL	103.8	464.1	42.7
CLIs	64.3	436.1	40.1

It is found from the above results that the process is out of statistical control when the control limits of Shewhart 3 – Sigma, IRD and IRD using process capability are adopted but in the case of the control limits interval of IRD using process capability is smaller than the control limits interval of Shewhart and IRD. It is clear that the product/service is not in good quality as expected, accordingly a modification and improvement is needed in the process/system.

V. CONCLUSION

The outcome of numerical example shows that the proposed method leads better to the performance in the presence of Inverse Rayleigh distribution, such as many points fall outside the control limits than the existing control charts and the control limits interval of IRD using process capability is smaller than the control limits interval of Shewhart and IRD. It is clear that the product/service is not in good quality as expected, accordingly a modification and improvement is needed in the process/system.

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