Analysis of linear System Performance by Sliding Mode Control Approach

N. M. Gujar
Department of Electrical Engineering
Mahatma Bashweshwar Education Society's
College of Engineering, Ambajogai, Beed India

S. S. Sankeswari
Department of Electrical Engineering
Mahatma Bashweshwar Education Society's
College of Engineering, Ambajogai, Beed India

R. H. Chile
Department of Electrical Engineering
SGGS Institute of Engineering and Technology
Nanded-431606, India

***Abstract*—This paper includes, a method to design sliding mode control law for typical single-input-single-output systems. In real time applications, the model of the systems obtained by emperical technique gives higher order model. In the proposed method, there is no need to use model order reduction technique as the higher order model can be directly used to design the sliding law. The sliding surface is $(n-1)^{th}$ order differential equation of error-obtained from output and desired reference trajectory, where n is order of the system whose performance has to improve. As per the theory of sliding mode, the total control law is obtained using the proposed method by selecting sliding surface and smooth hyperbolic tangent function. The hyperbolic tangent function is used to reduce chattering phenomenon that occurs in many cases when there are use of sliding mode strategy. The single variable processes with delay time, lower or higher order, monotonic or oscillatory behavior are effectively controlled by the proposed sliding mode algorithm. The Lyapunov function is used to ensure the stability and simulation examples are incorporated to show the effect of proposed method.**

***Index Terms*—Higher order systems; error based sliding law; single variable; stability; robustness, uncertain systems.**

I. INTRODUCTION

In control system Engineering, nowadays an important role is played by the sliding mode control (SMC) or sliding mode controller (SMCr) because the theory of SMC stabilize the linear parametric uncertain systems in addition to provide the features of disturbance rejection. The SMC is applicable in case of systems having parameter variations, in other words, the SMC is insensitive to parameter uncertainty [1]. In conventional controllers, such as proportional-integral-derivative (PID), lead-lag compensator or Smith predictors in delay time systems, are sometimes not applicable and useful to compensate for effects which arises due to difficulty in development of a complete model, lack of knowledge of some hidden process parameters or broadly, it can be say that, in case of mismatch in actual systems and its model. Therefore, SMCr could be designed to control non-linear systems with the assumption that the ability of the sliding mode controller to compensate for modeling errors that may arise from the linearization of the non-linear model of the process[2]. The SMC approach is undoubt-fully applicable and useful for certain upper bound, case to case, of modeling errors those arising after linearization. The modeling errors mainly occurs as the model of the system is varies from point-to-operating points, but the model used for the design of the controller obtained around an operating point.

An advantages of SMC not limited to robustness to parameter uncertainty, insensitivity to bounded disturbances, fast dynamic response, but it has a remarkable computational simplicity with respect to other robust control approaches, and easy in implementation of the controller [3], [4]. The control principle is that the controller brings states (error) to a sliding surface and then onwards the states are bound to remain on the sliding surface. The process of SMC can be divided into two phases, first sliding phase and second, reaching phase [5]. In this two types of control laws can be derived separately, the equivalent control and the switching control that corresponds to two phases [6]. In literature, the SMC has been widely studied and it has been shown that SMC play an important role in the application of control theory to practical problems [7]. Some of the remarkable applications in practical includes applications of SMC's to underwater vehicles [8], robots [9], [10], electric drives [11], [12], automotive transmissions and engines, and power systems [13], induction motor [14], human neuromuscular process [15], elevator velocity [16] etc.

An important step in the design of sliding mode controllers is to introduce a proper sliding surface so that tracking errors and output deviations are reduced to a satisfactory level in practical applications. However, implementation of the traditional SMC method introduces some drawbacks such as chattering effect, limited flexibility for the designer with a sliding function and constant gain as the error variable. In this paper, the objective is to obtain a first-order sliding mode control law. Equivalent control approach is used in solution based on the higher or original plant model. Stability of the closed-loop system is assured by selecting proper coefficient

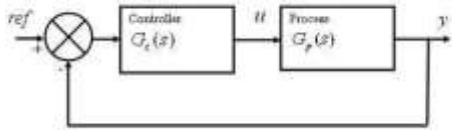

Fig. 1. Structure of closed-loop plants with controller

values in sliding surface. The various systems have been simulated in *M athworks*$^{T\ M}$ MATLAB 7.0.1 to show the effectiveness of the proposed method. The paper is organized as follows. The formal descriptions of systems and design approach of sliding mode controller are included in Section II. Simulation examples and their results are given in Section III while Section IV includes concluding remarks.

II. Sliding mode controller design approach

The structure of the SISO plants with conventional controller $tt_c(s)$ is as shown in Fig. 1, where, y is plant output, u is controller output, w is measurement noise, v plant-input noise and r is set-point. Let the higher order models with delay time are represented as

$$tt^1_P(s) = \frac{b_0}{s^q + a^1_1 s^{q-1} + a^1_2 s^{q-2} + \ldots + a^1_q} e^{-Ls}, \quad (1)$$

where, a^1_j $(j = 1, 2, \cdot, q)$ are constant coefficients of the polynomial, L is delay time. The first order Taylor approximation replaces delay time term e^{-Ls} with $1/(1 + Ls)$. The model given by equation (1) can be written as with Taylor approximation for delay time,

$$tt_p(s) = \frac{Y(s)}{U(s)} = \frac{\beta_0}{s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \ldots + \alpha_n}, \quad (2)$$

where α_j $(j = 0, 1, 2, \cdot, n)$ are constant coefficients. In sliding mode approach, the SMCr replaces conventional controller $tt_c(s)$ shown in Fig. 1.

A. Sliding surface

The synthesis of the SMCr using the differential error structures is presented in this subsection. The SMC law is developed using the error coefficients that are developed from the higher order process. In this the control law is dissimilar to conventional methods studied in the literature [1]. In the proposed method there is no need to reduce the higher order models into approximated first or second order model. The reduction of the model into first or second order is common practice in the controller design which can be causes to loss of some of the major properties of the original systems.

Consider the system of higher order with n poles and let the sliding surface for n^{th} order plant is selected as

$$s(t) = \frac{\alpha_1}{\zeta_1} e(t) + \frac{\alpha_2}{\zeta_2} \dot{e}(t) + \frac{\alpha_3}{\zeta_3} \ddot{e}(t) + \cdots + \frac{\alpha_n}{\zeta_n} \dot{e}^{(n-1)}(t) \quad (3)$$

or

$$s(t) = \sum_{i=1} \frac{\alpha_i}{\zeta_i} \frac{d^{i-1}}{dt^{i-1}} e(t) \quad (4)$$

where, the term $e(t)$ represents error signal, that is, difference between desired input $r(t)$ and actual output $y(t)$. The terms $\alpha_1, \cdot\ \alpha_n$ are the coefficients of the polynomial given by Eq. 2 and $\zeta_i \in (0, 1)$ are the tuning parameters, which helps to define sliding surface $S(t)$ and selected by the designer. The performance improvement can be obtained by selection of tuning parameters ζ, which also determines the performance of the system on the sliding surface. The main objective of any type of the controller is to make improvement in the transient response in terms of time domain parameters and should have less steady state time and error. The objective to design the control law is to ensure that the plant output $y(t)$ be equal to its reference value $r(t)$ at all times. This clearly indicate that the error signal and its all derivatives must be zero. The purpose of sliding mode control law is to force error $e(t)$ to approach the sliding surface and then move along the sliding surface to the origin. Therefore it is necessary that the sliding surface to be stable, which means $lim_{t\to\infty} e(t) = 0$; then the error will die out asymptotically. This implies that the system dynamics will track the desired trajectory asymptotically [13].

First derivative of the defined sliding surface can be given written as,

$$\dot{s}(t) = \frac{\alpha_1}{\zeta_1} \dot{e}(t) + \frac{\alpha_2}{\zeta_2} \ddot{e}(t) + \cdots + \frac{\alpha_n}{\zeta_n} \dot{e}^n(t) \quad (5)$$

In sliding mode, it is necessary that the output trajectory should remain on the sliding surface $s(t)$. This can be fulfilled if $\dot{s}(t) = 0$ [1], that is,

$$\dot{s}(t) = \frac{\alpha_1}{\zeta_1} \dot{e}(t) + \frac{\alpha_2}{\zeta_2} \ddot{e}(t) + \cdots + \frac{\alpha_n}{\zeta_n} \dot{e}^n(t) = 0. \quad (6)$$

The tuning parameters $\zeta_1, \zeta_2, \cdots \zeta_n$ are chosen such that the characteristic polynomial in Eq. 4 is strictly Hurwitz, that is, a polynomial whose roots lie strictly in the open left half of the complex plane. This Hurwitz condition implies that $lim_{t\to\infty} e(t) = 0$.

In the sliding mode approach, the sliding law is obtained by assumption that the measurement noise w and plant input noise v are zero, thus the error signal can be written as

$$e(t) = r(t) - y(t) \quad (7)$$

or n^{th} derivative is

$$\dot{e}^n(t) = \dot{r}^n(t) - \dot{y}^n(t) \quad (8)$$

From Eqs. 2, 6and 8, $\dot{s}(t)$ can be written as,

$$\frac{\alpha_1}{\zeta_1} \dot{e}(t) + \frac{\alpha_2}{\zeta_2} \ddot{e}(t) + \cdots + \frac{\alpha_n}{\zeta_n} [\dot{r}^n + a_1 \dot{y}^{(n-1)} + \cdots + a_n y - \beta_0 u] = 0. \quad (9)$$

The equivalent control u_{eq} is obtained from Eq. 9 as the solution of the problem $\dot{s}(t) = 0$ and given by

$$u_{eq}(t) = \left(\frac{\alpha_n}{\zeta_n} \beta_0\right)^{-1} \left[\frac{\alpha_1}{\zeta_1} \dot{e}(t) + \cdots + \frac{\alpha_n}{\zeta_n} (\dot{r}^n + a_1 \dot{y}^{(n-1)} + \cdots + a_n y)\right]. \quad (10)$$

$i=1$

B. Total control input

The desired performance of the systems can be achieved if the error trajectory translate from reaching phase to sliding phase. To satisfy this reaching condition, the equivalent control u_{eq} given in Eq.10 is augmented by a hitting control term u_{sw} which need to be determined. The chattering is due to the inclusion of the switching term and it can cause the control input to start oscillating around the zero sliding surface, resulting in unwanted wear and tear of the actuators. In general two approaches have been proposed in the literature to solve the problem. The first is to smoothen the switching term as the sliding surface gets closer to zero (soft switching). Therefore total control input can be given as [13]

$$u(t) = u_{eq}(t) + u_{sw}(t), \quad (11)$$

where $u_{sw}(t)$ is switching control. In this paper, the following switching function is used to get

$$u_{sw}(t) = k_{sw} tanh \left(\frac{s(t)}{\beta} \right), \quad (12)$$

where, $k_{sw}, \beta \in R$ are constants and *tanh* represent hyperbolic tangent function. Put $u = u(t) = u_{eq}(t) + u_{sw}(t)$ in 9 with $u_{eq}(t)$ given by Eq. 10 and $u_{sw}(t)$ given by Eq. 12, it can be written as

$$\dot{s}(t) = -\frac{\alpha_n}{\zeta_n}\beta_0 u_{sw}(t). \quad (13)$$

C. Stability

The direct Lyapunov stability approach is employed to investigate the stability property of the proposed sliding mode controller. Lyapunov function can be chosen to prove the stability as [13],

$$V(t) = \frac{1}{2}s^2(t) \quad (14)$$

with $V(0) = 0$ and $V(t) > 0$ for $s(t) \neq 0$. The reaching condition can be guaranteed if

$$\dot{V}(t) = s(t)\dot{s}(t) < 0 \quad (15)$$

From Eq. 13, Eq. 15 can be written as,

$$\dot{V}(t) = -s(t)\frac{\alpha_n}{\zeta_n}\beta_0 u_{sw}(t) < 0 \quad (16)$$

$$= -s(t)\frac{\alpha_n}{\zeta_n}\beta_0 \frac{|s(t)|}{s(t)} < 0$$

$$= -\frac{\alpha_n}{\zeta_n}\beta_0 |s(t)| < 0$$

or

$$\frac{\alpha_n}{\zeta_n}\beta_0 > 0 \quad (17)$$

or simply, the value of the ζ_n shall be selected such that

$$\alpha_n \beta_0 > \zeta_n \quad (18)$$

Design procedure:

1) First take a open-loop transfer function $tt_p(s)$ of the system for which the SMC is to be designed. Obtain the polynomial coefficients such as β_0, $\alpha_1, \alpha_2 \cdots \alpha_n$ and order $'n'$ of the transfer function.
2) Select the sliding surface given in Eq. 3 or 4 which is based on the order of the plant $'n'$. Choose the tuning parameters ζ_1, ζ_2 ζ_n such that the characteristic polynomial in Eq. 6 is strictly Hurwitz.
3) Obtain the switching control given in Eq. 12 by selecting the real values of k_{sw} and β. These values are slected by designer to minimize the chattering phenomenon.Total control law using Eqs. 10 and 12.
4) Verify the reaching and stability conditions using Eqs. 15-17.

The proposed approach has superior features, including: simplicity; easy implementation; and good computational efficiency. Fast computation of the SMC tuning parameters yields high-quality results.

III. Simulation examples

The higher order processes are simulated in MATLAB 7.0.1 to show the effectiveness of the proposed method. The proposed method is compared with prevalent tuning strategies such as sliding approach of Camacho et al. [18] and PID controller of Wang et al. [19] to show the usefulness. These methods are the most suitable candidates for comparison since these are the best and latest compared to the most of other tuning formulas.

A. Example 1

Consider the highly oscillatory process given in [19] with OLTF

$$tt_p(s) = \frac{1}{s^2 + s + 5}e^{-0.1s}.$$

After first order Taylor series for delay time, the OLTF of the process can be written as

$$tt_p(s) = \frac{10}{s^3 + 11s^2 + 15s + 50}.$$

The term $n = 3$ and polynomial coefficients are $b = 10, a_1 = 11, a_2 = 15$ and $a_3 = 50$. The sliding surface given in Eq. 3 or 4 for this example is

$$s(t) = \frac{\alpha_1}{\zeta_1}e(t) + \frac{\alpha_2}{\zeta_2}\dot{e}(t) + \frac{\alpha_3}{\zeta_3}\ddot{e}(t).$$

The tuning parameters $\zeta_1 = 0.8, \zeta_2 = 0.65$ and $\zeta_n = \zeta_3 = 3\zeta_1$ are selected such that the characteristic polynomial in Eq. 4 is strictly Hurwitz. The switching input given in Eq. 12 is used with $k_{sw} = 2$ and $\beta = 20$. It is clear that the reaching and stability conditions are satisfied. The tuning parameters of the Camacho et al's[18] are selected as: $\lambda = 0.4$, $K_d = 15$, $\delta = 0.68$, $K_m = 0.2$ and $T_m = 0.4695$ and those of Wang et al's [19] are selected such that the overshoot is to be not more than 10% and the settling time to be less than 15 sec. The unit step is applied to the system for time $t \geq 0$ and the behavior with all the prevalent laws and proposed control law are observed. The step responses of the closed-loop system, input signals and sliding surfaces for various controllers strategies are as

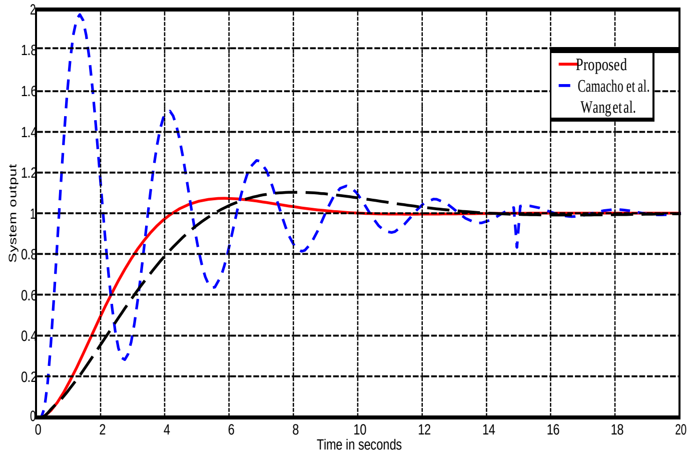

Fig. 2. Closed-loop step response (example 1)

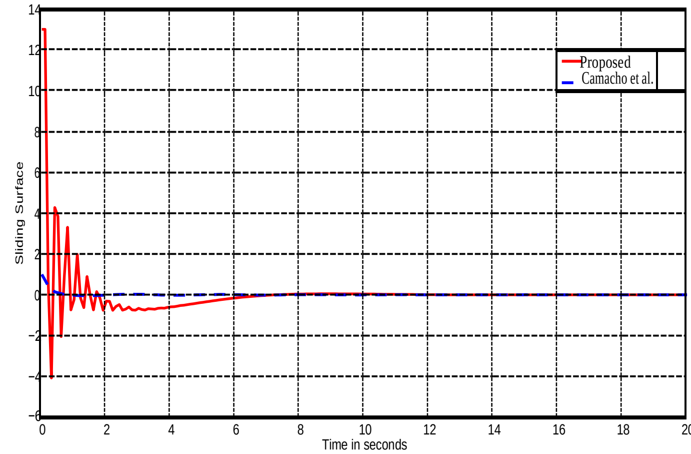

Fig. 4. Sliding surface (example 1)

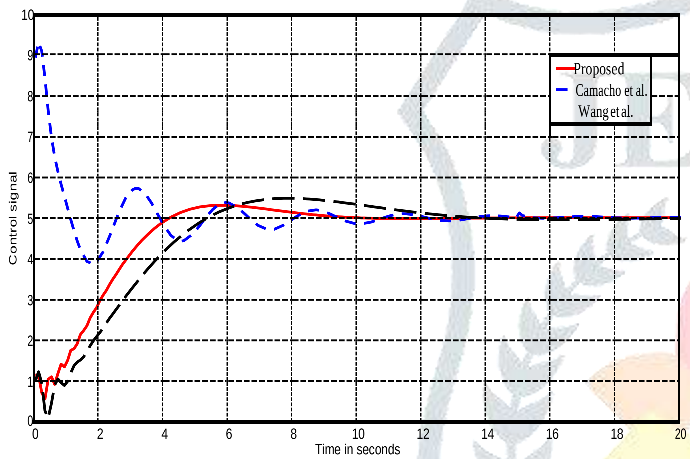

Fig. 3. Controller action (example 1)

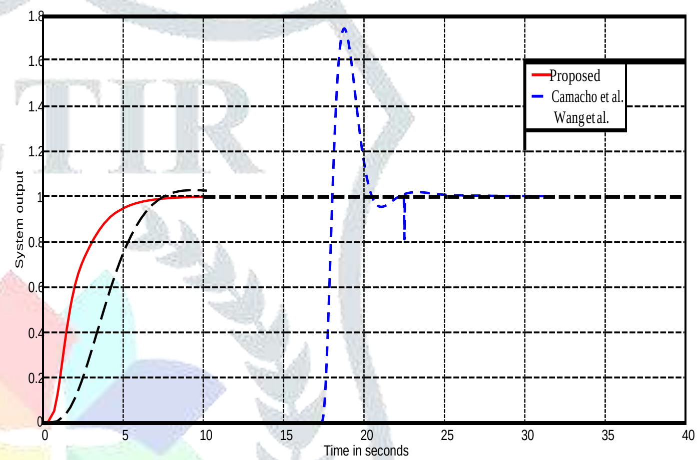

Fig. 5. Closed-loop step response (example 2)

shown in Figs. 2, 3 and 4, respectively. It can be observed that the response given by the proposed law is smooth without peak overshoot. The Camacho et al.'s response is decaying oscillatory and fast with large overshoot whereas Wang et al.'s method gives comparatively slow response. The control action required by proposed method is comparatively smooth whereas Camacho et al's method require large action at initial stage.

B. Example 2

Consider a high order process given in [17] with transfer function

$$tt_p(s) = \frac{1}{s^4 + 4s^3 + 6s^2 + 4s + 1}.$$

The parameters of the proposed law are: $\zeta_1 = 0.12$, $\zeta_2 = 0.6$, $\zeta_3 = 0.65$, $\alpha_4 = 1$ $k_{sw} = 30$ and $\beta = 50$. The parameters of the Camacho et al's control law are: $\lambda = 0.1$, $K_d = 25$, $\delta = 1.48$, $K_m = 0.2$ and $T_m = 0.4695$. The controller of Wang et al's [19] are designed such that the overshoot is to be within 2% tolerance band and the settling time to be less than 7 sec., which gives PID controller,

$$tt_c(s) = 0.76 + \frac{0.2725}{s} + 0.5532s.$$

The unit step responses of the closed-loop system, input signals and sliding surfaces for various control laws are as shown in Figs. 5, 6 and 7, respectively. It can be observed that the response given by the proposed law is smooth without peak overshoot. The Camacho et al.'s response is decaying oscillatory and fast with large overshoot whereas Wang et al.'s method gives comparatively slow response. The control action required by proposed method is comparatively smooth whereas Camacho et al's method require large action at initial stage.

***C.* Example 3**

In this work, the model used by Furat and Eker [20] of electromechanical system is used extensively for study purpose. The model parameters as per Furat and Eker [20] for system as

$$tt(s) = \frac{Y(s)}{U(s)} = \frac{0.86e^{-0.0035s}}{0.145s + 1} \quad (19)$$

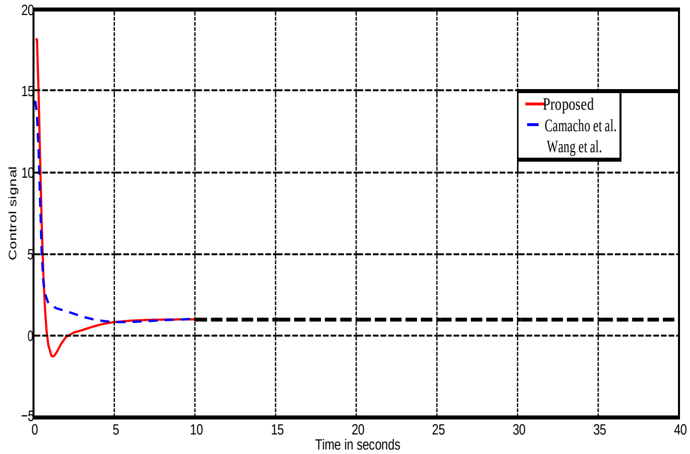

Fig. 6. Controller action (example 2)

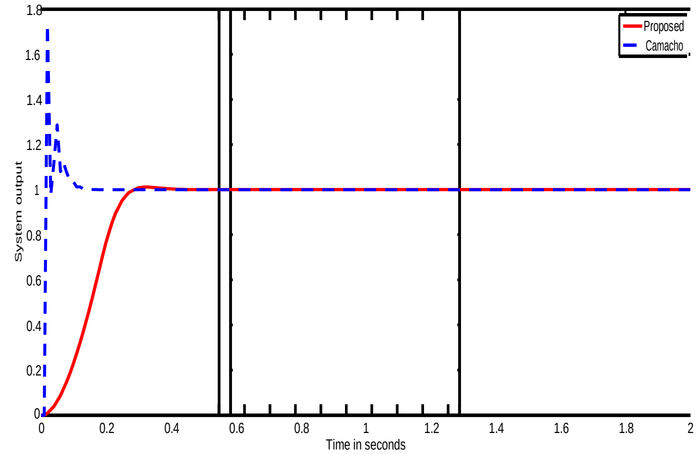

Fig. 8. Closed-loop step response (example 3)

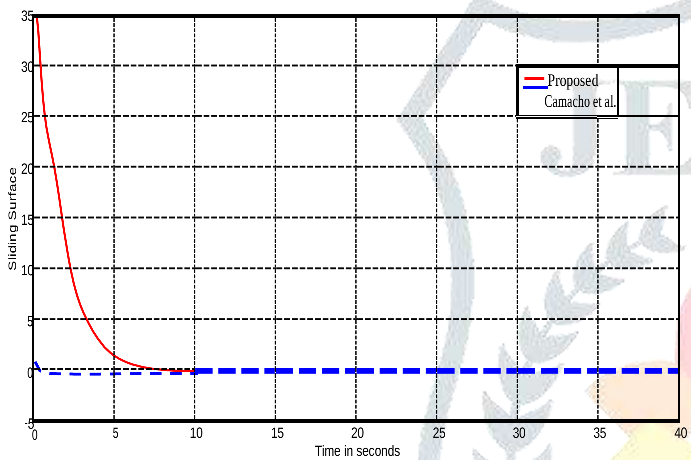

Fig. 7. Sliding surface (example 2)

or after delay approximation

$$\frac{Y(s)}{U(s)} = \frac{0.86}{(0.145s+1)(0.0035s+1)} = \frac{1694.6}{s^2+292.6s+1970} \quad (20)$$

The transfer function of the system is a second-order over-damped transfer function with two poles located on the left half part of the complex plane. Rearranging the system model as

$$Y(s)[s^2+292.6s+1970] = 1694.6U(s) \quad (21)$$

The above model can be represented in time domain, and given as

$$\ddot{y}(t) + 292.6\dot{y}(t) + 1970y(t) = 1694.6u(t) \quad (22)$$

where $y(t)$ is the output and $u(t)$ is the control input. The tuning parameters for the system is $\zeta_1 = 0.035$ and $\zeta_2 = 3\zeta_1$. The performance of the proposed controller is shown in Fig. 8

From the Fig. 8, it is clear that the performance of the proposed controller is far superior than the approach of Camacho's controller. The rise time is less and therer is no overshoot by the proposed controller. The response given by the camocho is very attracking with larger overshoot while the response of the proposed approach is acceptable.

The summary of simulation examples are given in table I. The settling time is measured in *seconds* with 2% tolernace band and peak overshoot is measured in terms of percnetage peak overshoot. The integral square error (ISE) is also included as a performance index.

TABLE I
SUMMARY OF SIMULATION RESULTS

Process	Control law	Settling time	% overshoot	ISE
Example-1	Proposed	8.5	7.2	15.14
–	Camacho et al.	15.8	95	17.91
–	Wang et al.	12.4	10	19.23
Example-2	Proposed	6.5	0	13.65
–	Camacho et al.	13	74.5	28.13
–	Wang et al.	11.1	2.8	29.74
Example-3	Proposed	0.27	2	25.65
–	Camacho et al.	0.11	170	138.13

IV. CONCLUSIONS

A simple method of sliding mode control law is proposed for lower-and-higher order systems. The error based derivative sliding surface offers the systems tracking ability. The method can be applicable to the systems with delay time, lower or higher, monotonic or oscillatory behavior systems. The observations can be made that the simple, still useful sliding mode parameters can be achieved based on Hurwitz criteria. The term useful refers to the controllers that have the best possible trade-off between output performance and control activity when the demanded specifications are obtained. The presented method can be applicable for various process including those have large time delay and have continuous oscillations in open-loop. The simulation examples show the effectiveness of the method to attain the satisfactory closed-loop response of the processes. The performance measure criteria such as ISE concludes that the presented sliding law is comparable with methods available in the literature. In this work, the method presented is restricted to the systems with all poles and no zeros in the open loop transfer function of the systems, even though the delay term is approximated using Taylor approximation and converted in the form of pole to obtain the control law. The method would be extended for the systems with zeros and poles and the term delay can be considered with an appropriate methodology.

REFERENCES

[1] V. I. Utkin, Sliding Modes in Control and Optimization, *Springer*, Berlin, 1992.
[2] O. Camacho and C. A. Smith, Sliding mode control an approach to regulate chemical processes, ISA Transaction, 39, 205-218, 2000.
[3] Ferrara A. and Vecchico C. Second order sliding mode control of vehicles with distributed collision avoidance capabilities, Mechatronics, 19,471-477, 2009.
[4] L. M. Capisani , A. Ferrara and L. Magnani, Design and experimental validation of a second-order sliding-mode motion controller for robot manipulators, International Journal Control, 82, 365-377, 2009.
[5] J. J. Slotine and W. Li, Applied nonlinear control, *Englewood Cliffs*, New Jersey: Prentice Hall Inc., 1991.
[6] M. Mihoub, A. S. Nouri and R. R. Abdennour, Real-time application of discrete second order sliding mode control to a chemical reactor, Control Eng Practice, 17, 1089-1095, 2009.
[7] F. J. Lin and R. J. Wai, Adaptive and fuzzy neural network sliding mode controllers for motor-quick return servomechanism. Mechatronics 13, 477-506, 2003.
[8] K. J. Walchko, D. Novick and M. C. Nechyba, Development of a sliding mode control system with extended Kalman filter estimation for subjugator, Florida Conference on Recent Advances in Robotics FCRAR, Dania Beach, Florida, May 8-9, 2003.
[9] C. H. Liang and J. P. Su, A new approach to the design of a fuzzy sliding mode controller, Fuzzy Sets Systems, 139, 111-124, 2003.
[10] K. B. Park and J. J. Lee, Sliding mode controller with filtered signal for robot manipulators using virtual plat/controller, Mechatronics, 7, 277-286, 1997.
[11] Utkin, V. I., Sliding mode control design principles and applications to electric drives, IEEE Transaction on Industrial Electronics, 40, 23-26, 1993.
[12] R. J. Wai, C. M. Lin, and C. F. Hsu, Adaptive fuzzy sliding mode control for electrical servo drive, Fuzzy Sets Systems, 143, 295-310, 2004.
[13] J. J. Slotine and W. Li, Applied Nonlinear Control, Prentice-Hall, Englewood Cliffs, NJ, 1991.
[14] F. J. Lin, R. J. Wai, R. H. Kuo and D. C. Liu, A comparative study of sliding mode and model reference adaptive observers for induction motor drive, Electrical Power Systems Research, 44, 163-174, 1998.
[15] C. L. Lim, N. B. Jones, S. K. Spurgeon, and J. Scott, Reconstruction of human neuro-muscular control signals using a sliding mode control technique, Simulation Modelling Practice and Theory, 11, 222-235, 2003.
[16] D. Sha, V. B. Bajic and H. Yang, New model and sliding mode control of hydraulic elevator velocity tracking system. Simulation Practice and Theory 9, 365-385, 2002.
[17] M. Shamsuzzoha and S. Skogestad, The setpoint overshoot method: A simple and fast closed-loop approach for PID tuning, Journal of Process Control, 20, 1220-1234, 2010.
[18] O. Camacho, R. Rojas, and W. G.-Gabin, Some long time delay sliding mode control approaches, ISA Transactions, 46, 95-101, 2007.
[19] Q. Wang, Z. Zhang, K. Astrom, L. Chek, Guaranteed dominant pole placement with PID controllers, Journal of Process Control, 19, 349-352, 2009.
[20] Furat M. and Eker I., Second-order integral sliding mode control with experimental application, ISA Transactions, 2014.