Fibonacci Antimagic Labeling of Some Special Graphs

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Abstract: In this paper we introduce Fibonacci Antimagic labeling. A bijective function
\( f : V(G) \rightarrow \{F_0, F_1, \ldots, F_n\} \) where \( F_j \) is the \( j \)th Fibonacci number (\( j = 0, 1, \ldots, n \)) is said to the Fibonacci Antimagic labeling if
the induced function
\( f^* : E(G) \rightarrow \{1, 2, \ldots, 2P\} \) is defined by \( f^*(uv) = (f(u) + f(v)) \) and all these edge labelings are distinct is called Fibonacci Antimagic labeling and a graph which admits Fibonacci Antimagic labeling is called a Fibonacci Antimagic graphs. Here we investigate Fibonacci Antimagic labeling of some special kind of graphs.

Keywords: Fibonacci Antimagic labeling, Tree, Path, Spider, Caterpillar, Unicyclic graph.

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1. Introduction
In this paper we consider finite, connected, undirected and simple graphs only. The vertex set and edge set of a graph \( G \) are denoted by \( V(G) \) and \( E(G) \) respectively. For various graph theoretic notation and terminology by D. B. West [7]. Fibonacci graceful labeling was introduced by Kathiresan and Amutha [5] and different kind of Fibonacci labeling were studied by [1], [2], [3] and [6]. Antimagic labelings was introduce by N. Hartsfield and G. Ringel [4]. In this paper we introduce a new concept called Fibonacci Antimagic labelings.

Definition 1.1
The Fibonacci sequence is a set of numbers that starts with zero followed by a one and proceed based on the rule that each number is equal to the sum of the proceeding two numbers is called a Fibonacci labelings.

Definition 1.2
A graph \( G \) is called antimagic if the \( n \) edges of \( G \) can be distinctly labeled 1 through \( n \) in such a way that when taking the sum of the edge labels incident to each vertex, the sums will all be different.

Definition 1.3
A Tree is an undirected graph in which any two vertices are connected by exactly one path.

Definition 1.4
A Walk is a path if all its vertices and also edges are distinct. In addition, if all the vertices are distinct then the trail is a path.

Definition 1.5
A Caterpillar is a tree with all vertices either on a single central path or distance 1 away from it. The central path may be considered to be the largest path in the caterpillar so that both end vertices have valency 1.

Definition 1.6
A spider is a tree with one vertex of degree atleast 3 and all others with degree atmost 2.

Definition 1.7
The graph \( C_n \Theta mK_1 \) is a unicyclic graph with \( p = q = n \) (\( m+1 \)) obtained from the cycle \( C_n \) by attaching \( m \) - pendent edges at each vertex of the cycle \( C_n \).

2. Main Results
Definition 2.1
Let \( G \) be a \( (p,q) \) graph. An bijective function \( f : V(G) \rightarrow \{F_0, F_1, \ldots, F_n\} \) where \( F_j \) is the \( j \)th Fibonacci number (\( j = 0, 1, \ldots, n \)) is said to the Fibonacci Antimagic labeling if the induced function \( f^* : E(G) \rightarrow \{1, 2, \ldots, 2P\} \) is defined by \( f^*(uv) = (f(u) + f(v)) \) and all these edge labelings are distinct.

Theorem 2.2
Every path \( P_n \), \( n \geq 2 \) admits a Fibonacci Antimagic labeling.

Proof
Let $G = P_n$ be a graph with $n$ vertices and $n - 1$ edges.
Define a vertex labeling by $f: V(G) \rightarrow \{F_0, F_1, ... , F_n\}$ and the induced edge labeling $f^*: E(G) \rightarrow \{1, 2, ... , 2P\}$ defined by $f^*(uv) = (f(u) + f(v))$ and all these edge labeling are distinct. Therefore the resulting graph every path $P_n$, $n \geq 2$ admits a Fibonacci Antimagic labeling.

**Example 2.3**
The following is an example of the Fibonacci Antimagic labeling of the Path $P_4$ and $P_5$.

Theorem 2.4
All trees are Fibonacci Antimagic labelings.
Proof
A tree has $n$ vertices and $n - 1$ edges.
Define a vertex labeling by $f: V(G) \rightarrow \{F_0, F_1, ... , F_n\}$ and the induced edge labeling $f^*: E(G) \rightarrow \{1, 2, ... , 2P\}$ defined by $f^*(uv) = (f(u) + f(v))$ and all these edge labeling are distinct.

Therefore the resulting all trees are Fibonacci Antimagic labeling.

**Example 2.5**
The following is an example of the Fibonacci Antimagic labeling of the tree.

Theorem 2.6
The caterpillar graph admits a Fibonacci Antimagic labeling.

Proof
Let $G$ be a caterpillar graph.
Define a vertex labeling by $f: V(G) \rightarrow \{F_0, F_1, ... , F_n\}$ and the induced edge labeling $f^*: E(G) \rightarrow \{1, 2, ... , 2P\}$ defined by $f^*(uv) = (f(u) + f(v))$ and all these edge labeling are distinct.
Therefore the resulting caterpillar graph admits a Fibonacci Antimagic labeling.

**Example 2.7**
The following is an example of the Fibonacci Antimagic labeling of the caterpillar graph.

![Caterpillar Graph](image1)

**Theorem 2.8**
The spider graph admits a Fibonacci Antimagic labeling.

**Proof**
Let $G=S_{n,m}$ be a spider graph with $n$ spokes in which each spoke is a path on length $m$.
Define a vertex labeling by $f : V(G) \rightarrow \{F_0,F_1,...,F_n\}$ and the induced edge labeling $f' : E(G) \rightarrow \{1,2,...,2P\}$ defined by $f'(uv) = (f(u) + f(v))$ and all these edge labeling are distinct. Therefore the resulting spider graph admits a Fibonacci Antimagic labeling.

**Example 2.9**
The following is an example of the Fibonacci Antimagic labeling of the spider graph.

![Spider Graph](image2)

**Theorem 2.10**
Every unicyclic graph admits a Fibonacci Antimagic labeling.

**Proof**
Let $G$ be a unicyclic graph with $p$ vertices and $q$ edges.
Define a vertex labeling by $f : V(G) \rightarrow \{F_0,F_1,...,F_n\}$ and the induced edge labeling $f' : E(G) \rightarrow \{1,2,...,2P\}$ defined by $f'(uv) = (f(u) + f(v))$ and all these edge labeling are distinct.
Therefore the resulting every unicyclic graph admits a Fibonacci Antimagic labeling.

Example 2.11
The following is an example of the Fibonacci Antimagic labeling of the unicyclic graph.

![Graph Image](image)

Fig 1.9 (C₅ at P₄)

3. Conclusion
In this paper we have shown that path, tree, spider, caterpillar and unicyclic graphs are fibonacci Antimagic labeling. In future, the same process will be analysed for other graphs.

References