Cordial Labeling Of One Point Union Of Double -Tail (C₃,tP₂) Garphs and their invariance

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1. Abstract: We discuss graphs of type G(k) i.e. one point union of k-copies of G for cordial labeling. We take G as double-tail graph. A double-tail graph is obtained by attaching a path Pₘ to a pair of adjacent vertices of given graph. It is denoted by double-tail(G,Pₘ) where G is given graph. We take G as C₃ and restrict our attention to m = 3 in Pₘ and consider up to t paths Pₜ each attached at a pair of adjacent vertices. Further we consider all possible structures of G(k) by changing the common point in one point union and obtain non-isomorphic structures. We show all these structures as cordial graphs. This is called as invariance of different structures of G(k) under cordial labeling.

Key words: cordial, one point union, double-tail graph, tail graph, cycle, labeling

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2. Introduction

The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Holton [5] Graph Theory by Harary [6], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West.[9].I.Cahit introduced the concept of cordial labeling[5]. f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f(u)−f(v)|. Further number of vertices labeled with 0 i.e v(f)(0) and the number of vertices labeled with 1 i.e.v(f)(1) differ at most by one .Similarly number of edges labeled with 0 i.e.e(f)(0) and number of edges labeled with 1 i.e.e(f)(1) differ by at most one. Then the function f is called as cordial labeling. Cahit has shown that : every tree is cordial; Kn is cordial if and only if n ≤ 3; Kₘₙ is cordial for all m and n; the friendship graph C₃(t) (i.e., the one-point union of t copies of C₃) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel Wₙ is cordial if and only if n is not congruent to 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [8].

Our focus of attention is on one point unions on different graphs. For a given graph there are different one point unions (up to isomorphism) structures possible. It depends on which point on G is used to fuse to obtain one point union. We have shown that for G = bull on C₃,bull on C₄,C₅* ,e the different path union Pₘ(G) are cordial [3].It is called as invariance under cordial labeling. We use the convention that v(t)(0,1) = (a,b) to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b. Further e(t)(0,1) = (x,y) we mean the number of edges labeled with o are x and number of edges labeled with 1 are y. The graph whose cordial labeling is available is called as cordial graph. In this paper we define double-tail graph and obtain one point union graphs on it. Let G be a (p,q) graph. To one of it’s pair of adjacent vertices we fuse t number of paths Pₘ. We denote this by double –tail(G,tPₘ). We choose m = 2 and t = 1,2,3, 4 and discuss their one point union graph at different vertices of G and it’s invariance under cordial labeling.

3. Preliminaries

3.1 Tail Graph: A (p,q) graph G to which a path Pₘ is fused at some vertex. This also can be explained as take a copy of graph G and at any vertex of it fuse a path Pₘ with it’s one of the pendent vertex. It’s number of vertices are P+m-1 and edges are by q + m-1. It is denoted by tail(G, Pₘ). In this paper we fix G as C₃ and take Pₘ for m =2, 3, 4, 5.

3.2 Double tail graph: To any graph G we attach paths of equal length to adjacent pair of vertices. When...
these paths are just an edge each then it is referred as bull graph. This graph is denoted by double-tail(G, P_m) when both tails are identical and equal to p_m. If tails are p_m and p_n then the graph is denoted by double-tail(G, p_m, p_n). It has p+m+n-2 vertices and q+m+n-1 edges where G is a (p,q) graph.

3.3 Fusion of vertices. Let u ≠ v be any two vertices of G. We replace these two vertices by a single vertex say x and all edges incident to u and v are now incident to x. If loop is formed then it is deleted. [6]

3.4 G^k it is One point union of k copies of G is obtained by taking k copies of G and fusing a fixed vertex of each copy with same fixed vertex of other copies to create a single vertex common to all copies. If G is a (p,q) graph then |V(G^k)| = k(p-1)+1 and |E(G)| = k.q

4. Results

Proved:

Theorem 4.1 All non-isomorphic one point union on k-copies of graph obtained on G = double-tail(C_3, P_2) given by G^k are cordial graphs for all k.

Proof: From figure 4.1 it follows that there are three different structures on one point union of k copies of G are possible depending on if we use vertex a, b, or c as common point.

To take one point union at vertex a or b the type A or Type B label is used alternately. In G^k where k = 2n the type A label and type b label each is used for n times. If k = 2n+1 (n = 0, 1, 2,..) then type A label is used for n+1 times and type B label is used for n times. The label number distribution is v_t(0,1) = (3+4x, 2+4x) e_t(0,1) = (3+5x, 2+5x) for k = 2n+1 , n = 0, 1, 2, ..If k = 2n , n = 1, 2, 3, ..then v_t(0,1) = (5+4(x-1), 4+4(x-1)) e_t(0,1) = (5x, 5x). When one point union is taken at point c then the graph G^k has no cordial labeling when k = 2(mod 4).

For all other values of k we proceed as follows.

Case k = 4x. We use type D label repeatedly. The label number distribution is v_t(0,1) = (1+8x, 8x) e_t(0,1) = (10x, 10x).
Case $k = 4x+1$.

We first obtain labeling for $k = 4x$ as shown above. With this labeled copy we fuse type A label at vertex c. The label number distribution is $v(t,1) = (3+8x, 2+8x)$ $e(t,1) = (3+10x, 2+10x)$.

Case $k = 4x+3$. We first obtain labeling for $k = 4x$ as shown above. With this labeled copy we fuse type C label at vertex c. The label number distribution is $v(t,1) = (7+8x, 6+8x)$ $e(t,1) = (7+10x, 8+10x)$.

Thus the graph is cordial.

Theorem 4.2 All non-isomorphic one point union on $k$-copies of graph obtained on $G = \text{double-tail}(C_3, 2P_2)$ given by $G^{(k)}$ are cordial graphs.

Proof: From Fig 4.6 it follows that we can take one point union at four vertices ‘a’, ‘b’, ‘c’. For the one point union at any of these vertices we fuse the type A and Type B label at respective vertex. In $G^{(k)}$ where $k = 2x$ the type A label and type B label each is used for $x$ times. If $k = 2x+1$, $x = 0, 1, 2, …$ then type A label is used for $x+1$ times and type B label is used for $x$ times.

The label number distribution is $v(t,1) = (3+6x, 4+6x)$ $e(t,1) = (3+7x, 4+7x)$ for $k = 2x+1$, $x = 0, 1, 2, …$ If $k = 2x$, $x = 1, 2, 3, …$ then $v(t,1) = (6+6(x-1), 7+6(x-1))$ $e(t,1) = (7x, 7x)$. Thus the graph is product cordial even if we change the common point in $G^{(k)}$.

Theorem 4.3 All non-isomorphic one point union on $k$-copies of graph obtained on $G = \text{double-tail}(C_3, 2P_2)$ given by $G^{(k)}$ are cordial graphs.

Proof: From Fig 4.9 it follows that we can take one point union at three vertices ‘a’, ‘b’, or ‘c’. For the one point union at vertex a or vertex b we use Type A label and Type B label alternately in $G^{(k)}$. In $G^{(k)}$ where $k = 2x$ the type A label and type B label each is used for $x$ times. If $k = 2x+1$, $x = 0, 1, 2, …$ then type A label is used for $x+1$ times and type B label is used for $x$ times. The label number distribution is given by $v(t,1) = (5+8x, 4+8x)$, $e(t,1) = (4+9x, 5+9x)$ where $k = 2x+1$, $x = 0, 1, 2, …$ If $k = 2x$, $x = 1, 2, …$ then we have, $v(t,1) = (9+8(x-1), 8+8(x-1))$, $e(t,1) = (9k, 9k)$. Thus the graph is cordial.
Then we have, \( v \) given by \( v = (9,12) \), \( e = (11,11) \)

![Image](https://via.placeholder.com/150)

Thus the graphs are cordial.

**Theorem 4.4** All non-isomorphic one point union on \( k \)-copies of graph obtained on \( G = \text{double-tail}(C_3, 4P_2) \) given by \( G^{(k)} \) are cordial graphs.

Proof: From Fig 4.15 it follows that we can take one point union at three vertices ‘a’, ‘b’, or ‘c’. For the one point union at vertex a or vertice b we use Type A label and type B label alternately in \( G^{(k)} \). In \( G^{(k)} \) where \( k = 2x \) the type A label and type B label each is used for \( x \) times. If \( k = 2x+1 \) ( \( x = 0,1,2,.. \)) then type A label is used for \( x+1 \) times and type B label is used for \( x \) times. The label number distribution is given by \( v = (6+10x, 5+10x) \), \( e = (5+11x, 6+11x) \) where \( k = 2x+1, x=0,1,2,.. \) If \( k = 2x; x= 1,2,.. \) then we have, \( v = (11+10(x-1), 10+10(x-1)), e = (11k, 11k) \). Thus the graph is cordial.
Thus the graph is cordial.

Conclusions:  In this paper we define some new families obtained from $C_3$ and fusing to two adjacent vertices with pendant edges upto four. We show that

1) All non- isomorphic one point union on k-copies of graph obtained on $G =$double - tail($C_3$, $P_2$) also called $G^{(k)}$ are cordial graphs.

2) All non- isomorphic one point union on k-copies of graph obtained on $G =$double- tail($C_3$, $2P_2$) given by $G^{(k)}$ are cordial graphs.

3) All non- isomorphic one point union on k-copies of graph obtained on $G =$double- tail($C_3$, $3P_2$) given by $G^{(k)}$ are cordial graphs.

4) All non- isomorphic one point union on k-copies of graph obtained on $G =$double- tail($C_3$, $4P_2$) given by $G^{(k)}$ are cordial graphs.
References:


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