A Note On Cordial Labeling of Cycle Related Mixed Double Path Union

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1. Abstract: Let $G_1$ and $G_2$ be two graphs. On alternate vertices of $P_m$ both $G_1$ and $G_2$ are fused. Resultant graph is mixed double path union on $G_1$ and $G_2$. We study it for cordial labeling by taking case 1: $G_1 = C_3$ and $G_2 = \text{flag of } C_3$. Case 2: $G_1 = C_4$ and $G_2 = \text{flag of } C_4$. Case 3: $G_1 = C_5$ and $G_2 = \text{flag of } C_5$. Case 4: $G_1 = C_4$ and $G_2 = \text{flag of } C_3$. All these structures are observed to be cordial.

Key words: cordial, labeling, double path union, cycle, mixed path

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1. Introduction. The graphs we consider are simple, finite and connected. For terminology and definitions we depend on Harary[5], Clark and Holton[4] and Dynamic survey of graph labeling[7]. I. Cahit introduced concept of cordial [3] labeling. $f: V(G) \rightarrow \{0,1\}$ be a function. From this label of any edge $(uv)$ is given by $|f(u) - f(v)|$. Further number of vertices labeled with 0 i.e $v_0(0)$ and the number of vertices labeled with 1 i.e $v_0(1)$ differ at most by one. Similarly number of edges labeled with 0 i.e $e_0(0)$ and number of edges labeled with 1 i.e $e_0(1)$ differ by at most one. Then the function $f$ is called as cordial labeling. Cahit has shown that: every tree is cordial; $K_n$ is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all $m$ and $n$; the friendship graph $C_3^t$ (i.e., the one-point union of $t$ copies of $C_3$) is cordial if and only if $t$ is not congruent to 2 (mod 4); all fans are cordial; the wheel $W_n$ is cordial if and only if $n$ is not congruent to 3 (mod 4). A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [7]. We use the convention that $v_i(0,1) = (a,b)$ to indicate the number of vertices labeled with 0 are $a$ in number and that number of vertices labeled with 1 are $b$. Further $e_i(0,1) = (x,y)$ we mean the number of edges labeled with 0 are $x$ and number of edges labeled with 1 are $y$. The graph whose cordial labeling is available is called as cordial graph.

A mixed double path union on $G_1$ and $G_2$ is denoted by $P_m(G_1, G_2)$. We take $G_1$ as $C_k$ and $G_2$ as $\text{flag of } C_k$, $k = 3, 4, 5$.

3. Definitions: Fusion of vertex. Let $G$ be a $(p,q)$ graph. let $u \neq v$ be two vertices of $G$. We replace them with single vertex $w$ and all edges incident with $u$ and that with $v$ are made incident with $w$. If a loop is formed is deleted. The new graph has $p-1$ vertices and at least $q-1$ edges. If $ucG_1$ and $vcG_2$, where $G_1$ is $(p_1,q_1)$ and $G_2$ is $(p_2,q_2)$ graph. Take a new vertex $w$ and all the edges incident to $u$ and $v$ are joined to $w$ and vertices $u$ and $v$ are deleted. The new graph has $p_1 + p_2 - 1$ vertices and $q_1 + q_2$ edges. Sometimes this is referred as $u$ is identified with $v$.

Path union of $G$ i.e. $P_m(G)$ is obtained by taking a path $P_m$ and $m$ copies of graph $G$. Fuse a copy each of $G$ at every vertex of path at given fixed point on $G$. It has $mp$ vertices and $mq + m - 1$ edges, where $G$ is a $(p,q)$ graph. If we change the vertex on $G$ that is fused with vertex of $P_m$ then we generally get a path union non-isomorphic to earlier structure.

Flag of a graph $G$ denoted by $FL(G)$ is obtained by taking a graph $G = G(p,q)$. At suitable vertex of $G$ attach a pendent edge. It has $p + 1$ vertices and $q + 1$ edges.
Double path union on $G_1 = (p_1,q_1)$ and $G_2 = (p_2,q_2)$ graphs. It is denoted by $P_m(G_1,G_2)$. At each vertex of a path a copy of $G_1$ and a copy of $G_2$ is fused at same fix vertex of $G_1$ and $G_2$. It has $m(p_1+p_2)$ vertices and $m(q_1+q_2)+(m-1)$ edges.

4. Main Results:

Theorem 1. Mixed double path union $G = P_m(G_1,G_2)$ is cordial where $G_1 = C_3$ and $G_2 = \text{flag}(C_3)$.

Proof: First we fuse $C_3$ and $\text{flag}(C_3)$ to obtain the structure as follows. This structure is actually $P_1(G_1,G_2)$ and is fused at each vertex of path $P_m$ at vertex $a$ on it.

Define a function $f: V(G) \rightarrow \{0,1\}$. It produces labeled copy as in figure 4.1 below.

![Figure 4.1](image1.png)

Fig 4.1 $G = P_1(G_1,G_2)$ A labeled copy, Vertex $a$ is fusion vertex on $G$. $v_f(0,1) = (3,3)$, $e_f(0,1) = 3,4$)

Note that label of each vertex on path $P_m$ is 0.

Label distribution on vertices is $v_f(0,1) = (3m,3m)$ for all $m$. For edges we have $e_f(0,1) = (3+4(m-1), 4+4(m-1))$

Thus the graph is cordial.

Theorem 2. Mixed double path union $G = P_m(G_1,G_2)$ is cordial where $G_1 = C_4$ and $G_2 = \text{flag}(C_4)$.

Proof: First we fuse $C_4$ and $\text{flag}(C_4)$ to obtain the structure as follows. This structure is actually $P_1(G_1,G_2)$ and is fused at each vertex of path $P_m$ at vertex $a$ on it.

Define a function $f: V(G) \rightarrow \{0,1\}$. It produces labeled copy as in figure 4.3 below.

![Figure 4.2](image2.png)

Fig 4.2 labeled copy of $G = P_m(G_1,G_2)$
Note that label of each vertex on path $P_m$ is 0.

Label distribution on vertices is $v_f(0,1) = (4m,4m)$ for all $m$. For edges we have $e_f(0,1) = (4+5(m-1), 4+5(m-1))$

Thus the graph is cordial.

Theorem 3. Mixed double path union $G = P_m(G_1,G_2)$ is cordial where $G_1 = C_5$ and $G_2 = \text{flag}(C_5)$.

Proof: First we fuse $C_4$ and $\text{flag}(C_4)$ to obtain the structure as follows. This structure is actually $P_1(G_1,G_2)$ and is fused at each vertex of path $P_m$ at vertex $a$ on it.

Define a function $f$: $V(G) \to \{0,1\}$. It produces labeled copy as in figure 4.5 below.

![Diagram 4.3 labeled copy of $G = P_m(C_4,\text{flag}(C_4))$](image1)

$v_f(0,1) = (4,4)$, $e_f(0,1) = (4,5)$

![Diagram 4.4 labeled copy of $G = P_m(C_5,\text{flag}(C_5))$](image2)

$v_f(0,1) = (5,5)$, $e_f(0,1) = (5,6)$
Note that label of each vertex on path $P_m$ is 0.

Label distribution on vertices is $v_f(0,1) = (5m,5m)$ for all $m$. For edges we have $e_f(0,1) = (5+6(m-1), 5+6(m-1))$

Thus the graph is cordial

Theorem 4. Mixed double path union $G = P_m(G_1,G_2)$ is cordial where $G_1 = C_4$ and $G_2 = \text{flag}(C_3)$.

Proof: First we fuse $C_4$ and $\text{flag}(C_3)$ to obtain the structure as follows. This structure is actually $P_1(G_1,G_2)$ and is fused at each vertex of path $P_m$ at vertex $a$ on it.

Define a function $f: V(G) \to \{0,1\}$. It produces labeled copy as in figure 4.6 below.

We take a path $P_m = (v_1,v_2,\ldots,v_m)$. At vertex $v_i$ we fuse type A label if $i \equiv 0,1 \pmod{4}$ and type B label if $i \equiv 2,3 \pmod{4}$. The fusion is taken at point $a$ on both type of labels. The label number distribution is $v_f(0,1) = (4+14x,3+14x)$ if $m$ is of type $4x+1$, $x = 0,1,2,..$
\( v_f(0,1) = (7x,7x) \) if \( m \) is of type \( 2x \), \( x = 1, 2, 3.. \)

\( v_f(0,1) = (10+14x,11+14x) \) if \( m \) is of type \( 3+4x \), \( x = 0, 1, 2, 3.. \)

\( v_f(0,1) = (7x,7x) \) if \( m \) is of type \( 2x \), \( x = 1, 2, 3.. \)

On edges we have \( e_f(0,1) = (4+18x,4+18x) \) if \( m \) is of type \( 4x+1 \), \( x = 0, 1, 2, 3.. \)

\( e_f(0,1) = (8+18x,9+18x) \) if \( m \) is of type \( 4x+2 \), \( x = 0, 1, 2, 3.. \)

\( e_f(0,1) = (13+18x,13+18x) \) if \( m \) is of type \( 4x+3 \), \( x = 0, 1, 2, 3.. \)

\( e_f(0,1) = (17+18x,18+18x) \) if \( m \) is of type \( 4x \), \( x = 1, 2, 3.. \)

Thus the graph is cordial.

Conclusions

In this paper we obtain path union on mixed graph. This family of graph is constructed by fusing copy each of \( G_1 \) and \( G_2 \) at fixed vertex of path vertex. This graph is denoted by \( P_m(G_1,G_2) \).

We have proved that
1) Mixed double path union \( G = P_m(G_1,G_2) \) is cordial where \( G_1 = C_3 \) and \( G_2 = \text{flag}(C_3) \).
2) Mixed double path union \( G = P_m(G_1,G_2) \) is cordial where \( G_1 = C_4 \) and \( G_2 = \text{flag}(C_4) \).
3) Mixed double path union \( G = P_m(G_1,G_2) \) is cordial where \( G_1 = C_5 \) and \( G_2 = \text{flag}(C_5) \).
4) Mixed double path union \( G = P_m(G_1,G_2) \) is cordial where \( G_1 = C_4 \) and \( G_2 = \text{flag}(C_3) \).

Further it is necessary to investigate this type of families for general cases such as \( G_1 \) and \( G_2 \) are \( C_n \) etc.

References:


\(^1\) Mukund V. Bapat, Hindale, Tal: Devgad, Sindhudurg
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