RESULTS ON THE EQUITABLE DOMINATION AND INVERSE EQUITABLE DOMINATION NUMBERS FOR SOME CLASSES OF GRAPHS

[1] K. Ameenal Bibi, [2] G. Vinu Priya, [3] P. Rajakumari
[1] Associate Proffesor,[2] Assistant Professor,[3] Research Scholar
[1,2,3] P.G. and Research Department of Mathematics,
D.K.M.College for Women (Autonomous),
Vellore-632001, Tamilnadu, India.

Abstract : ***Let G = (V, E) be a simple, finite, undirected and connected graph. A non-empty subset D of a graph V (G) is called an equitable dominating set of a graph G if for every $v \epsilon V - D$, there exists a vertex $u \epsilon D$ such that $uv \epsilon E(G)$ and $|deg(u) - deg(v)| \leq 1$. The minimum cardinality of a minimal equitable dominating set is called an domination number of G and it is denoted by $\gamma_e(G)$.***

Let G = (V, E) be a graph with no isolated vertices. If D is the equitable dominating set G then $V - D$ is also an equitable dominating set of G. A non-empty subset $D' \subseteq V - D$ is an inverse equitable dominating set of G if D' is an equitable dominating set of G. The inverse equitable domination number of G is the minimum cardinality among all the minimal inverse equitable dominating sets of G. In this paper, we established equitable and inverse equitable domination numbers for some families of graphs.

IndexTerms - Dominating set, equitable domination, inverse equitable domination, Andrasfai graph, Cocktail Party graph, Musical graph, Antiprism graph, Coxeter graph, Cubic symmetric graph, Doyle graph, Folkman graph, Levi graph, Icosahedron graph, Octahedron graph..

I. INTRODUCTION

The domination in graphs is one of the major areas in graph theory which attracts many researchers because it has the potential to solve many real life problems involving design and analysis of communication network, social network as well as Defense surveillance. The literature on domination and related parameters has been surveyed and beautifully presented in the two books by Haynes, Hedetniemi and slater [7]

Swaminathan et al [11] introduced the concept of equitable domination in graphs, by considering the following real world problems. In a network nodes with nearly equal capacity may interact with each other in a better way. In this society, persons with nearly equal status, tend to be friendly. In an industry, employees with nearly equal powers form association and move closely. Equitability among citizens in terms of wealth, health, status etc is the goal of a Democratic Nation.

The concept of the inverse domination number was introduced by kulli and sigarkanti . Let D be the dominating set (γ-set) of G. A dominating set D' contained in V – D is called an inverse dominating set of G with respect to D. The smallest cardinality among all the minimal dominating sets in V-D is called the inverse domination number denoted by $\gamma'(G)$.

Sivakumar.S and N.D.Soner introduced the inverse equitable domination in graphs. We used the following definitions in the subsequent sections.

Definition 2.1 [12]

A Circular graph is a graph which has a circular adjacency matrix. It is a graph of n vertices in which the i^{th} graph vertex is joined to $(i - j)^{th}$ and $(i + j)^{th}$ vertices for each j by the cyclic group of symmetry.

Definition 2.2[2]

The n-Andrasfai graph [6] is a circular graph on (3n-1) vertices whose indices are given space by the integers 1,2,3...(3n-1) that are congruent to 1 (mod3). The Andrasfai graphs have diameter 2 for $n \geq 2$ and is denoted by A_n.

Definition 2.3 [2]

The Cocktail Party graph is a graph consisting of two rows of paired vertices in which all the vertices except the paired ones are joined by an edge and is denoted by CP_K, where k=2n, for all $n \geq 2$. It is also called Hyper Octahedral graph or Robert's graph.

Definition 2.4 [2]

The Musical graph $n \geq 3$ of order n consists of two parallel copies of cycle graph C_n in which all the paired vertices and the neighborhood vertices are connected and is denoted by M_{2n}. $\forall n \geq 3$.

Definition 2.5 [2]

The n-Antiprism graph is a graph which can be obtained by joining two parallel copies of cycle graph C_n by an alternative band of triangles. It is denoted by AP_n.

Definition 2.6 [9]

The Coxetor graph is a non -hamiltonian cubic symmetric graph on 28 vertices and 42 edges.

Definition 2.7[9]

Truncated tetrahedron (or) the Cayley graph is the graph comprises of 12 vertices each of degree three with number of edges equal to 18.

Definition 2.8 [9]

A Cubic symmetric graph is a symmetric cubic, since cubic graphs must have an even number of vertices.

Definition 2.9 [9]

The Doyle graph also known as Halt graph, is the quadratic symmetric graph on 27 vertices.

Definition 2.10 [9]

The Folkman graph is a semi symmetric graph that has the minimum possible number of vertices 20. It has 30 vertices and 45 edges.

Definition 2.11 [9]

The Levi graph is a generalized polygon which is the vertex/edge incidence graph of the generalized quadrangle.

Definition 2.12

Octahedron is a 4-regular graph with 6 vertices.

Definition 2.13

The Icosahedron graph has 12 vertices and 30 edges. Since the icosahedron graph is regular and Hamiltonian, it has generalized notation.

2. MAIN RESULTS

EQUITABLE DOMINATION NUMBER FOR SOME FAMILIES OF GRAPHS.

In this section, we investigated the equitable domination numbers for some graphs.

Theorem: 2.1

Let G=A_k be the Andrasfai graph with k vertices where k=3n-1 then the equitable domination number of A_k is given by $\gamma_e(A_k) = n, \forall\ n \geq 2, n \in n$.

Proof

Let G = A_K be the Andrasfai graph of (3n-1) vertices. Since G is a circulant graph, all the vertices are adjacent in a symmetrical manner and hence every vertex has the same degree. Let V(G) ={v_1,v_2,v_3…v_{3n-1}} be the vertex set of G. Since G is a regular graph of A_k its degree n .

The non-empty subset D of V(G) is called an equitable dominating set of a graph G if for every $v \in V - D$, there exists a vertex u ϵ D such that $uv \in E(G)$ $|\deg(u) - \deg(v)| \leq 1$. The Andrasfai graph satisfying equitable condition (i.e) $|\deg(u) - \deg(v)| = 0$. Therefore $\gamma_e(A_k) = n$, if $n \geq 2$.

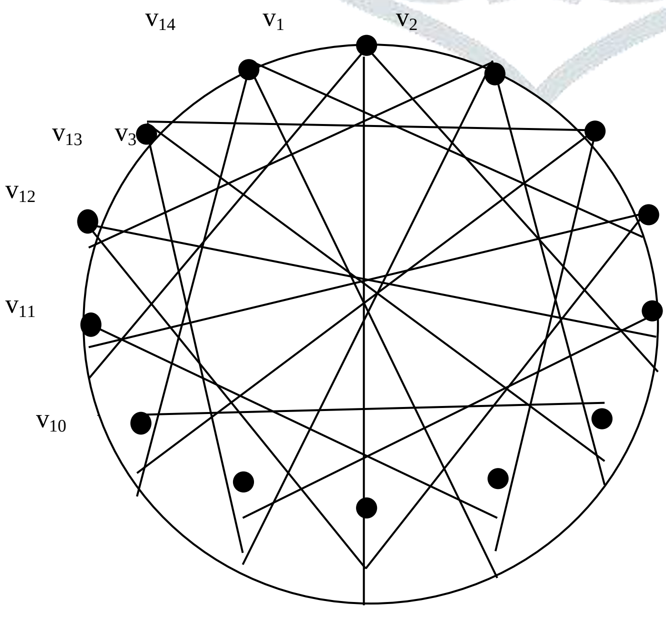

Fig:1 Andrasfai Graph A_{14} When n=5

Theorem2.2

Suppose G is the Cocktail Party graph with 2n vertices then $\gamma_e(\mathrm{CP_k}) = 2$, for $n \geq 2$.

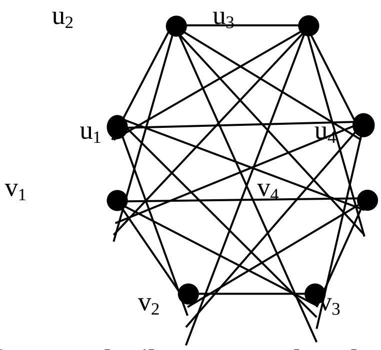

Fig : 2 Cocktail Party Graph CPk with k=8 and n=4

Proof

If G is the Cocktail Party graph with k=2n vertices. So by the definition of Cocktail Party graph, the vertex set of G is partitioned into two subsets v_1 and v_2 such that $v_1 \cup v_2 = V$ and $v_1 \cap v_2 = \phi$.

Let $V_1=\{u_1,u_2,u_3\ldots u_n\}$ and $V_2=\{v_1,v_2,v_3\ldots v_n\}$ be the two subsets of V(G). To dominate we need one vertex among V_1 and one more vertex among V_2. i.e. D = { v_1,u_1} forms the minimum dominating set, since the degree of any vertex of CP_k is k-2.

The non-empty subset of $V(CP_k)$ is called an equitable dominating set of a graph G if for every $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(CP_k)$ and $|\deg(u) - \deg(v)| \leq 1$.The Cocktail Party graph satisfying equitable domination condition and $|\deg(u) - \deg(v)| = 0$. Therefore, $\gamma_e(\mathrm{CP_k}) = 2, \forall\, n \geq 2$.

Theorem 2.3

For the Musical graph G=M_k with K=2n $\forall\, n \geq 3$, equitable domination number is given by $\gamma_e(\mathrm{M_k}) = \left\lceil\frac{k}{6}\right\rceil + 1$.

Proof

Let G=M_k be the Musical graph of order 2n and n≥3. Let $V_1=\{v_1, v_2, v_3 \ldots. v_n\}$ be the vertex set of the interior cycle and $V_2=\{v_{n+1},v_{n+2},v_{n+3}\ldots v_{2n}\}$ be the vertex set of the exterior cycle. Since M_k is regular, the degree of any vertex of M_k is 5. For every $v \in V - D$, there exists a vertex $u \in D$ such that $uv \in E(M_k)$ and $|\deg(u) - \deg(v)| \leq 1$.The musical graph satisfying equitable dominating condition i.e. $|\deg(u) - \deg(v)| = 0$. Therefore $\gamma_e(\mathrm{M_k}) = \left\lceil\frac{k}{6}\right\rceil + 1$.

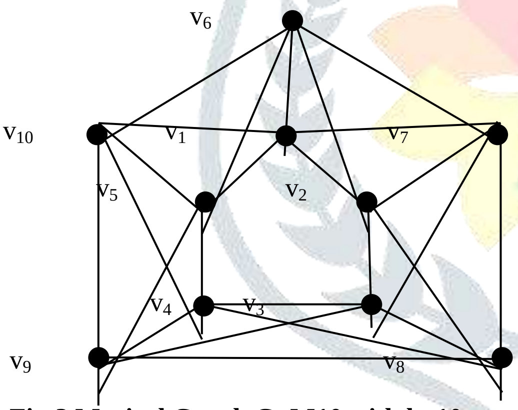

Fig:3 Musical Graph G=M10 with k=10

Theorem:2.4

If G is a Coxeter graph, then $\gamma_e(\mathrm{G}) = 7$.

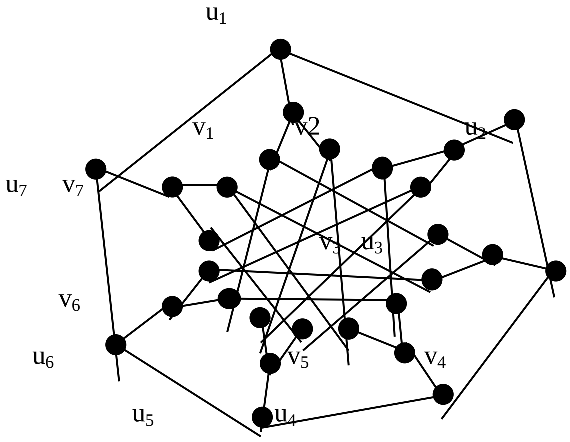

Fig:4 Coxeter graph G with (G) = 7

Proof

Let G be the Coxeter graph with 28 vertices of degree 3. Let U={u_1,u_2,u_3…u_7}, V={v_1,v_2,v_3…v_7} and W={w_1,w_2,w_3…w_{14}} be the three sets of vertices of G. The vertex set {v_1,v_2,v_3…v_7} forms the minimum dominating set. If for every $v \epsilon V - D$ there exists a vertex $u \epsilon D$ such that $uv \epsilon E(G)$ and $|\deg(u) - \deg(v)| \leq 1$. The Coxeter graph satisfying the equitable domination condition and $|\deg(u) - \deg(v)| = 0$. Therefore $\gamma_e(\mathrm{G}) = 7$.

Theorem 2.5

Let G be the Cayley graph then $\gamma_e(\mathrm{G}) = 3$.

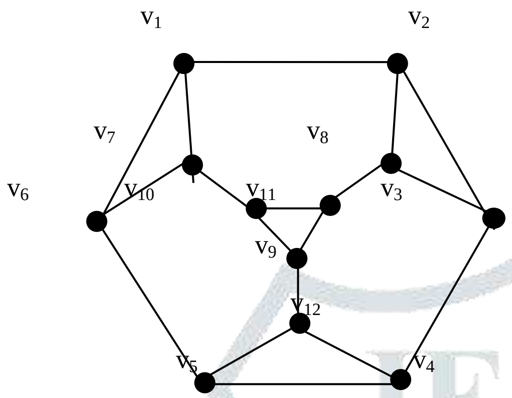

Fig:5 Cayley graph

Proof

By the definition, the truncated tetrahedron (or) the Cayley graph is the graph comprises of 12 vertices each of degree 3 with number of edges 18. We needed three vertices to dominate all the remaining vertices. Every vertex $v\epsilon V - D$, there exists a vertex $u \epsilon D$ such that $uv \epsilon E(G)$ and $|\deg(u) - \deg(v)| \leq 1$. The Cayley graph satisfying the equitable condition and $|\deg(u) - \deg(v)| = 0$. Therefore $\gamma_e(\mathrm{G}) = 3$.

Theorem 2.6

The Cubic symmetric graph is a regular graph of order 3. Let G be a cubic symmetric graph then $\gamma_e(\mathrm{G}) = 2$.

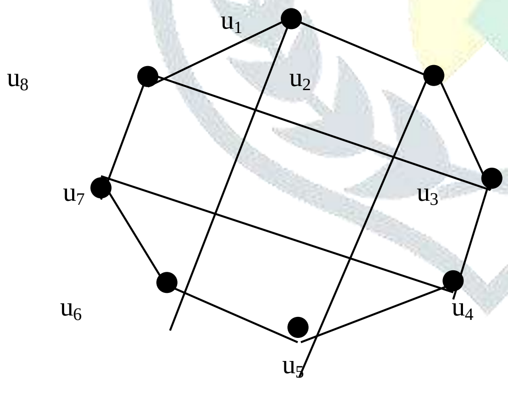

Fig: 6 Cubic Symmetric Graph G

Proof

By the definition of the Cubic Symmetric graph, G consists of 8 vertices each of degree 3. Only two vertices are enough to dominate all the remaining vertices of G. Let D=={u_1,u_4}from the minimum dominating set of G. For every vertex $v \epsilon V - D$, there exists $u \epsilon D$ such that $|\deg(u) - \deg(v)| \leq 1$. This satisfies the equitable domination condition. Therefore $\gamma_e(\mathrm{G}) = 2$.

Theorem: 2.7

Let G be a Doyle graph then $\gamma_e(\mathrm{G}) = 9$.

Proof:

Let G be a Doyle graph with 27 vertices. Let V={ v_1,v_2,v_3…v_n} the set of vertices of the graph. The vertex set V can be partitioned into three vertex sets V_1,V_2 and V_3 and degree of each vertex being four. If V_1 is taken as the dominating set whose vertices dominate the other two sets in which $|\deg(u) - \deg(v)| = 0$, Thus satisfying the equitable condition$|\deg(u) - \deg(v)| \leq 1$. Therefore, $\gamma_e(\mathrm{G}) = 9$.

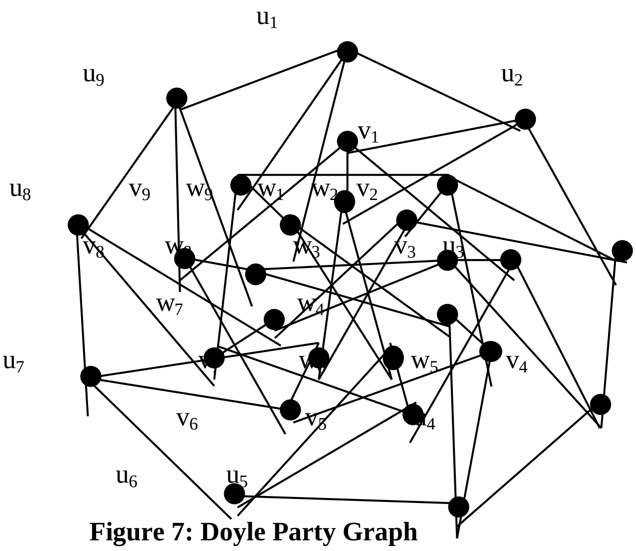

Figure 7: Doyle Party Graph

Theorem: 2.8

Let $G_{2,n}$ be a Double Cycle graph then $\gamma_e(G_{2,n}) = \left\lceil \frac{2n-3}{3} \right\rceil$ $\forall$ n ≥ 3.

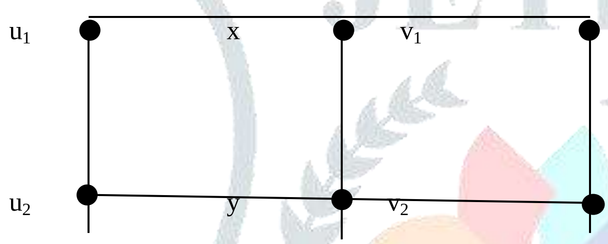

Fig:8 Double Cycle

Proof

Let $G_{2,n}$ denote the graph consisting of two cycles each of size n with a shared edge. We know that C_n is regular, the degree of any vertex of C_n is 2. Any dominating set in C_n is clearly equitable.

Now joining two cycles, the degree of any vertex of $G_{2,n}$ is 2 except the middle vertices x & y. Therefore $\gamma_e(G_{2,n}) = \left\lceil \frac{2n-3}{3} \right\rceil$ for n ≥ 3.

Observation 2.9

Let G be the Octahedron graph, then $\gamma_e(G) = 2$.

Observation 2.10

Let G be a Icosahedron graph. Then $\gamma_e(G) = 2$.

Observation 2.11

(i) Let G be a Folkman graph. Then $\gamma_e(G) = 6$.

(ii) Let G be a Frucht graph. Then $\gamma_e(G) = 3$.

(iii) Let G be a Levigraph. Then $\gamma_e(G) = 10$.

(iv) Let G be a Antiprism graph then $\gamma_e(G) = \left\lceil \frac{k}{3} \right\rceil$

3. INVERSE EQUITABLE DOMINATION NUMBER OF SOME FAMILIES OF GRAPHS.

In this section, we established the inverse equitable domination number of some families of graphs.

Theorem:3.1

Let CP_k be the Cocktail Party graph. Then $\gamma_e'(CP_k) = 2$.

Proof

Let V={v_1,u_1,v_2, u_2…$v_{k/2}$, $u_{k/2}$} be the vertices of CP_k.

Let V_1={ v_1,v_2,v_3…$v_{k/2}$} and

$V_2 = \{u_1, u_2, u_3 \ldots u_{k/2}\}$ be the two vertex sub sets of CP_k.

The necessary and sufficient condition for the existence of atleast one inverse dominating set of G is that G contains no isolated vertices. To have a dominating set D' we need to have one vertex from V_1 and another from V_2. Since the set of vertices $\{v_1, u_1\}$ forms a dominating set of CP_k, the induced sub graph is a connected graph.

Let $D' = \{v_i, u_i \,/\, (v_{i+1}, u_{i+1}) \in D\}$ be an inverse dominating set of CP_k. It satisfies the equitable domination that condition, if for every $v \in V - D'$, there exists u $\in D'$ such that $uv \in E(CP_k)$ and $|\deg(u) - \deg(v)| \le 1$. Therefore $\gamma_e'(CP_k) = 2 \ \forall$ n ≥ 2.

Theorem 3.2

Let M_k be the Musical graph. Then $\gamma_e'(M_k) = \left\lceil \frac{k}{6} \right\rceil + 1$.

Proof

We know that $\gamma_e(M_k) = \left\lceil \frac{k}{6} \right\rceil + 1$. Let D$\subseteq V$. Hence $\langle V - D \rangle$ is a connected graph. The necessary and sufficient condition for the existence of atleast one inverse dominating set of G is that G contains no isolated vertices.

A graph without isolated vertices contains an inverse equitable dominating set. Since D is the minimum equitable dominating set then V-D is also an equitable dominating set. If V – D contains a dominating set D' with $|\deg(u) - \deg(v)| = 0$ then D' satisfies equitable domination condition. Therefore $\gamma_e'(M_k) = \left\lceil \frac{k}{6} \right\rceil + 1$.

Theorem 3.3

If G is a Coxeter graph then $\gamma_e'(G) = 9$.

Proof

Let D= $\{v_1, v_2, v_3 \ldots v_7\}$ form the minimum equitable dominating set. Here $\langle V - D \rangle$is connected. We know that $\gamma_e(G) = 7$. Also there exists a subset $D' \subseteq V - D$ with $|D'| > |D|$ giving $|D'| = 9$. Therefore, there exists a vertex u $\in D'$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \le 1$. Hence $\gamma_e'(G) = 9$.

Theorem 3.4

Let G be the Cayley graph, then $\gamma_e'(G) = 3$.

Proof

We know that $\gamma_e(G) = 3$. Let D be the equitable dominating set. Let D$\subseteq V$. Here $\langle V - D \rangle$ is a connected graph. Suppose V is the set of all vertices and D⊆V is the equitable dominating set, then there exists another set D' which still dominates the vertices of V. For every $v \in V - D'$, there exists a vertex $u \in D'$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \le 1$. Thus D' is the inverse equitable dominating set of G with cardinality 3.(i.e) $\gamma_e'(G) = 3$. .

Observation 3.5

(i) Let G be a Cubic symmetric graph. Then $\gamma_e'(G) = 2$

(ii) Let G be a Doyle graph then $\gamma_e'(G) = 9$

(iii) Let G be a Octahedron graph then $\gamma_e'(G) = 2$

(iv) Let G be a Icosahedron graph then $\gamma_e'(G) = 2$

(v) For the Folkman graph, $\gamma_e'(G) = 6$

(vi) For the Frucht graph, $\gamma_e'(G) = 4$

(vii) For the Levigraph, $\gamma_e'(G) = {}^{n}/_{3}$.

3.6 Results

(i) For the Cocktail Party graph, then $\gamma(G) = \gamma_e(G) = \gamma_e'(G) = \gamma'(G)$

(ii) For the Musical graph, $\gamma(G) = \gamma_e(G) = \gamma_e'(G)$

(iii) For the Cayley graph, $\gamma(G) = \gamma_e(G) = \gamma_e'(G) = \gamma'(G)$

(iv)For the Cubic Symmetric graph, $\gamma(G) = \gamma_e(G) = \gamma_e'(G) = \gamma'(G)$

(v) If G is Coxeter graph, $\gamma_e'(G) > \gamma_e(G)$

(vi) For the Folkman graph, $\gamma(G) = \gamma_e(G) = \gamma_e'(G) = \gamma'(G)$

(vii) For the Frucht graph, $\gamma_e'(G) > \gamma_e(G)$

(viii) For the Octahedron graph, $\gamma(G) = \gamma_e(G) = \gamma_e'(G) = \gamma'(G)$

4.CONCLUSION

In this paper, we have established many numbers of results on equitable domination and inverse equitable domination of certain families of graphs.

5.REFERENCES

[1] Ameenal Bibi K, P.Rajakumari "*The Split Domination, Inverse Domination and Equitable Domination in the Middle and the Central graphs of the Path and the Cycle graphs*" International Journal of Mathematics Trends and Technology (IJMTT) – Volume 49 Number 3 September 2017.

[2] Ameenal Bibi K, and L.Jethruth Emelda Merry, " *A study on the domination number of circulant graphs*", Int. Journal of computational intelligence research.

[3] Arumugam.S and Ramachandran.S "*Invitation to Graph Theory*", Scitech publication (India Pvt. Ltd)-2010

[4] Bondy T.A and Murthy U.S.R "*Graph Theory with Applications*, London Macmillan- 1976".

[5] Garey M.R. and Johnson D.S "*Computer and Intractability*", W.H.Free Man & Co.1978.

[6] Godsil.C and Royle.G "The Andrasfai graphs", New York-Springer-Verking, Pp:118- 123,2001.

[7] Haynes T.W., S.T.Hedetniemi, P.J.Slater, "*Fundamentals of Domination in graphs*", Marcel Dekker, New York (1997).

[8] Haynes T.W, Hedetniemi S.T, Slater P.J, *Domination in graphs : Advanced Topics*, Marcel Dekker, New York (1997).

[9] Hemalatha G, Jeyanthi.P "*Results on domination number for some families of graphs*". IJPAM, volume 107, No.3 2016,565-577.

[10] Jayasree T.G & Radha.R Iyer "*Results on the inverse domination number of some graphs*", IJMET volume 9, Isssue, Jan 2018.

[11] V.Swaminathan and K.M.Dharmalingam, *Degree Equitable domination on graphs*, Kragujevak Journal of Mathematics, 35 (1) , 191-197 (2011)

[12] Wilfred.V, Balakrishnan.R, Sethuraman.G, Wilson Robin.J, "*On circulant Graphs" in graph theory and its applications* (Anna university, Chennai, March)2001, Alpha, Science, Pp:34-36.