PERFORMANCE OF LDPC CODES IN RAYLEIGH CHANNELS

[1]V. Bhanupriya, [2] Dr. C. Subhas
[1]M.Tech student, [2]Professor
[1] Department of ECE(CMS),
[1]Sree Vidyanikethan Engineering College, A. Rangampet, Near Tirupati, India

Abstract : Forward Error Correction (FEC) codes are used for error detection and correction in communication systems. Low Density Parity Check (LDPC) codes are used as powerful FEC codes in long distance communication systems which work close to the Shannon limit. Linear LDPC block codes will have sparse parity-check matrices. The decoding algorithm used for LDPC codes is an iterative message passing algorithm (MPA). LDPC codes are used to reduce bit rate error (BER). This paper discusses BER performance of LDPC codes using Binary Phase Shift Keying (BPSK) modulation with different combining techniques used for mitigating the fading effects in Rayleigh channels. Maximal Ratio Combining (MRC) provides excellent reduction in BER as compare to Selection Combining (SC) and Equal Gain Combining (EGC). The simulation results show that with LDPC codes the performance of diversity combining techniques has been improved when compared with uncoded diversity combining techniques. Finally, it is shown that the BER performance of LDPC codes in Rayleigh channels has improved dramatically in the higher SNR region.

***IndexTerms* - LDPC codes, diversity combining, BPSK modulation, Rayleigh fading channel, message passing algorithm.**

I. INTRODUCTION

Low Density Parity Check (LDPC) codes provide high coding gain with low bit error rate (BER). Like Turbo codes, LDPC codes work close to Shannon limit. But decoding complexity is high in Turbo codes than LDPC codes [1]. These codes decoded with Belief Propagation (BP) algorithm achieve the optimum performance when the channel is completely known at the receiver [1]. LDPC codes have certain advantages, such as simple descriptions of their code structure and fully parallelizable decoding implementations. With iterative message passing decoders, LDPC codes exhibit an interesting noise threshold effect, if the noise level of the channel is smaller than a certain noise threshold, the bit error probability goes to zero as the block size goes to infinity; if the noise level is above the noise threshold, the probability of error is always bounded away from zero. Gallager [2] first presented this result for regular LDPC codes for the binary symmetric channel (BSC). Luby et al. [3] showed that the noise threshold effect also exists for irregular LDPC codes, and they designed some irregular LDPC codes whose performance is very close to the Shannon limit on the erasure channel. Richardson, etal. [4] generalized this idea to a variety of message passing (BP) decoding algorithms, including its full version [5] which can be applied to a very broad class of BSCs. An estimation of the waterfall performance of LDPC codes is derived that reduces the gap between simulated and asymptotic performance [6]. In [7], irregular LDPC codes of larger lengths were analyzed in a flat uncorrelated as well as in a flat correlated Rayleigh fading channel. The channel model assumed in [7] was a slow memoryless frequency non-selective (i.e., flat) Rayleigh fading channel with a large coherence time. This effectively means that during the entire transmission the channel stays coherent.

The code optimization of irregular LDPC codes is a nonlinear cost function minimization problem, a problem where differential evolution has been shown to be effective [8]. In the year 1995, LDPC codes were rediscovered by Mackay at al. [9] whose performance is closest to the Shannon limit based on irregular graphs. Mackay at al. investigated constructions of regular and irregular Gallager codes that allow more rapid encoding and have smaller memory requirements in the encoder and found that these fast encoding Gallager codes have equally good performance.

In this paper, we evaluate the performance of LDPC codes with SC, EGC ad MRC techniques. The paper is organized as follows. Section II briefly reviews the basic concepts of LDPC codes. In Section III, we review the encoding of LDPC codes. We discuss the decoding of LDPC codes in Section IV. In Section V, we present the results on BER performance of LDPC codes in Rayleigh fading channel with and without combining techniques. Section VI concludes our work.

II. LOW DENSITY PARITY-CHECK CODES

LDPC is a linear block code in which the parity check matrix has sparse property. The number of 1s in the H matrix is very less compared to number of zeros. LDPC codes were first introduced by Gallager in 1962 [2]. But at that time because of the computational complexity, it was largely neglected. In the meantime, the field of forward error correction was dominated by highly structured algebraic block and convolutional codes. After the discovery of Turbo codes, the LDPC codes were eventually revisited by MacKay, Neal, Sipser and Spielman and Richardson and Urbanke. MacKay and Neal verified the performance of LDPC close to shannon limit [9]. Sipser and Spielman proved that with N tends to ∞ linear decoding complexity was sufficient to decode capacity approaching codes. The mathematical tool to estimate the performance of codes and to build capacity reaching codes was developed by Richardson and Urbanke [4]. The rediscovery of the LDPC gives a drastic change in error correction coding field. LDPC codes are block codes with parity-check matrices that contain only a very small number of non-zero entries

[10]. It is the sparseness [11] of H which guarantees both a decoding complexity which increases only linearly with the code length and a minimum distance which also increases linearly with the code length. LDPC codes are represented in two ways. One is matrix form of its H matrix and second is graphical form. The graphical representation of LDPC codes are known as the Tanner graph, which was introduced by Tanner [12]. Tanner graph is a bipartite graph, which means the graph is separated into two partitions. These partitions are called by different names: sub code nodes and digit nodes, variable nodes and check nodes, message nodes and check nodes [13]. The matrix representation and corresponding tanner graph of a sample parity check matrix is shown Fig.1.

Here C1, C2, ... C8 are the variable nodes and f1, f2, f3, f4 are the check nodes. Tanner is known as the originator of the codes based on graphs. The tanner graph [14] have an important role in the development of decoding algorithm of LDPC. In decoding of LDPC codes, iterative probabilistic decoding algorithms are widely used. McEliece have shown that these decoding techniques can be derived from Pearls belief propagation algorithm, or message passing algorithm [15].

The connection between the message node and check node is known as edge. The number of edges in the tanner graph equal to number of ones in parity check matrix. A code is said to be systematic, if and only if the message bit node can be distinguishable from the parity bit nodes by placing them on separate side of the graph [16]. A sequence of connected vertices which start and end at the same vertex in the graph and which contain other vertices no more than once is known as a cycle in a tanner graph. The number of the edges in a tanner graph gives the length of the cycle. Size of the smallest cycle is known as girth of a graph.

III. LDPC ENCODING

Let u be a message block, G is generator matrix, H is parity check matrix. parity-check matrix H can be found by performing Gauss-Jordan elimination on H to obtain it in the form

$$H = [A, I_{n-k}] \quad (1)$$

where A is a (n-k)Xk binary matrix and In-k is the size (n - k)identity matrix. The generator matrix is then

$$G = [I_k, A^k] \quad (2)$$

By applying proper elementary operation and convert H matrix into row reduce echelon from. This gives the sparse property for H matrix. Sparse property means number of 1s is less than the number of 0s. Sparse property make the LDPC less complex. Code word is generated by modulo 2 addition of message bits u with generator matrix G.

$$C = uG \quad (3)$$

C is code word, u is input message bits and G is generator matrix.

IV. LDPC DECODING

LDPC code decoding is performed through iterative processing based on the Tanner graph, to satisfy the parity check conditions. The condition that $CH^T = 0$ is known as parity check condition. If $CH^T = 0$ then the received code word is said to be valid, that is the received code word is similar to the transmitted code word. Iterative decoding has two variations namely hard decision and soft decision decoding algorithms. The decision made by the decoder based on the received information is called a hard-decision if the value of a single bit can either be 0 or 1. Example for hard decision decoding in LDPC is Bit Flipping Algorithm. If the decoder is able to distinguish between a set of quantized values between 0 and 1, then it is called a soft-decision decoder. These values give the probability of a particular bit in a node. The sum product algorithm is a soft decision message-passing algorithm.

A Message Passing Algorithm (MPA) [17] based on Pearls belief algorithm describes the iterative decoding steps. The reason for the name message passing algorithm is that at each round of the algorithms messages are passed from message nodes to check nodes, and from check nodes back to message nodes in the tanner graph. Different message-passing algorithms are named for the type of messages passed or for the type of operation performed at the nodes. In some algorithms, such as bit flipping decoding, the messages are binary and in others, such as belief propagation decoding, the messages are probabilities which represent a level of belief about the value of the code word bits [18]. It is often convenient to represent probability values as log likelihood ratios, and when this is done belief propagation decoding is often called sum-product decoding since the use of log likelihood ratios allows the calculations at the bit and check nodes to be computed using sum and product operations [19].

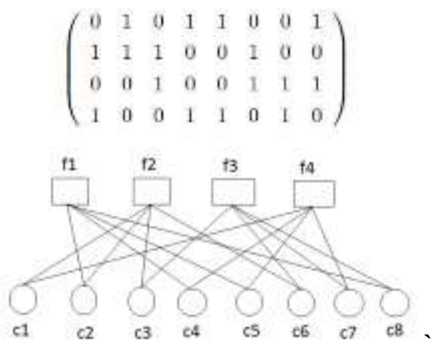

Figure 1: Tanner graph

Hard decision decoding

Bit flipping algorithm is the best example for hard Decision decoding. In the bit-flipping algorithm the messages are passed along the Tanner graph edges. A message node sends a message declaring if it is a one or a zero, and then each check node sends a message to each connected message node by finally declaring that what value the bit is based on the information available to the check node [20]. The algorithm is explained on the basis of an example code word $C = [11001000]^T$. Suppose that the received cord word is $Y = [11001000]^T$. So C2 was flipped. The figure1 shows the tanner graph which is used for decoding algorithm. The steps involved in the bit flipping algorithm is given in Tabel:1 and Tabel:2.

Soft decision decoding

Soft-decision decoding is based on the idea of belief propagation. The sum-product algorithm is a soft decision message passing algorithm. In the case of sum product decoding these probabilities are expressed as log-likelihood ratios. Log likelihood ratios (LLR) are used to represent the matrix for a binary variable by a single value. The sum-product algorithm is a soft decision message-passing algorithm [21] which is similar to the bit-flipping algorithm. The main difference between SPA and bit flipping algorithm is that each decision is represented with probabilities of the information bits.

$$P_i = P_r(C_i = Y_i)$$

q_{ji} is a message sent by the message node C_i to the check node f_j. Every message contains always the pair $q_{ij}(0)$ and $q_{ij}(1)$ which stands for the amount of belief that Y_i is a 0 or 1.

r_{ji} is a message sent by the check node f_j to the variable node C_i. Again there is a $r_{ji}(0)$ and $r_{ji}(1)$ that indicates the (current) amount of belief that Y_i is a 0 or 1.

The steps involved in sum product algorithm

Step 1: All message nodes send their q_{ij} messages.

$$q_{ij}(1) = P_i \text{ and } q_{ij}(0) = 1 - P_i$$

Step 2: the check nodes calculate their response messages r_{ji},using the formula shown below (4)

$$r_{ji}(0) = \frac{1}{2} + \frac{1}{2}\prod_{i \in \pi\upsilon_{j/i}} \left(1 - 2q_{ij}(1)\right) \qquad (4)$$

and

$$r_{ji}(1) = 1 - r_{ji}(0) \qquad (5)$$

Step 3: The message nodes update their response message to the check nodes using

$$q_{ji}(0) = k_{ij}(1 - p_i)\prod_{j' \in \frac{C_i}{j}} \left(r_{j'i}(0)\right) \qquad (6)$$

And

Table 1: Overview of messages received and sent by the check nodes

Check nodes	Activities				
f_1	Receive	$C_2 \to 0$	$C_4 \to 0$	$C_5 \to 0$	$C_8 \to 0$
	Send	$1 \to C_2$	$1 \to C_4$	$1 \to C_5$	$1 \to C_8$
f_2	Receive	$C_1 \to 0$	$C_2 \to 0$	$C_3 \to 0$	$C_6 \to 0$
	Send	$1 \to C_1$	$1 \to C_2$	$1 \to C_3$	$1 \to C_6$
f_3	Receive	$C_3 \to 0$	$C_6 \to 0$	$C_7 \to 0$	$C_8 \to 0$
	Send	$1 \to C_3$	$1 \to C_6$	$1 \to C_7$	$1 \to C_8$
f_4	Receive	$C_1 \to 0$	$C_4 \to 0$	$C_5 \to 0$	$C_7 \to 0$
	Send	$1 \to C_1$	$1 \to C_4$	$1 \to C_5$	$1 \to C_7$

Table 2: Message nodes decisions for hard decision decoder

Message nodes	Y_i	Message from check node		Decision
C_1	1	$f_2 \to 0$	$f_4 \to 1$	1
C_2	0	$f_2 \to 1$	$f_1 \to 1$	1
C_3	0	$f_2 \to 1$	$f_3 \to 0$	0
C_4	0	$f_1 \to 1$	$f_4 \to 0$	0
C_5	1	$f_1 \to 0$	$f_4 \to 1$	1
C_6	0	$f_2 \to 1$	$f_3 \to 0$	0
C_7	0	$f_3 \to 0$	$f_4 \to 0$	0
C_8	0	$f_1 \to 1$	$f_3 \to 0$	0

$$q_{ji}(1) = k_{ij}(p_i) \prod_{j' \in C_i / j} \left(r_{j'i}(1)\right) \qquad (7)$$

Constant k_{ij} are chosen in such a way to ensure that

$$q_{ij}(0) + q_{ij}(1) = 1 \qquad (8)$$

At this point the message nodes also update their current estimation C_i' of their C_i. This is done by calculating the probabilities for 0 and 1 and voting for the bigger one.

$$Q_i(0) = k_i(1 - p_i) \prod_{j \in C_i} \left(r_{ji}(0)\right) \qquad (9)$$

and

$$Q_i(1) = k_i(p_i) \prod_{j \in C_i} \left(r_{ji}(1)\right) \qquad (10)$$

Step 4: C_i is 1 if $Q_i(1) > Q_i(0)$. Otherwise C_i is zero.

Go to step 2.

SPA algorithm provides low complexity in log domain than in the probability domain. Using log ratio turns multiplication into additions. This makes algorithm simpler.

V. RESULTS AND DISCUSSION

The simulations were carried with the parameters shown in Table 3.

The simulation results on BER performance of LDPC codes with selection combining for various values of E_b/N_0 are tabulated in Table:4.

Table 3: Parameters

Parameter	Value
Number of input data	1000 bits
Modulation technique	BPSK
Channel model	Rayleigh fading
Diversity combining	SC, MRC, EGC
FEC coding	LDPC codes

Performance evaluation of LDPC codes with Selection Combining:

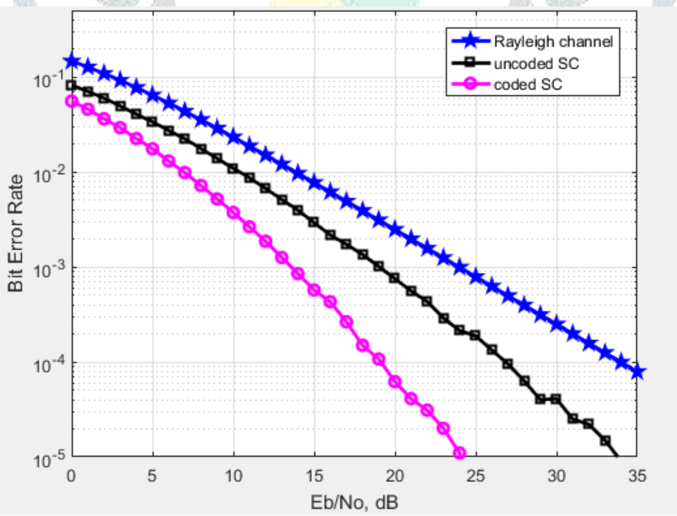

Figure 2: BER performance of LDPC codes with Selection Combining in Rayleigh Channel

Table 4: BER performance of LDPC codes with Selection Combining at different E_b/N_0 values

E_b/N_0	BER for uncoded SC	BER for coded SC
4	$4.11 * 10^{-2}$	$2.267 * 10^{-2}$
5	$3.335 * 10^{-2}$	$1.75 * 10^{-2}$
7	$2.194 * 10^{-2}$	$9.639 * 10^{-3}$
9	$1.409 * 10^{-2}$	$5.226 * 10^{-3}$

The simulation results on BER performance of LDPC codes with Equal Gain Combining for various values of E_b/N_0 are tabulated in Table:5.

Performance evaluation of LDPC codes with Equal Gain Combining:

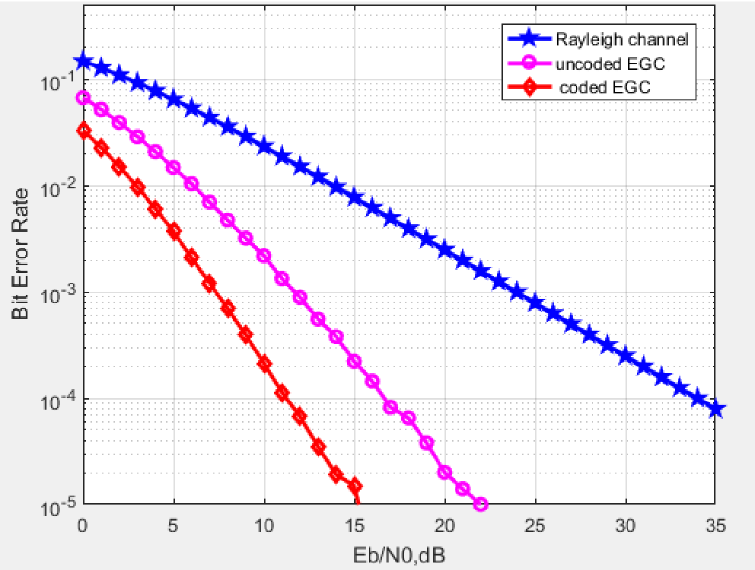

Figure 3: BER performance of LDPC codes with Equal Gain Combining in Rayleigh channel

Table 5: BER performance of LDPC codes with Equal Gain Combining at different E_b/N_0 values

E_b/N_0	BER for uncoded EGC	BER for coded EGC
4	$2.072 * 10^{-2}$	$6.183 * 10^{-3}$
5	$1.461 * 10^{-2}$	$3.635 * 10^{-3}$
7	$6.824 * 10^{-3}$	$1.237 * 10^{-3}$
9	$3.034 * 10^{-3}$	$3.91 * 10^{-4}$

Performance evaluation of Maximal Ratio Combining with LDPC codes:

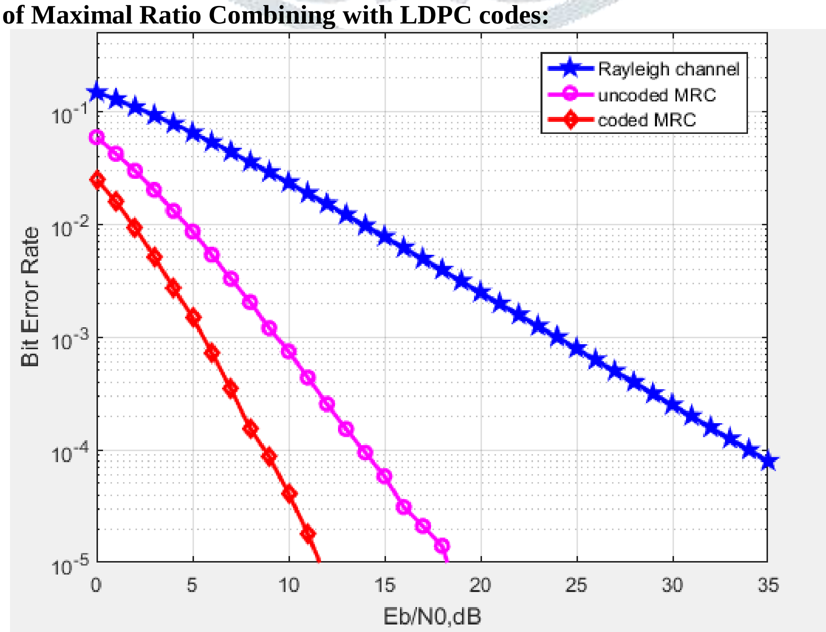

Figure 4: BER performance of LDPC codes with Maximal Ratio Combining in Rayleigh channel

The simulation results on BER performance of LDPC codes with Maximal Ratio Combining for various values of E_b/N_0 are tabulated in Table:6.

The simulation results on BER performance of LDPC codes with various combining techniques at E_b/N_0 value of 10 dB are tabulated in Table:7.

Table 6: BER performance of LDPC codes with Maximal Ratio Combining at different E_b/N_0 values

E_b/N_0	BER for uncoded MRC	BER for coded MRC
4	$1.327 * 10^{-2}$	$2.855 * 10^{-3}$
5	$8.416 * 10^{-3}$	$1.505 * 10^{-3}$
7	$3.402 * 10^{-3}$	$3.9 * 10^{-4}$
9	$1.279 * 10^{-3}$	$8.2e^{-05}$

Table 7: The simulation results on BER performance of LDPC codes with various combining techniques at Eb/N_0 value of 10 dB

At 10 dB E_b/N_0		
Diversity combining techniques	Without LDPC	With LDPC
SC	$1.089 * 10^{-2}$	$3.72 * 10^{-3}$
EGC	$2.065 * 10^{-3}$	$2.03 * 10^{-4}$
MRC	$7.82 * 10^{-4}$	$3.3e^{-2}$

VI. CONCLUSION

Diversity combining techniques such as Selection Combining(SC), Maximal Ratio Combining(MRC) and Equal Gain Combining(EGC) are very important part of the wireless communication system. They are used to mitigate the effect of fading. LDPC codes are the most powerful error control codes. Their performance is within 0.2dB of Shannon capacity. In this paper, BER performance of LDPC codes is evaluated with various combining techniques over the independent and identically distributed Rayleigh fading environment with BPSK modulation. It is concluded that the BER performance of LDPC codes with diversity combining techniques is superior to uncoded diversity combining techniques. More specifically, performance of LDPC codes with MRC is the best.

REFERENCES

[1] R.G.Gallager, “Low-Density Parity-Check Codes”, IRE Trans. Information Theory, pp. 21-28, January 1962.

[2] David J.C. MacKay, “Good Error-Correcting Codes Based on Very Sparse Matrices”, IEEE Trans. Information Theory, vol.45, no.2, pp. 399-431, March 1999.

[3] Beng Soon Tan, Kwok Hung Li and Kah Chan Teh, “Efficient BER Computation of LDPC Coded SC/MRC Systems Over Rayleigh Fading’’, IEEE Trans. Communications, 2010.

[4] Kwok Hung Li, Beng Soon Tan and Kah Chan Teh, “Performance Analysis of LDPC Codes with Maximum-Ratio Combining Cascaded with Selection Combining over Nakagami-m Fading”, IEEE Trans. Wireless Communications, vol.10, no.6, pp. 1886-1894, June 2011.

[5] D.G.Brennan, “Linear Diversity Combining Techniques”, Proc.IRE, vol.47, pp.1075-1102,June 1959.

[6] M.S.Alouini and M.K.Simon, “An MGF-Based Performance Analysis of Generalized Selection Combining over Rayleigh Fading Channels”, IEEE Trans. Communications, vol.48, no. 3, pp. 401-415, March 2000.

[7] J. Hou, P. H. Siegel, and L. B. Milstein, “Performance analysis and code optimization of low density parity-check codes on Rayleigh fading channels,” in IEEE Journal on Selected Areas in Communications, no. 15, May 2001, pp. 924–934.

[8] Yibo Jiang, Alexei Ashikhmin and Naresh Sharma, “LDPC Codes for Flat Rayleigh Fading Channels with Channel Side Information”, IEEE Trans. Communications, Vol. 56, No.8, pp. 1207-1213, August 2008.

[9] Rinu Jose, “Analysis of Hard Decision and Soft Decision Decoding Algorithms of LDPC Codes in AWGN “, IEEE Trans. Information Theory, 2015.

[10] Nauman F. Kiyani, “Analysis of Random Regular LDPC Codes on Rayleigh Fading Channels”, IEEE Trans. Communications, pp. 69-76, 2001.

[11] Zhen Mei, Martin Johnston, “Performance Analysis of LDPC-Coded Diversity Combining on Rayleigh Fading Channels with Impulsive Noise”, IEEE Trans. Communications, 2016.

[12] Alberto Benvegn, Samuele Bandi and Velio Tralli, “On the performance and the optimization of LDPC Codes on Fading Channels with Imperfect CSI”, IEEE Trans. Communications, 2008.

[13] Joseph J. Boutros, “Design and Analysis of Low-Density Parity-Check Codes for Block-Fading Channels”, IEEE Trans. Communications, 2013.

[14] A. Sudhir Babu, “Evaluation of BER for AWGN, Rayleigh and Rician Fading Channels under Various Modulation Schemes”, IJCA, Vol. 26, No.9, July 2011.

[15] Ashwin Pananjady, “Generation of LDPC codes and Analysis of Decoders for Various Channels”, IEEE Transactions communications, vol. 47, no. 2, pp. 619–637, 2001.

[16] Joydev Ghosh, Abhishek Dey and Subham Bachhar, “Performance Analysis by Improving Bit Error Rate (BER) through various Diversity Techniques in Wireless Communication”, International Journal of Computer Applications (0975 – 8887) Vol. 95, No. 9, June 2014.

[17] Aleksandar, “An Overview and Analysis of BER for Three Diversity Techniques in Wireless Communication systems”, yugoslav journal of operations research, no. 2, pp. 251-269, 2015.

[18] Er. Sonia, Er. Swati Gupta, “Hard Decision and Soft Decision Decoding Algorithms for LDPC and QC-LDPC Codes”, IJCSMC, Vol.4, pg. 182-191, September 2015.

[19] M. Fossorier and N. Miladinovic, “Irregular LDPC codes as concatenation regular LDPC codes,” IEEE Commun. Lett., vol. 9, pp. 628-630, July 2005.

[20] Bernhard M.J. Leiner, LDPC Codes A Brief Tutorial, IEEE Trans. Information Theory, April 2005.

[21] S. Niranjayan and N. C. Beaulieu, “The BER Optimal Linear Rake Receiver for signal Detection in Symmetric Alpha-Stable Noise”, IEEE Trans. Communications, vol. 57, no. 12, pp. 3585-3588, December 20.