CONSTRUCTION OF NON-PARAMETRIC CONTROL CHART FOR MEAN USING GROUPED SIGNED RANK

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Abstract: ***Control charts play a crucial role in detecting whether a process is in-control or out-of-control. The standard Shewhart-type control charts (Shewhart, 1931) are based on inspecting samples at equally spaced time intervals and issuing an alarm (a signal) if the "result of the sample" is considerably worse (i.e. larger or smaller) than what one can expect if the process was operating on target. In this article we construct the non- parametric control limits for mean based on grouped signed rank with process capability for the data on balance with the presently available control limits.***

Key words: Control chart, Grouped signed rank, Non-parametric test and process capability.

I. INTRODUCTION

A shortcoming of a Shewhart control chart is that its conclusion is merely based on the present sample which means that it pays no attention to the past data resulting into a relatively bad performance for small disturbances in the process (Montgomery, 2013). Due to this feature these charts are more efficient to detect small and moderate shifts. The present study is based on providing new memory control charting techniques (by modifying the existing structures and also by designing some new structures) that perform relatively better than the existing ones, especially for small and moderate shifts in the process parameters. In the article, we provide the detailed structures of non-parametric mean chart for monitoring the process parameters with an example.

II. CONCEPTS AND TERMINOLOGIES

a. Upper specification limit (USL)

It is the greatest amount specified by the producer for a process or product to have the acceptable performance.

b. Lower specification limit (LSL)

It is the smallest amount specified by the producer for a process or product to have the acceptable performance.

c. Tolerance level (TL)

It is a statistical interval within which, with some confidence level, a specified proportion of a sampled population falls. It is the difference between USL and LSL, TL = USL-LSL

d. Process capability (CP)

Process capability compares the output of an in-control process to the specification limits by using capability indices (Gupta and Kapoor, 2001). The comparison is made by forming the ratio of the spread between the process specifications to the spread of the process values, as measured by 6 process standard deviation units.i. e. $C_p = \frac{TL}{6\sigma} = \frac{USL-LSL}{6\sigma}$.

III. METHODS AND MATERIALS

A mean control scheme (for individual observations) is easy to implement and interpret with upper control limits (UCL) and lower control limits (LCL).

Grouped Signed Rank (GSR) is a nonparametric procedure based on the Wilcoxon-Signed-Rank statistic which was introduced by Wilcoxon (1945). The SRt , t=1,2,… represents the signed-rank statistic for the t^{th} sample of g sequentially recorded observations. The starting value, Z0, is often taken to be the process target value, although other starting value can be used (Saccucci et. al., 1989).

IV. CONSTRUCTION OF NON-PARAMETRIC CONTROL CHART FOR MEAN

A shoe company wants to know if three groups of workers have different salaries (in Rs.):
Women : 23K, 41K, 54K, 66K, 78K.
Men : 45K, 55K, 60K, 70K, 72K
Minorities : 18K, 30K, 34K, 40K, 44K.

a. Construction of Shewhart control chart for Mean

The 3σ control limits suggested by Shewhart (1931) are $\bar{\bar{X}} \pm 3\left(\sigma/\sqrt{n}\right)$

$$UCL_{\bar{\bar{X}}} = \bar{\bar{X}} + 3\left(\sigma/\sqrt{n}\right) = 49 + \left(\frac{3 \times 14}{\sqrt{3}}\right) = 73$$

$$CL_{\bar{\bar{X}}} = \bar{\bar{X}} = 49$$

$$UCL_{\bar{\bar{X}}} = \bar{\bar{X}} + 3\left(\sigma/\sqrt{n}\right) = 49 - \left(\frac{3 \times 14}{\sqrt{3}}\right) = 24$$

and the control limit interval

$$CLI_{\bar{\bar{X}}} = \left(6\sigma/\sqrt{n}\right) = \left(\frac{6 \times 14}{\sqrt{3}}\right) = 48.50$$

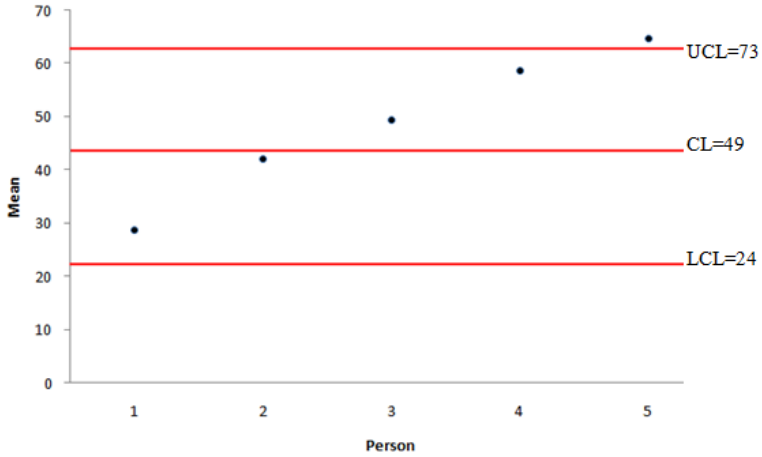

Figure 1: Construction of control chart for mean

From the result, it is clear that the process is in control with the control limit interval is 48.50 for n=3.

b. Construction of non parametric control chart for mean based on grouped signed rank

The control limits based on grouped signed rank are $\bar{\bar{X}}_{GSR} \pm 3\left(\sigma_{GSR}/\sqrt{n}\right)$

$$UCL_{GSR} = \bar{\bar{X}}_{GSR} + 3\left(\sigma_{GSR}/\sqrt{n}\right) = 40 + 3\left(18.33/\sqrt{3}\right) = 72$$

$$CL_{GSR} = \bar{\bar{X}}_{GSR} = 40$$

$$LCL_{GSR} = \bar{\bar{X}}_{GSR} - 3\left(\sigma_{GSR}/\sqrt{n}\right) = 40 - 3\left(18.33/\sqrt{3}\right) = 8$$

and the control limit interval

$$CLI_{GSR} = \left(6\sigma/\sqrt{n}\right) = \left(\frac{6 \times 18.33}{\sqrt{3}}\right) = 63.50$$

From the result, it is clear that the process is in control and the control limit interval is 63.50 for n=3.

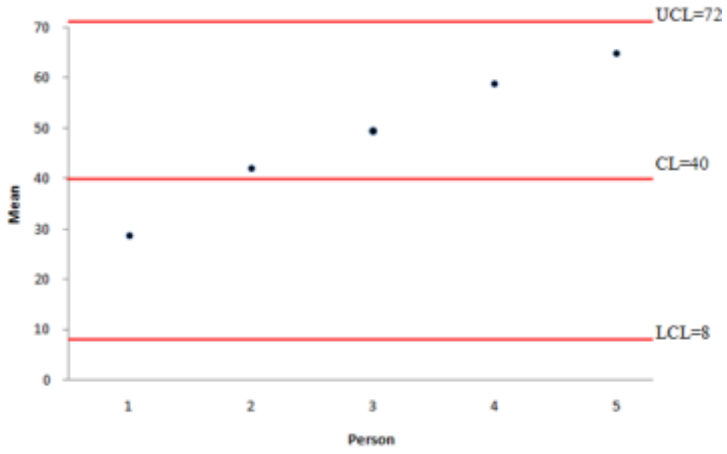

Figure 2: Construction of non parametric control chart for mean based on grouped signed rank

c. Construction of non parametric control chart for mean based on grouped signed rank using process capability

From the given data the difference between upper specification and lower specification limits is 36, which termed as tolerance level (TL) and choose the process capability (Radhakrishnan and Balamurugan, 2012) is 2.0, it is found that the value of σ_{GSR} is 3.00. The non-parametric control limits of using process capability for mean as follows:

$$UCL_{Cp} = \bar{\bar{X}}_{Cp} + 3\left(\sigma_{Cp}/\sqrt{n}\right) = 40 + 3\left(3/\sqrt{3}\right) = 45$$

$$CL_{Cp} = \bar{\bar{X}}_{Cp} = 40$$

$$LCL_{Cp} = \bar{\bar{X}}_{Cp} - 3\left(\sigma_{Cp}/\sqrt{n}\right) = 40 - 3\left(3/\sqrt{3}\right) = 35$$

and the control limit interval

$$CLI_{Cp} = \left(6\sigma/\sqrt{n}\right) = \left(\frac{6 \times 3}{\sqrt{3}}\right) = 10.39$$

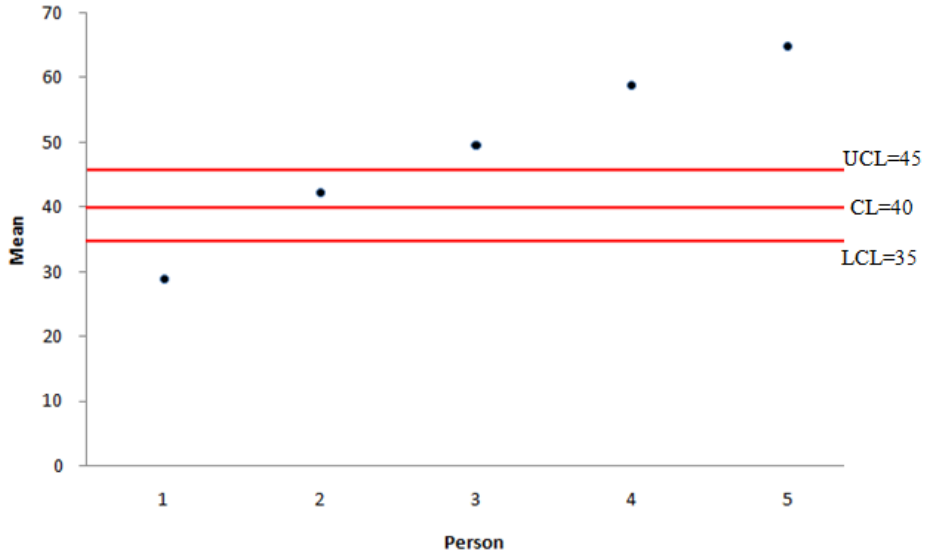

Figure 3: Construction of non parametric control chart for mean based on grouped signed rank using process capability

From the result, it is clear that the process is out of control, since the person numbers 4 and 5 go above the upper control limit and the person number 1 goes below the lower control limit with the control limit interval is 10.39 for n=3.

V. CONCLUSION

Our results make a strong case for using nonparametric charts in practice, particularly with the computing resources available today. We believe that Shewhart control chart research has not received enough attention, since the vast majority of the statistical process control literature concerns GSR non-parametric control charting techniques (Nanthakumar et.al 2015). Thus further work on nonparametric control chart based on grouped signed rank using process capability would be welcome. More research needs to be done regarding GSR using process capability analysis for small subgroup sizes and even for individual observations because, although traditional statistical process control applications of control charts involve sub-grouped data, recent advances have led to more and more instances where individual measurements are collected over time.

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