APPLICATION OF FUZZY VERTEX COLOURING OF FUZZY GRAPH IN STORAGE PROBLEM

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Abstract:

Fuzzy Colouring of fuzzy graph has several applications.In this paper, we have described a new concept of solving storage problem using fuzzy vertex colouring.

Keywords:

Fuzzy graph,Fuzzycolouring, Fuzzy chromatic number.

1.1 INTRODUCTION

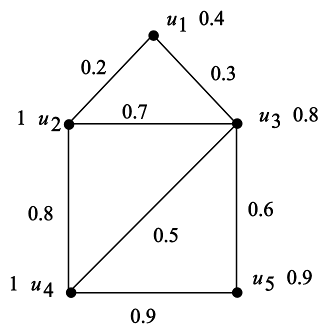
The fuzzy colouring is used to solve problems of combinatorial optimizations like traffic control,Exam scheduling, Register allocationetc.RakeshJaiswal,ShwetaRai [13] who have proposed a paper on research level journal about the topic on application of fuzzy graph coloring in traffic light problem.

Here all the graphs considered are simple, finite, undirected and connected graph .This paper deals with an application on fuzzy colouring of fuzzy simple graph using storage problem. It is used to partition the incompatible chemicals based on their exploding property and toxic gas formation and also to store incompatible chemicals in different compartments for precautionary measure safety. The storage problem is referred briefly in the book "Graph theory with applications" written by J.A.Bondy and U.S.R.Murty[4].

Definition 1.1

Let *V* be a finite non empty set. The triple $G=(V,\sigma,\mu)$ is called a **fuzzy graph** on *V* where σ and μ are fuzzy sets on *V* and *E* respectively such that $\mu(u,v)\leq\sigma(u)\Lambda\sigma(v)$ for all $u,v\in V$ and (u,v)€E. For fuzzy $G=(V,\sigma,\mu)$, the elements *V* and *E* are called set of vertices and set of edges of *G* respectively.

Example 1.1



G

Definition 1.2

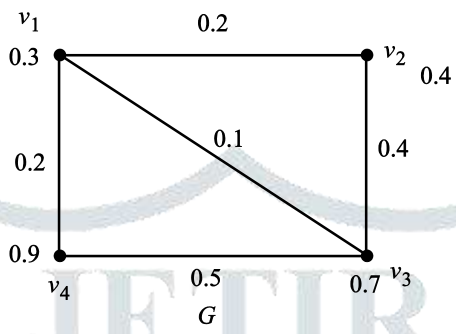
A family $\Gamma = \{\gamma_1, \gamma_2, ..., \gamma_k\}$ of fuzzy sets on *V* is called a ***k*-fuzzy coloring** of $G = (V, \sigma, \mu)$ if

a) $\vee\Gamma = \sigma$

b) $\gamma_i \wedge \gamma_j = 0$

c) For every strong edge (u, v) of G, (i.e.) $(\mu((u, v) > 0))$ $\min(\gamma_i(u) \wedge \gamma_i(v)) = 0$ for $1 \le i \le k$.

Example1.2



The fuzzy colouring is $\Gamma = \{\gamma_1, \gamma_2, \gamma_3\}$

$$\gamma_1(v_i) = \begin{cases} 0.3 & i = 1 \\ 0 & \text{otherwise} \end{cases}$$

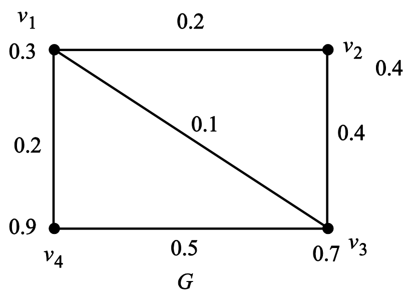
$$\gamma_2(v_i) = \begin{cases} 0.9 & i = 2 \\ 0.4 & i = 4 \\ 0 & \text{otherwise} \end{cases}$$

$$\gamma_3(v_i) = \begin{cases} 0.7 & i = 3 \\ 0 & \text{otherwise} \end{cases}$$

Definition 1.3

The minimum number *k* for which there exists a *k*-fuzzy colouring is called the fuzzy **chromatic number** of *G*, denoted as $\chi^f(G)$.

Example 1.4



$$\chi^f(G) = 3$$

Definition 1.5:

The chromatic number of fuzzy graph $G=(V,\sigma,\mu)$ is defined as $\chi(G)$ $\chi(G)=\max\{(\chi_\alpha / \alpha \in L)\}$ where $\chi_\alpha = \chi(G_\alpha)$.

2.PRELIMINARIES

2.1Storage Problem:

A company manufacturers *n*chemicals $C_1, C_2, ..., C_n$. Certain pairs of these chemicals are incompatible and would cause explosions if brought into contact with each other. As a precautionary measure the company wishes to partition its warehouse into compartments and store incompatible chemicals in different compartments.

We obtained a graph *G* on the vertex set $\{v_1, v_2, ..., v_n\}$ by joining two vertices v_i and v_j if and only if the chemicals C_i and C_j are incompatible. It is easy to see that the least number of compartments in which the warehouse should be partitioned is equal to the chromatic number of *G*.

Unfortunately no good algorithm is known for determining the chromatic number. Here we described a systematic

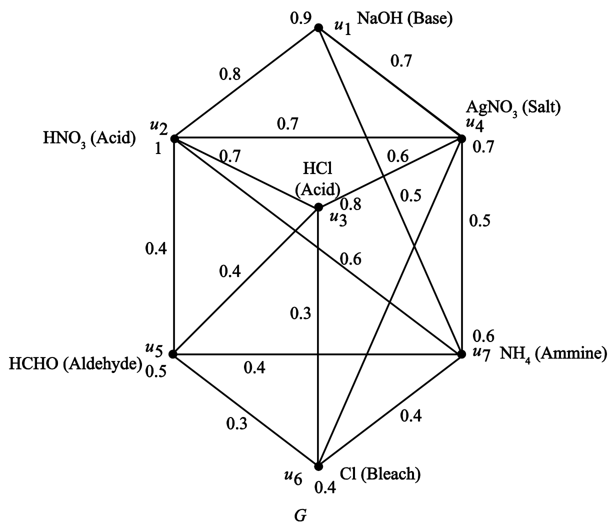
2.2:Storage of Incompatible Chemicals in Warehouse

Procedure to solve storage problem:

- **FindingMinimum covering**
- **Algebraic device**
- **Finding Maximal independent set**
- **Fuzzy Vertex colouring**
- **Canonical form**

procedure which is basically enumerative in nature. It is not very efficient for larger graphs.

Now we obtained a fuzzy simple graph $G=(V,\sigma,\mu)$ on the vertex set $\sigma(u_i)$ and edge set $\mu(u_i,u_j)\ \forall\ i=1,2,...n;\ j=1,2,...n$ by joining two vertices $\sigma(u_i)$ and $\sigma(v_j)$ iff the chemicals C_i and C_j are incompatible and we considered fuzzy set whose membership value depends on chemical exploding and toxic gas formation on chemical based values are assigned toeach edges in *G*.Here we applied fuzzy vertex colouring in storage problem and found the fuzzy chromatic number of a fuzzy simple graph.



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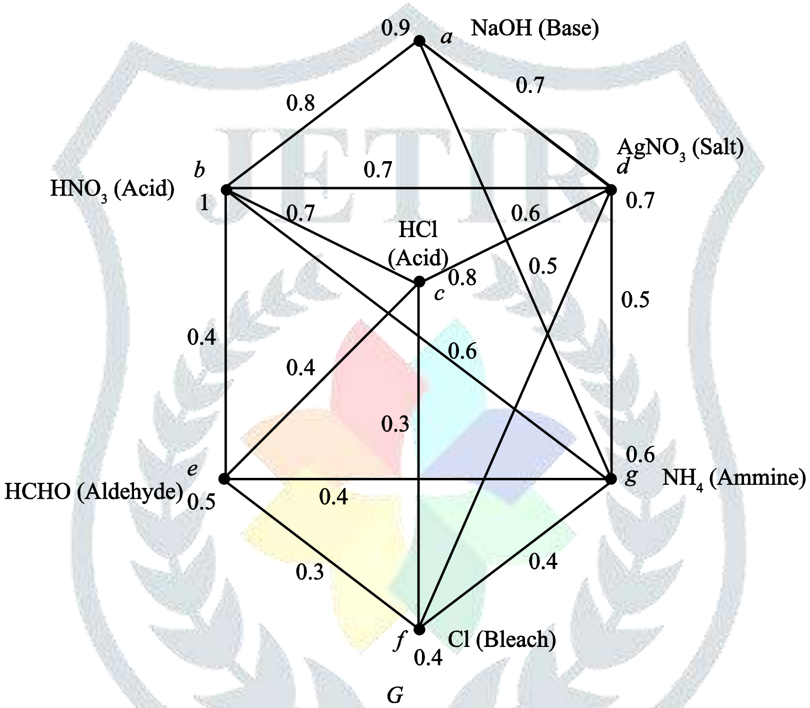
In the above figure, the vertices are $\sigma(u_1)=0.9,\ \sigma(u_2)=1,\ \sigma(u_3)=0.8,\ \sigma(u_4)=0.7,$ $\sigma(u_5)=0.5,\ \sigma(u_6)=0.4,\ \sigma(u_7)=0.6$ and the edge sets are $\mu(u_1,u_2)=0.8,$ $\mu(u_1,u_4)=0.7,\ \mu(u_2,u_4)=0.7,\ \mu(u_2,u_3)=0.7,\ \mu(u_3,u_4)=0.6,\ \mu(u_3,u_5)=0.4,\ \mu(u_2,u_5)=0.4,$ $\mu(u_2,u_7)=0.6,\ \mu(u_3,u_6)=0.3,\ \mu(u_4,u_7)=0.5,\ \mu(u_4,u_6)=0.3,\ \mu(u_6,u_7)=0.4,\ \mu(u_5,u_6)=0.3,$ $\mu(u_7,u_1)=0.5$.

we joined two vertices $\sigma(u_i)$ and $\sigma(u_j)$ if the chemicals C_i and C_j are incompatible.and C_i and C_j are compatible if $\sigma(u_i)$ and $\sigma(u_j)$ are non adjacent.

Every chemical is taken as the vertex of thefuzzy graph.For our convenience label the vertices as $\{a,b,c,d,e,f,g\}$.



This graph shows incompatible chemicals formation.

Here the vertex '*a*' and its corresponding chemical is NaOHwhich reacts with the verticesb,d and g results in either explosive or toxic gas formation.

ie). *a* is adjacent to b,d,g .

Similarly we get

$b(HNO_3)$ reacts with the vertices (d,c,g,e) ie) *b*is adjacent to (d,c,g,e).

$e(HCHO)$ reacts with the vertices (b,c,g,f) ie)*e is* adjacent to (b,c,g,f)

$f(Cl)$ reacts with the vertices (e,c,g,d) ie) *f is* adjacent to (e,c,g,d)

$g(NH_4)$ reacts with (f,e,b,a,d) ie)*g is* adjacent to (f,e,b,a,d)

But certain pairs of these chemicals are compatible if the vertices are not adjacent. These compatible chemicals form an Independent set .Since the chromatic number of a graph is the least number of Independent sets into which its vertex set can be partitioned.

3. VERTEX COLOURING IN STORAGE PROBLEM

We listed all the independent sets in a graph because every independent set is a subset of a maximal independent set. First, determine complement of a maximal independent set which is a minimal covering. Observe that a subset *K* of *V* is a minimal covering of *G* iff for each vertex *v*, either *v* belongs to *k* or all the neighbours of *v* belong to *k* (but not both). This provides us with a procedure for finding minimal covering: For each vertex *v*, choose either *v* or all the neighbours of *v* ………… (1).To implement this procedure effectively we make use of an algebraic device and logical product, logical sum. We used these two operations to simplify logical expressions.

$$a*a=a$$
$$a+a=a \qquad (2)$$
$$a+(b*a)=a$$

Consider the fuzzy simple graph *G* of figure given below. Our objective (1) is to find the minimal coverings of*G*

$$(a+bdg)(b+adcge)(c+bdef)(d+abcfg)(e+bcgf)(f+egdc)(g+adebf) \qquad (3)$$

$$(a+bdg)(b+adcge)=(ab+aadcge+bbdg+abddcgge)$$

$$=(ab+adcge+bdg+abdcge)$$

$$=(ab+bdg+(adcge+adcge*b))$$

$$=(ab+bdg+adcge) \qquad \text{(by 2)}$$

$$(ab+bdg+adcge)(c+bdef)=(acb+bcdg+adccge+abbdef+bbddefg+abddcgeef)$$

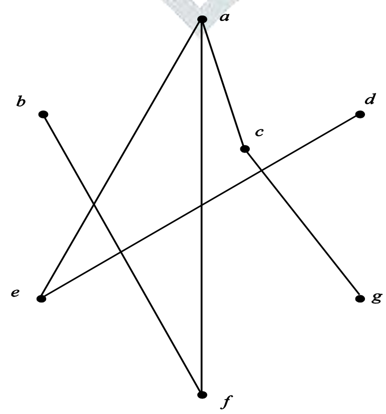
$$=(acb+bcdg+adcge+abdef+bdefg+abdcgef)$$

$$=(acb+bcdg+adcge+abdef+(bdefg+ac*bdefg)) \text{(by 2)}$$

$$=(acb+bcdg+adcge+abdef+bdefg)$$

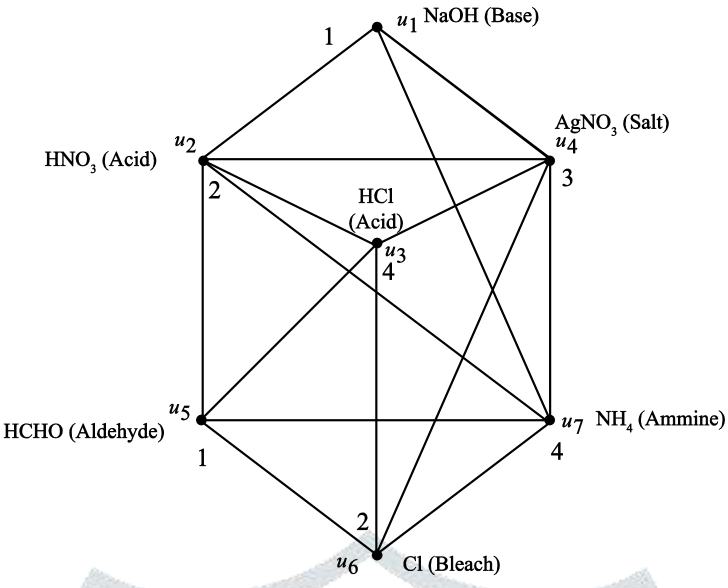
By simplification, Equation (3) reduced to $bcdeg+adcge+aebdf+bdefg+abcfg+bcdfg$.

Thus $\{bcdeg\},\{adcge\},\{aebdf\},\{bdefg\},\{abcfg\},\{bcdfg\}$ are the minimal coverings of *G*. On complementation. we obtained the list of all maximal independent sets of $G:\{af\}\{bf\}\{cg\}\{ac\}\{de\}\{ae\}$ .It is shown in the figure below.-



$\overline{G}$

Now consider the problem of determining the fuzzy chromatic number and chromatic number of the above graph. Let us colour the vertices by using proper vertex colouring.



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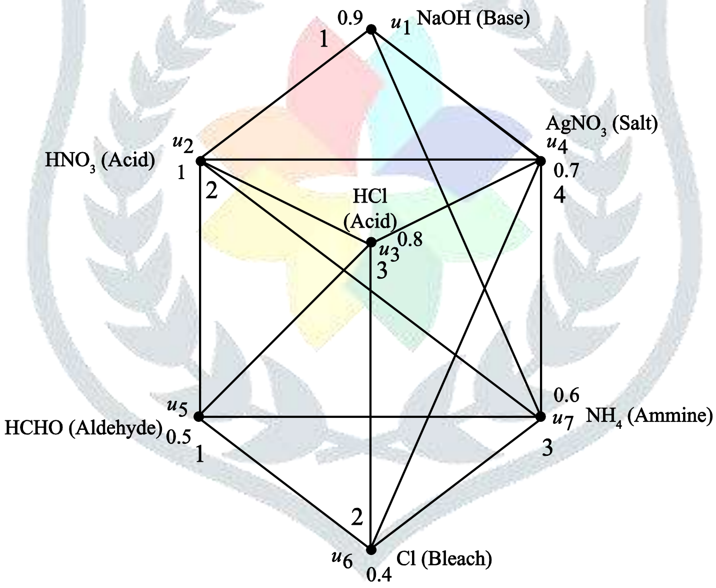
The chromatic number is $\chi(G)=4$.

Now let us colour the vertices by using fuzzy vertex colouring.



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The fuzzy colouring is $\Gamma=\{\gamma_1,\gamma_2,\gamma_3,\gamma_4\}$

$$\gamma_1(u_i)=\begin{cases}0.9 & i=1\\ 0.5 & i=5\\ 0 & \text{otherwise}\end{cases}$$

$$\gamma_2(u_i)=\begin{cases}1 & i=2\\ 0.4 & i=6\\ 0 & \text{otherwise}\end{cases}$$

$$\gamma_3(u_i)=\begin{cases}0.8 & i=3\\ 0.6 & i=7\\ 0 & \text{otherwise}\end{cases}$$

$$\gamma_4(u_i)=\begin{cases}0.7 & i=4\\ 0 & \text{otherwise}\end{cases}$$

Here $\Gamma=\{\gamma_1,\gamma_2,\gamma_3,\gamma_4\}$

$\gamma_1(u_i)=\{(u_1,0.9)\cup(u_5,0.5)\}$

$=\{(u_1,0.9),(u_5,0.5)\}$

Hence $\gamma_1^*=(u_1,u_5)$

$\gamma_2(u_i)=\{(u_2,1)\cup(u_6,0.4)\}$

$=\{(u_2,1),(u_6,0.4)\}$

$\gamma_2^*=(u_2,u_6)$

$\gamma_3(u_i)=\{(u_3,0.8)\cup(u_7,0.6)\}$

$=\{(u_3,0.8),(u_7,0.6)\}$

$\gamma_3^*=(u_3,u_7)$

$\gamma_4(u_i)=\{(u_4,0.7)\cup(u_5)\}$

$\gamma_4^*=(u_7)$

Hence $S=\{u_1,u_5\}\{u_2,u_6\}\{u_3,u_7\}$. Here we assigned colours C_1,C_2,C_3 to γ_1^*,γ_2^* and γ_3^* respectively. Since $|S|=1$ there is only one colour required to colour $G=\langle S\rangle$ .Hence $\Gamma'=\{\gamma\}$ where $\gamma=\{u_4,0.7\}$.

The fuzzy colouring of fuzzy graph G is shown in the table below.

Vertices	γ_1	γ_2	γ_3	γ_4	$\wedge\gamma=\sigma$ (**max** γ_i)	$\gamma_i\wedge\gamma_j$ $i,j=1,2,3,4$
u_1	0.9	0	0	0	0.9	0
u_2	0	1	0	0	1	0
u_3	0	0	0.8	0	0.8	0
u_4	0	0	0	0.7	0.7	0

u_5	0.5	0	0	0	0.5	0
u_6	0	0.4	0	0	0.4	0
u_7	0	0	0.6	0	0.6	0

The proper colouring of a graph naturally induces a partitioning of the vertices into different subsets. From the above the figure fuzzy colouring reduces into 4 partition. They are $\gamma_1^* = \{u_1, u_5\}$, $\gamma_2^* = \{u_2, u_6\}$, $\gamma_3^* = \{u_3, u_7\}$, $\gamma_4^* = \{u_4\}$. Here no two vertices in any of these four subsets are adjacent so is called an independent set.

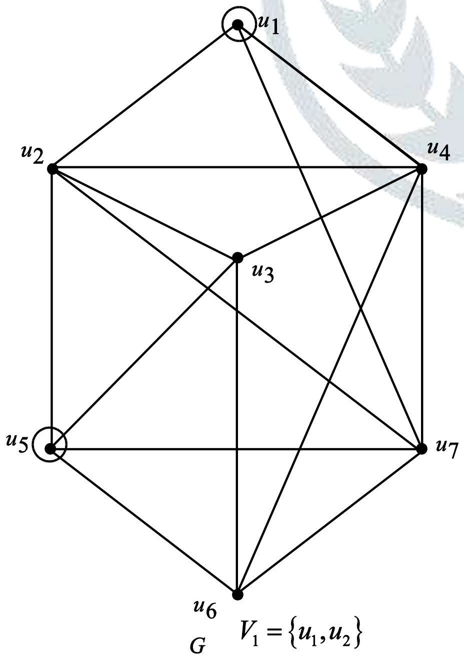
$$\left|\gamma_1^*\right| = |0.9 + 0.5| = 1.4$$

$$\left|\gamma_2^*\right| = |1 + 0.4| = 1.4$$

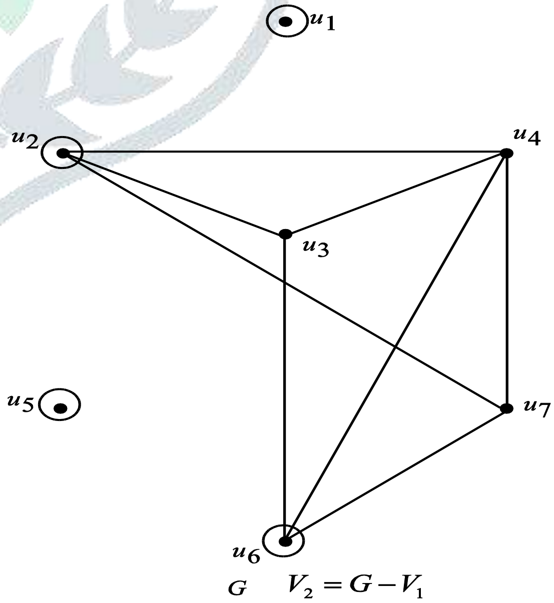
$$\left|\gamma_3^*\right| = |0.8 + 0.6| = 1.4$$

$$\left|\gamma_4^*\right| = |0.7| = 0.7$$

A single vertex in any graph constitutes an independent set so here $\gamma_4^* = \{u_4\}$ is also a independent set. The cardinality of maximal independent set is an independent number to which no other vertex can be added without destroying its independence property. This graph has three maximal independent sets. Thus $\gamma_1^* = \{u_1, u_5\}$, $\gamma_2^* = \{u_2, u_6\}$, $\gamma_3^* = \{u_3, u_7\}$, $\gamma_4^* = \{u_4\}$. By definition of fuzzy independent sets. The number of vertices in the largest independent set of a graph *G* is called the independent number and it is denoted by $\alpha(G)$. Here $\alpha(G) = 1.4$ .A *k*-colouring $\{V_1, V_2, \ldots, V_k\}$ of *G* is said to be canonical .Now consider $\gamma_1^* = V_1$, $\gamma_2^* = V_2$, $\gamma_3^* = V_3$, $\gamma_4^* = V_4$ if V_1 is a maximal independent set of *G*, V_2 is a maximal independent set of $G - V_1$, V_3 is a maximal independent set of $G - (V_1 \cup V_2)$. V_4 is a maximal independent set of $G - (V_1 \cup V_2 \cup V_3)$ and so on. All these can be seen in the following graphs.

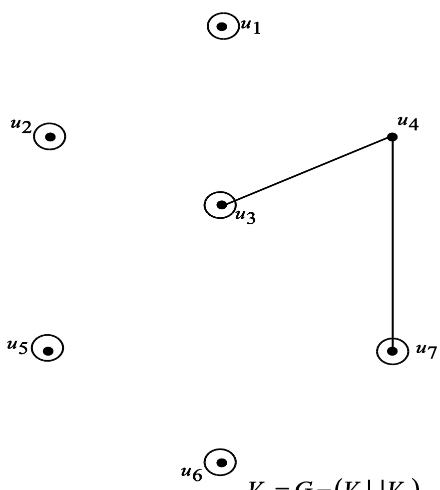


(1)



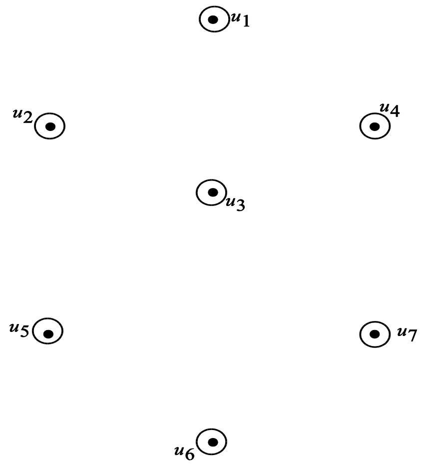
(2)

(3)



G $V_3 = G-(V_1 \cup V_2)$

(4)



G $V_4 = G-(V_1 \cup V_2 \cup V_3)$

If G is k-colouring then there exists a Canonical k-colouring of G . One can determine all proper colourings of G by using canonical form.

The minimum number of colours used in such colouring is called the chromatic number of $G$and $\chi^f(G)=4$.

The colouring correspond to each canonical form are

$\gamma_1^* = \{u_1,u_5\}$, $\gamma_2^* = \{u_2,u_6\}$, $\gamma_3^* = \{u_3,u_7\}$, $\gamma_4^* = \{u_4\}$.

Therefore the canonical form in fuzzy colouring partition the vertex sets which gives the chromatic number of G and equal to the least number of components .

This graph show that the chemicals $\sigma(u_1)$ and $\sigma(u_5)$ can be preserved in one compartment and the chemicals $\sigma(u_2)$ and $\sigma(u_6)$ can be preserved in another compartment.

Similarly the chemicals $\{\sigma(u_3),\sigma(u_7)\}$ and $\{\sigma(u_4)\}$ can be placed in distinct compartments.

Each colour classes of chemicals are preserved in distinct compartment in the warehouse .From this discussion, we observed that the chemical $\{\sigma(u_4)\}$ .i.e. $AgNO_3$ is the most dangerous chemical and it has to be placed in a separate compartment. Hence we concluded that the incompatible chemicals or the most dangerous chemicals are preserved in separate compartment to avoid explosion.

Fuzzy colouring is used in storage problem of combinatorial optimization and fuzzy colouring in storage problem gives optimal solution. Thus ,it is helpful to preserve incompatible chemicals to distinct compartments in warehouse.

4.CONCLUSION

In this paper, we discussed the relation between the chromatic number and fuzzy chromatic number**.**Thus,Storage problem is solved here by colouring of fuzzy graph which gives an optimal solution.

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