E-cordial labeling of path union on mixed graph.

mukund bapat

Abstract: On alternate vertex of a path $P_m$ graphs $G_1$ and $G_2$ are fused resulting in mixed path union of $G_1$ and $G_2$ denoted by $P_m(G_1, G_2)$. We chose $G_1$ and $G_2$ from $C_3$, $C_4$ and $C_5$ and show that $P_m(G_1, G_2)$ are $e$-cordial.

In general $P_m(G_1, G_2)$ is not same as $P_m(G_2, G_1)$ (upto isomorphism)

Key words: E-cordial, labeling, bull, one point union, cycle.

Subject Classification: 05C78.

2. Introduction:

The graphs we consider are finite, connected, un directed and simple. For terminology and definitions we refer Harary [3] E.J.of G. Theory, Dynamic survey of graph labeling [2]. In 1997 Yilmaz and Cahit [4] introduced a weaker version of edge graceful labeling called $E$-cordial. The word cordial was used first time in this paper. Let $G$ be a graph with vertex set $V$ and edge set $E$. Let $f$ be a function that maps $E$ into $\{0,1\}$. Define $f$ on $V$ by $f(v) = \sum (f(uv))/(uv) \in E \pmod{2}$. The function $f$ is called as $E$-cordial labeling if $|e(0)-e(1)| \leq 1$ and $|v(0)-v(1)| \leq 1$. Where $e(i)$ is the number of edges labeled with $i = 0,1$ and $v(i)$ is the number of vertices labeled with $i = 0,1$. We also use $v(0,1) = (a,b)$ to denote the number of vertices labeled with $0$ are $a$ and that with $1$ are $b$ in number. Similarly $e(0,1) = (x,y)$ to denote number of edges labeled with $0$ are $x$ and that labeled with $1$ are $y$ in number respectively. A lot of work has been done in this type of labeling and the above mentioned paper gave rise to number of papers on cordial labeling. A graph that admits $E$-cordial labeling is called as $E$-cordial graph. Yilmaz and Cahit has shown that Trees $T_n$ with $n$ vertices and Complete graphs $K_n$ on $n$ vertices are $E$ - cordial iff $n$ is not congruent to 2 (modulo 4). Friendship graph $C_e(n)$ for all $n$ and fans $F_n$ for $n$ not congruent to 1 (mod 4). They have observed that the graph with $n$ vertices where $n \equiv 2 \pmod{4}$ is not $e$-cordial. One may refer A Dynamic survey of graph labeling [3] for more details on completed work

3. Preliminaries:

3.1 Fusion of vertex. Let $G$ be a $(p,q)$ graph. let $u \neq v$ be two vertices of $G$. We replace them with single vertex $w$ and all edges incident with $u$ and that with $v$ are made incident with $w$. If a loop is formed is deleted. The new graph has $p-1$ vertices and at least $q-1$ edges.[5]

3.2 Path union of $G$ i.e. $P_m(G)$ is obtained by taking a path $P_m$ and $m$ copies of graph $G$. Fuse a copy each of $G$ at every vertex of path at given fixed point on $G$. It has $mp$ vertices and $mq + m-1$ edges, where $G$ is a $(p,q)$ graph. If we change the vertex on $G$ that is fused with vertex of $P_m$ then we generally get a path union non isomorphic to earlier structure. In this paper we define a $e$-cordial function $f$ that does not depends on which vertex of given graph $G$ is used to obtain path union. This allows us to obtain path union in which the same graph $G$ is fused with vertices of $P_m$ at different vertices of $G$, as our choice and the same function $f$ is applicable to all such structures that are possible on $P_m(G)$.

4. Main results:

Theorem 4.1 $P_m(G_1, G_2)$ where $G_1 = C_3$ and $G_2 = C_4$ is $e$-cordial for all $m$ not congruent to $3,4 \pmod{8}$

Proof. Define a function $f: E(G) \rightarrow \{0,1\}$. That gives us labeled structures as follows. In all diagrams below numbers shown are edge labels.
Obtain labeled copy of $P_8(C_3,C_4)$ by Joining point $x$ on Type A with point $y$ on Type B by an edge with label ‘0’. We have $v_f(0,1) = (14,14)$ $e_f(0,1) = (17,18)$. We repeat this procedure on $P_8(C_3,C_4)$ each time adding a new edge with label ‘0’ to obtain path union on $8x$ vertices where we have $v_f(0,1) = (14x,14x)$ $e_f(0,1) = (18x-1,18x)$. To obtain a path on $8x+t$ length $x = 0,1,2,3,..$ and $t = 1,2,3,4,5,6,7$ we first obtain $P_{8x}(C_3,C_4)$ and with an edge join it with $P_f(C_3,C_4)$ obtained below. The edge label of new edge is ‘0’.

For $t=1$ use Type C label at vertex $x$. The new edge is $(yx)$ with label ‘0’. Resultant label distribution for $P_{8x+1}$ is $v_f(0,1) = (14x+1,14x+2)$, $e_f(0,1) = (18x+2,18x+1)$. (If $x = 0$ just Type C label will work)

For $t = 2$ use Type D label at vertex $x$. The new edge is $(yx)$. Resultant label distribution is for $P_{8x+2}$ is $v_f(0,1) = (14x+3,14x+4)$, $e_f(0,1) = (18x+4,18x+4)$.

For $t = 3$ and $t = 4$, the desired labeling does not exist.

For $t = 5$ use Type B join with Type C label at vertex $y$ on Type B and vertex $x$ on Type C. The new edge is $(yx)$. Resultant label distribution for $P_{8x+5}$ is $v_f(0,1) = (14x+9,14x+8)$, $e_f(0,1) = (18x+10,18x+10)$.

For $t = 6$ use Type B and Type D label at vertex $y$ on Type B and $x$ on Type D with the new edge $(yx)$. Resultant label distribution for $P_{8x+6}$ is $v_f(0,1) = (14x+11,14x+10)$, $e_f(0,1) = (18x+12,18x+13)$. For $t = 7$ use Type B join with Type C label join with Type D label at vertex $y$ on Type B and vertex $x$ on Type C, further $x$ on C with $y$ on D. The new two new edges are $(yx)$. Resultant label distribution for $P_{8x+5}$ is $v_f(0,1) = (14x+12,14x+12)$, $e_f(0,1) = (18x+14,18x+15)$.

Theorem 4.2 $P_m(G_1,G_2)$, $G_1 = C_4$ and $G_2 = C_3$ is e-cordial for all $m$ not congruent to 4, 5 (mod 8). Proof. Define a function $f: E(G) \rightarrow \{0,1\}$. We obtain a labeled copy of $P_8(C_3,C_4)$ by joining Type B and Type A with vertex $x$ on Type B and vertex $y$ on Type A with new edge $(xy)$ with label ‘0’ given in above theorem 4.1. We have $v_f(0,1) = (14x,14x)$ $e_f(0,1) = (18x-1,18x)$. To obtain a path on $8x+t$ length $x = 0,1,2,3,..$ and $t = 1,2,3,4,5,6,7$ we first obtain $P_{8x}(C_3,C_4)$ and with an edge join it with $P_f(C_3,C_4)$ obtained below. The edge label of new edge is ‘0’. For $t=1$ use Type X label at vertex $x$. The new edge is $(yx)$ with label ‘0’. Resultant label distribution for $P_{8x+1}$ is $v_f(0,1) = (14x+2,14x+2)$, $e_f(0,1) = (18x+3,18x+2)$. (If $x = 0$ just Type X label will work)

For $t = 2$ use Type D label at vertex $y$. The new edge is $(yy)$ with label ‘0’. Resultant label distribution is for $P_{8x+2}$ is $v_f(0,1) = (14x+3,14x+4)$, $e_f(0,1) = (18x+4,18x+4)$. 

For $t = 5$ use Type B and Type D label at vertex $y$ on Type B and vertex $x$ on Type C. The new edge is $(yx)$. Resultant label distribution for $P_{8x+5}$ is $v_f(0,1) = (14x+9,14x+8)$, $e_f(0,1) = (18x+10,18x+10)$. For $t = 6$ use Type B and Type D label at vertex $y$ on Type B and $x$ on Type D with the new edge $(yx)$. Resultant label distribution for $P_{8x+6}$ is $v_f(0,1) = (14x+11,14x+10)$, $e_f(0,1) = (18x+12,18x+13)$. For $t = 7$ use Type B join with Type C label join with Type D label at vertex $y$ on Type B and vertex $x$ on Type C, further $x$ on C with $y$ on D. The new two new edges are $(yx)$. Resultant label distribution for $P_{8x+5}$ is $v_f(0,1) = (14x+12,14x+12)$, $e_f(0,1) = (18x+14,18x+15)$.

Theorem 4.2 $P_m(G_1,G_2)$, $G_1 = C_4$ and $G_2 = C_3$ is e-cordial for all $m$ not congruent to 4, 5 (mod 8). Proof. Define a function $f: E(G) \rightarrow \{0,1\}$. We obtain a labeled copy of $P_8(C_3,C_4)$ by joining Type B and Type A with vertex $x$ on Type B and vertex $y$ on Type A with new edge $(xy)$ with label ‘0’ given in above theorem 4.1. We have $v_f(0,1) = (14x,14x)$ $e_f(0,1) = (18x-1,18x)$. To obtain a path on $8x+t$ length $x = 0,1,2,3,..$ and $t = 1,2,3,4,5,6,7$ we first obtain $P_{8x}(C_3,C_4)$ and with an edge join it with $P_f(C_3,C_4)$ obtained below. The edge label of new edge is ‘0’. For $t=1$ use Type X label at vertex $x$. The new edge is $(yx)$ with label ‘0’. Resultant label distribution for $P_{8x+1}$ is $v_f(0,1) = (14x+2,14x+2)$, $e_f(0,1) = (18x+3,18x+2)$. (If $x = 0$ just Type X label will work)
For t=3 use Type Y label at vertex x. The new edge is (xx) has label ‘0’. Resultant label distribution for $P_{8x+3}$ is $v_i(0,1) = (14x+5,14x+6)$, $e_i(0,1) = (18x+7,18x+6)$.

For $t = 4$ and $t = 5$, the desired labeling does not exists.

For $t=6$ join Type Y and type C label with an edge with label number ‘0’.

Resultant label distribution for $P_{8x+3}$ is $v_i(0,1) = (14x+11,14x+10)$, $e_i(0,1) = (18x+13,18x+13)$.

For $t=7$ join Type Y and type C label and the Type C end with type K each by an edge with label ‘0’. Resultant label distribution for $P_{8x+3}$ is $v_i(0,1) = (14x+13,14x+12)$, $e_i(0,1) = (18x+16,18x+15)$. To complete the case $t=7$ we start with the case $t=7$ and join it with Type K label by an edge with label ‘0’. The resultant label distribution is $v_i(0,1) = (14,14)$, $e_i(0,1) = (17,18)$. Thus the graph is e-cordial.

Theorem 4.3 $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_4$ and $G_2 = C_5$ for all $m$ not congruent to 4, 5 (mod 8).

Proof: Define a function $f:E(G)\rightarrow \{0,1\}$. That gives us labeled structures as follows. In all diagrams below numbers shown are edge labels. Except Type C all are e-cordial.

We consider following cases on $t = 1, 2, 3, 4, 5, 6, 7, 8$ and obtain labeled copy of $P_t(G_1,G_2)$. The path under consideration will be $pt = (v_1, v_2, v_3, \ldots, v_t)$. Having obtained $P_{t+1}(G_1,G_2)$ we just fuse a labeled copy from above at vertex $v_t$ to obtain $P_t(G_1,G_2)$. Case $t = 1$ : Type A label. Case $t = 2$ : at $v_1$ fuse a copy of type A label and at $V_2$ fuse a copy of Type B label. The label number distribution will be $v_i(0,1) = (5,4)$, $e_i(0,1) = (5,5)$. Case $t = 3$ at vertex $V_3$ fuse Type A label. The label number distribution will be $v_i(0,1) = (7,6)$, $e_i(0,1) = (8,7)$. For $t = 4, 5$ (mod 8) the labeling does not exists. Case $t = 6$. We have already obtained $P_6(G_1,G_2)$. Add a new edge with label ‘0’ and extend $P_6(G_1,G_2)$ by Type C label. The label number distribution will be $v_i(0,1) = (13,14)$, $e_i(0,1) = (16,16)$. Case $t = 7$. We have already obtained $P_7(G_1,G_2)$. Add a new edge with label ‘0’ and extend $P_7(G_1,G_2)$ by Type E label. The label number distribution will be $v_i(0,1) = (15,16)$, $e_i(0,1) = (18,19)$. Case $t = 8$. We have already obtained $P_8(G_1,G_2)$. Add a new edge with label ‘0’ and extend $P_8(C_4,C_5)$ by Type C label. The label number distribution will be $v_i(0,1) = (18,18)$, $e_i(0,1) = (21,22)$. To obtain a path union of length $P_{8x+1}$ (x=0,1,2,..7) we first obtain a path union on $P_{8x}$ (take x copies of c and concatenate it each time by adding an edge with label ‘0’ and fuse with other copy of $P_8(C_4,C_5)$). This procedure repeated for x times will produce labeled copy of $P_{8x}(C_4,C_5)$). The label number distribution will be $v_i(0,1) = (18x,18x)$, $e_i(0,1) = (22x-1, 22x)$. The labeled copy of $P_{8x+1}(C_4,C_5)$ is obtained by joining a labeled copy of $P_1(C_4,C_5)$ with $P_{8x}(C_4,C_5)$ for $t = 1, 2, 3, 6, 7$. The label number distribution is given below.

<table>
<thead>
<tr>
<th>Sr. Number</th>
<th>$mx$ ($x = 0,1,2,..$)</th>
<th>$v_i(0,1)$</th>
<th>$e_i(0,1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8x</td>
<td>(18x,18x)</td>
<td>(22x-1,22x)</td>
</tr>
</tbody>
</table>
The label number distribution is given by $\lambda = 8x + 3$, $x = 0, 1, 2, \ldots$. At each vertex $v$, we fuse a copy of Type A if $i \equiv 1 \pmod{2}$ and copy of type B if $i \equiv 0 \pmod{2}$. The resultant label number distribution for vertices and edges is given by $\nu(0, 1) = (4x + 4x), e(0, 1) = (5x - 1, 5x)$ when $m = 2x$ and when $m = 2x - 1$ we have, $\nu(0, 1) = (4x + 1, 4x + 2), e(0, 1) = (5x + 1, 5x + 2)$. Thus the graph is e-cordial for all $m$.

Theorem 4.4 $P_m(G_1, G_2)$ is e-cordial where $G_1 = C_3$ and $G_2 = C_5$ for all $m$.

Proof: Define a function $f : E(G) \to \{0, 1\}$. That gives us labeled structures as follows. In all diagrams above numbers shown are edge labels. All are e-cordial. On the path $P_m = (v_1, v_2, v_3 \ldots v_m)$. All the labels on path are labeled as ‘0’. At vertex $v_i$ fuse a copy of Type A if $i \equiv 1 \pmod{2}$ and copy of type B if $i \equiv 0 \pmod{2}$. The resultant label number distribution for vertices and edges is given by $\nu(0, 1) = (4x + 4x), e(0, 1) = (5x - 1, 5x)$ when $m = 2x$ and when $m = 2x - 1$ we have, $\nu(0, 1) = (4x + 1, 4x + 2), e(0, 1) = (5x + 1, 5x + 2)$. Thus the graph is e-cordial for all $m$.

Theorem 4.5 $P_m(G_1, G_2)$ is e-cordial where $G_1 = C_5$ and $G_2 = C_3$ for all $m$.

Proof: Define a function $f : E(G) \to \{0, 1\}$. That gives us labeled structures same as Type A and Type B in fig 4.10 and 4.11 in theorem 4.3. On the path $P_m = (v_1, v_2, v_3 \ldots v_m)$ all the edges on path are labeled as ‘0’. At vertex $v_i$ of $P_m$ fuse a copy of Type B if $i \equiv 1 \pmod{2}$ and copy of type A if $i \equiv 0 \pmod{2}$. The resultant label number distribution for vertices and edges is given by $\nu(0, 1) = (4x + 4x), e(0, 1) = (5x - 1, 5x)$ when $m = 2x$ and when $m = 2x - 1$ we have, $\nu(0, 1) = (4x + 1, 4x + 2), e(0, 1) = (5x + 1, 5x + 2)$. Thus the graph is e-cordial for all $m (x = 0, 1, 2 \ldots)$.

Theorem 4.6

Proof: we define a function $f : E(G) \to \{0, 1\}$ as follows. It gives us following different types of labels which we combine to obtain $P_m(G_1, G_2)$.

Take a path on $m$ vertices say $P_m = (v_1, v_2, v_3, \ldots v_m)$. To obtain $P_m(G_1, G_2)$ label all the edges on it with ‘0’. At each vertex $v_i$ fuse a copy of type 1 if $i \equiv 1, 3, 7 \pmod{8}$. At each vertex $v_i$ fuse a copy of type 2 if $i \equiv 2, 4 \pmod{8}$. At each vertex $v_i$ fuse a copy of type 3 if $i \equiv 5 \pmod{8}$. At each vertex $v_i$ fuse a copy of type 4 if $i \equiv 6, 8 \pmod{8}$.

The label number distribution is given by $\nu(0, 1) = (3, 2), e(0, 1) = (2, 3)$ when $m = 1$. If $x \neq 0$ we have $\nu(0, 1) = (18x + 3, 18x + 2), e(0, 1) = (22x + 2, 22x + 3)$ when $m = 8x + 1$, $x = 0, 1, 2 \ldots$

The label number distribution is given by $\nu(0, 1) = (18x + 5, 18x + 4), e(0, 1) = (22x + 5, 22x + 5)$ when $m = 8x + 2$, $x = 0, 1, 2 \ldots$

The label number distribution is given by $\nu(0, 1) = (18x + 8, 18x + 6), e(0, 1) = (22x + 8, 22x + 8)$ when $m = 8x + 3$, $x = 0, 1, 2 \ldots$ In this case the graph is not e-cordial.

The label number distribution is given by $\nu(0, 1) = (18x + 10, 18x + 8), e(0, 1) = (22x + 11, 22x + 10)$ when $m = 8x + 4$, $x = 0, 1, 2 \ldots$ In this case the graph is not e-cordial.
The label number distribution is given by $v_f(0, 1) = (18x+11, 18x+12)$, $e_f(0, 1) = (22x+14, 22x+13)$ when $m = 8x+5$, $x = 0, 1, 2, ..$

The label number distribution is given by $v_f(0, 1) = (18x+13, 18x+14)$, $e_f(0, 1) = (22x+16, 22x+16)$ when $m = 8x+6$, $x = 0, 1, 2, ..$

The label number distribution is given by $v_f(0, 1) = (18x+16, 18x+18)$, $e_f(0, 1) = (22x+19, 22x+19)$ when $m = 8x+7$, $x = 0, 1, 2, ..$

The label number distribution is given by $v_f(0, 1) = (18x+18, 18x+18)$, $e_f(0, 1) = (22x+21, 22x+22)$ when $m = 8x+8$, $x = 0, 1, 2, ..$

Conclusions: In this paper we have discussed mixed path unions for $e$-cordial labeling. These path unions are obtained by fusing two graphs $G_1$ and $G_2$ at alternate nodes of path $P_m$. It is denoted by $P_m(G_1,G_2)$. If $G_1 \neq G_2$ (equality upto isomorphism.) $P_m(G_1,G_2) \neq P_m(G_2,G_1)$ We have shown this actually by taking $G_1$ and $G_2$ from $C_3$, $C_4$, $C_5$.

We prove that 1) $P_m(G_1,G_2)$ where $G_1 = C_3$ and $G_2 = C_4$ is e-cordial for all $m$ not congruent to 3,4 (mod 8).

2) $P_m(G_1,G_2)$, $G_1 = C_4$ and $G_2 = C_3$ is e-cordial for all $m$ not congruent to 4,5 (mod 8).

3) $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_4$ and $G_2 = C_5$ for all $m$ not congruent to 4,5 (mod 8).

4) $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_3$ and $G_2 = C_5$ for all $m$.

5) $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_5$ and $G_2 = C_3$ for all $m$.

6) $P_m(G_1,G_2)$ is e-cordial where $G_1 = C_5$ and $G_2 = C_4$ for all $m$ not congruent to 3,4 (mod 8). Another observation is that $P_m(C_3)$is not e-cordial when $m = 2$ (mod 4).And $P_m(C_4)$ is e-cordial for all $m$. But in mixed path union of $C_3$ and $C_4$ we get that mixed path union of $C_3$ and $C_4$ is e-cordial for all $m$ not congruent to 3,4 (mod 8).In this path union if we interchange position of $C_3$ and $C_4$ we get $P_m(C_4,C_3)$ which is e-cordial iff $m$ not congruent to 4,5 (mod 8).

References:


[3] Harary,Graph Theory,Narosa publishing ,New Delhi


1 Mukund V. Bapat, Hindale, Devgad, Sindhudurg, Maharashtra India: 416630 mukundbapat@yahoo.com