On g-binary Continuity

Nazir Ahmad Ahengar
J.K. Maitra
Department of Mathematics and Computer Sciences R.D.V.V Jabalpur 482001 (M.P) India

Abstract: In this paper we introduce and study the new concept of generalized continuity in generalized binary topological spaces simply called g-binary continuity. We examine some properties of g-binary continuous functions and investigate the relationships between these functions and their relationship with some other functions.

IndexTerms - **g-binary topology, g-binary open, g-α open, g-binary α-open, g-regular open, g-binary regular-open, g-semi open, g-binary semi open, g-pre open, g-binary pre open.**

1. Introduction

Recently the authors [5] introduced the concept of binary topology and discussed some of its basic properties, where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. The purpose is to introduce and study the new concept of generalized continuity in generalized binary topological spaces and examine some properties of generalized binary continuous functions and investigate the relationships between these functions. Section 2 deals with the basic concepts of g-binary topology. In section 3 g-binary continuous functions in g-binary topological spaces are studied. Throughout the paper $\wp(X)$ denotes the power set of X.

2. PRELIMINARIES

Let X and Y are any two non-empty sets. A generalized binary topology (or g-binary topology) from X to Y is a binary structure $M_g \subseteq \wp(X) \times \wp(Y)$ that satisfies the following axioms:

i) $(\emptyset, \emptyset)$ and $(X, X) \in M_g$

ii) If $\{(A_\alpha, B_\alpha); \alpha \in \Delta\}$ is a family of members of M_g, then $(\bigcup_{\alpha\in\Delta} A_\alpha, \bigcup_{\alpha\in\Delta} B_\alpha) \in M_g$

If M_g is a generalized binary topology from X to Y, then the triplet (X, Y, M_g) is called a generalized binary topological space (g-binary topological space) and the members of $M_g$are called the g-binary open subsets of the g-binary topological space (X, Y, M_g). The elements of $X \times Y$ are called the g-binary points of g-binary topological space (X, Y, M_g). Let $X = \{1,2\}$ and $Y = \{a, b, c\}$. Then $M_g = \{(\emptyset, \emptyset), (\{1\}, \{a, c\}), (\{2\}, \{a, b\}), (X, Y)\}$ is g-binary topological space.

Definition 2.1: Let X and Y be any two non-empty sets and (A, B), (C, D) belongs to $\wp(X) \times \wp(Y)$, we say (A, B) ⊆ (C, D) if A ⊆ C and B ⊆ D.

Definition 2.2: Let (X, Y, M_g) be a g-binary topological space and $A \subseteq X, B \subseteq Y$. Then (A, B) is g-binary closed in (X, Y, M_g) if $(X\backslash A, Y\backslash B) \in M_g$.

Proposition 2.1: Let (X, Y, M_g) be a g-binary topological space. Then

i) (X, Y) and $(\emptyset, \emptyset)$ are g-binary closed sets.

ii) If $\{(A_\alpha, B_\alpha): \alpha \in \Delta\}$ is a family of g-binary closed sets, then $(\bigcap_{\alpha\in\Delta} A_\alpha, \bigcap_{\alpha\in\Delta} B_\alpha)$ is g-binary closed.

Definition 2.3: Let (X, Y, M_g) be a g-binary topological space and $(A, B) \subseteq (X, Y)$. Let ${(A, B)^{1^*}}_g = \bigcap\{A_\alpha : (A_\alpha, B_\alpha)$ is g-binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$ and Let ${(A, B)^{2^*}}_g = \bigcap\{B_\alpha : (A_\alpha, B_\alpha)$ is g-binary closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$.Then

$(A,B)^{1^*}{}_g, (A,B)^{2^*}{}_g)$is g-binary closed and $(A,B) \subseteq (A,B)^{1^*}{}_g, (A,B)^{2^*}{}_g)$. The ordered pair $\left((A,B)^{1^*}{}_g, (A,B)^{2^*}{}_g)\right)$ is called g-binary closure of (A,B) and is denoted $gbcl(A,B)$ in the g-binary topology (X,Y,M_g) where $(A,B) \subseteq (X,Y)$.

Proposition 2.2: In a g-binary topological space (X,Y,M_g) if $(A,B) \subseteq (X,Y)$, then $gbcl(A,B)$ is smallest g-binary closed set containing (A, B).

Proposition 2.3: Let $(A,B) \subseteq (X,Y)$. Then (A,B) is g-binary closed in (X,Y,M_g) iff $(A,B) = gbcl(A,B)$.

Definition 2.4: Let (X,Y,M_g) be a g-binary topological space and $(A,B) \subseteq (X,Y)$. Let $(A,B)^{1^0}{}_g = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ is g-binary open and $(A_\alpha, B_\alpha) \subseteq (A,B)\}$ and Let $(A,B)^{2^0}{}_g = \cup\{B_\alpha : (A_\alpha, B_\alpha)$ is g-binary open and $(A_\alpha, B_\alpha) \subseteq (A,B)\}$.Then $((A,B)^{1^0}{}_g, (A,B)^{2^0}{}_g)$is g-binary open and $((A,B)^{1^0}{}_g, (A,B)^{2^0}{}_g) \subseteq (A,B)$. The ordered pair $((A,B)^{1^0}{}_g, (A,B)^{2^0}{}_g)$ is called g-binary interior of (A, B) and is denoted by $gbint(A,b)$.

Proposition 2.4: In a g-binary topological space (X,Y,M_g) if $(A,B) \subseteq (X,Y)$, then $gbint(A,B)$ is largest g-binary open set contained in (A, B).

Proposition 2.5: Let $(A,B) \subseteq (X,Y)$. Then (A,B) is g-binary open in (X,Y,M_g) iff $(A,B) = gbint(A,B)$.

3. g-binary continuity

Definition 3.1: Let (X,Y,M_g) be a g-binary topological space and (Z,τ) be a g-topological space. A function $f: Z \to X \times Y$ is called g-binary continuous at $z \in Z$ if for any g-binary open set $(A,B) \in (X,Y,M_g)$ with $f(z) \in (A,B)$ then there exists a g-open set G in (Z,τ) such that $z \in G$ and $f(G) \subseteq (A,B)$. The function f is called g-binary continuous if it is g-binary continuous at each $z \in Z$.

Proposition 3.1: Let (X,Y,M_g) be a g-binary topological space and (Z,τ) be a g-topological space. Let $f: Z \to X \times Y$ be a function. Then f is called g-binary continuous if $f^{-1}(A,B)$ is g-open in (Z,τ) for every g-binary open set (A, B) in (X,Y,M_g).

Proof: Let f be g-binary continuous and (A, B) be a g-binary open set in $\left(X,Y,M_g\right)$. If $f^{-1}(A,B) = \emptyset$. Then $\emptyset$ is g-open set. But if $f^{-1}(A,B) \neq \emptyset$, let $z \in f^{-1}(A,B)$. Then $f(z) \in (A,B)$. Since f is g-binary continuous at z, there exists a g-open set G in (Z,τ) such that $z \in G$ and $f(G) \subseteq (A,B)$. Hence $z \in G \subseteq f^{-1}(A,B)$. Therefore $f^{-1}(A,B)$ is g-open in (Z,τ).

Conversely to show that $f: Z \to X \times Y$ g-binary continuous. Let $z \in Z$ and (A, B) be a g-binary open set in (X,Y,M_g) with $f(z) \in (A,B)$. Then $z \in f^{-1}(A,B)$, where $f^{-1}(A,B)$ is g-open. Also $f(f^{-1}(A,B)) \subseteq (A,B)$. Hence f is g-binary continuous at z. Therefore f is g-binary continuous.

Definition 3.2: A subset A of a g-topological space (Z,τ) is called

i) g-semi open if $A \subseteq gcl(gint(A))$
ii) g-pre open if $A \subseteq gint(gcl(A))$
iii) g-α open if $A \subseteq gint(gcl(gint(A)))$
iv) g-regular open if $A = gint(gcl(A))$

Proposition 3.2:

i) Every g-open set is g-α open.
ii) Every g-α open set is g-semi open.
iii) Every g-α open set is g-pre open.
iv) Every g-regular open set is g-open.

Remark 3.1: The converse of the Proposition 3.2 need not be true as shown in Example 3.1, Example 3.2, Example 3.3 and Example 3.4.

Example 3.1: Let $Z = \{1,2,3,4\}$ and $\tau = \{\emptyset, \{2,3\}, \{1,3,4\}, Z\}$ be g-topological space. Then the set $\{1,2,3\}$ is g-α open but not g-open.

Example 3.2: Let $Z = \{1,2,3,4\}$ and $\tau = \{\emptyset, \{2\}, \{2,3\}, \{3,4\}, \{2,3,4\}, Z\}$ be g-topological space. Then the set $\{1,2\}$ is g-semi open but not g-α open.

Example 3.3: Let $Z = \{1,2,3,4\}$ and $\tau = \{\emptyset, \{1,2\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{2,3,4\}, Z\}$ be g-topological space. Then the set $\{1,3,4\}$ is g-pre open but not g-α open.

Example 3.4: Let $Z = \{1,2,3,4\}$ and $\tau = \{\emptyset, \{2\}, \{2,3\}, \{3,4\}, \{2,3,4\}, Z\}$ be g-topological space. Then the set $\{2,3\}$ is g-open but not g-regular open.

Definition 3.3: Let (X, Y, M_g) be a g-binary topological space and (Z, τ) be a g-topological space. A function $f: Z \to X \times Y$ is called

i) g-binary semi-continuous if $f^{-1}(A, B)$ is g-semi open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).

ii) g-binary pre-continuous if $f^{-1}(A, B)$ is g-pre open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).

iii) g-binary α-continuous if $f^{-1}(A, B)$ is g-α open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).

iv) g-binary r-continuous if $f^{-1}(A, B)$ is g-regular-open in (Z, τ) for every g-binary open set (A, B) in (X, Y, M_g).

Proposition 3.3:

i) Every g-binary continuous function in g-binary topology is g-binary semi-continuous.

ii) Every g-binary continuous function in g-binary topology is g-binary pre-continuous.

iii) Every g-binary continuous function in g-binary topology is g-binary α-continuous.

Proof: Let (A, B) be a g-binary open set in (X, Y, M_g). Since f is g-binary continuous, we have $f^{-1}(A, B)$ is g-open in (Z, τ). We know that every g-open set is g-semi-open. Hence $f^{-1}(A, B)$ is g-semi-open in (Z, τ). Thus f is g-binary semi-continuous. Similarly we can prove (ii) and (iii).

Remark 3.2: The converse of the Proposition 3.3 need not be true as shown in Example 3.5.

Example 3.5: Let $Z = \{1,2,3,4\}, X = \{a_1, a_2\}$ and $Y = \{b_1, b_2\}$. Then $\tau = \{\emptyset, \{1,2\}, \{2,3,4\}, Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_1\}, \{b_2\}), (\{a_1\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(1) = (a_1, b_1), = f(2) = f(3)$ and $f(4) = (\emptyset, b_1)$. Now $f^{-1}(\emptyset, \emptyset) = \emptyset$, $f^{-1}(\{a_1\}, \{b_1\}) = \{1,2,3\}, f^{-1}(\{a_1\}, \{b_2\}) = \{\emptyset\}$, $f^{-1}(\{a_1\}, \{Y\}) = \{1,2,3\}$, and $f^{-1}(X, Y) = Z$. This shows that the inverse image of every g-binary open set in (X, Y, M_g) is g-semi-open in (Z, τ). Hence f is g-binary semi-continuous but not g-binary continuous because $\{1,2,3\}$ is g-semi-open but not g-open in (Z, τ). In the similar way f is g-binary pre-continuous (g-binary α-continuous) but not g-binary continuous because $\{1,2,3\}$ is g-pre open (g-α open) but not g-open in (Z, τ).

Remark 3.2: The concepts of g-binary semi-continuous and g-binary pre-continuous in g-binary topology are independent of each others as shown in Example 3.6 and Example 3.7

Example 3.6: Let $Z = \{1,2,3,4\}, X = \{a_1, a_2\}$ and $Y = \{b_1, b_2\}$. Then $\tau = \{\emptyset, \{1\}, \{2\} \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, Z\}andM_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_1\}, \{Y\}), (\{a_2\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(1) = (a_1, b_1), f(2) = f(3) = f(4) = (a_2, b_2)$. Now $f^{-1}(\emptyset, \emptyset) = \emptyset$, $f^{-1}(\{a_1\}, \{b_1\}) = \{1\}$, $f^{-1}(\{a_1\}, \{Y\}) = \{\emptyset\}$, $f^{-1}(\{a_2\}, \{Y\}) = \{2,3,4\}$ and $f^{-1}(X, Y) = Z$. This shows that the inverse image of every g-binary open sets in (X, Y, M_g) is g-semi-open in (Z, τ). Hence f is g-binary semi-continuous but is not g-binary pre-continuous.

Example 3.7: Let $Z = \{1,2,3\}, X = \{a_1, a_2\}$ and $Y = \{b_1, b_2\}$. Then $\tau = \{\emptyset, \{1,2\}, \{2,3\}, Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_1\}, \{Y\}), (\{a_2\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \to X \times Y$ by $f(1) = (a_1, b_1) = f(3)$, $f(2) = (\emptyset, b_2)$. Now $f^{-1}(\emptyset, \emptyset) = \emptyset$, $f^{-1}(\{a_1\}, \{b_1\}) = \{1,3\}$, $f^{-1}(\{a_1\}, \{Y\}) = \{1,3\}$, $f^{-1}(\{a_2\},$

$\{Y\}) = \{\emptyset\}$ and $f^{-1}(X, Y) = Z$. This shows that the inverse image of every g-binary open sets in (X, Y, M_g) is g-pre open in (Z, τ). Hence f is g-binary pre-continuous but is not g-binary semi-continuous because $\{1,3\}$ is not g-semi open in (Z, τ).

Proposition 3.4:

i) Every g-binary α-continuous in g-binary topology is g-binary semi-continuous function.

ii) Every g-binary α-continuous in g-binary topology is g-binary pre-continuous function.

iii) Every g-binary r-continuous in g-binary topology is g-binary continuous function.

Proof: Let (A, B) be a g-binary open set in (X, Y, M_g). Since f is g-binary α-continuous, we have $f^{-1}(A, B)$ is g α-open in (Z, τ). We know that every g α-open set is g-semi-open (g-pre open). Hence $f^{-1}(A, B)$ is g-semi-open (g-pre open) in (Z, τ). Thus f is g-binary semi-continuous (g- binary pre continuous). Similarly let (A, B) be a g-binary open set in (X, Y, M_g). Suppose f is g-binary r-continuous, we have $f^{-1}(A, B)$ is g r-open in (Z, τ). We know that every g r-open set is g-open. Hence $f^{-1}(A, B)$ is g-open in (Z, τ). Thus f is g-binary continuous, which proves (iii).

Remark 3.3: The converse of the Proposition 3.4 need not be true as shown in Example 3.8, Example 3.9 and Example 3.10.

Example 3.8: Let $Z = \{1, 2, 3, 4\}$, $X = \{a_1, a_2\}, Y = \{b_1, b_2\}$ then $\tau = \{\emptyset, \{2\}, \{2,3\}, \{3,4\}, \{2,3,4\}, Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_1\}, \{Y\}), (\{a_2\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \rightarrow X \times Y$ by $f(1) = f(2) = (a_1, b_1)$, $f(3) = f(4) = (b_1, b_2)$. Now $f^{-1}(\emptyset, \emptyset) = \emptyset$, $f^{-1}(\{a_1\}, \{b_1\}) = \{1,2\}$, $f^{-1}(\{a_1\}, \{Y\}) = \{1,2\}$, $f^{-1}(\{a_2\}, \{Y\}) = \{3,4\}$ and $f^{-1}(X, Y) = Z$. This shows that the inverse image of every g-binary open sets in (X, Y, M_g) is g-semi open in (Z, τ). Hence f is g-binary semi-continuous but not binary g-α continuous because $\{1,2\}$ is not g-α open (Z, τ).

Example 3.9: Let $Z = \{1, 2, 3, 4\}$, $X = \{a_1, a_2\}, Y = \{b_1, b_2\}$ then $\tau = \{\emptyset, \{1,2\}, \{2,3\}, \{3,4\}, \{1,2,3\}, \{2,3,4\}, Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_1\}, \{Y\}), (\{a_2\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \rightarrow X \times Y$ by $f(1) = f(3) = f(4) = (a_1, b_1)$, $f(2) = (b_1, b_2)$. Now $f^{-1}(\emptyset, \emptyset) = \emptyset$, $f^{-1}(\{a_1\}, \{b_1\}) = \{1,3,4\}$, $f^{-1}(\{a_1\}, \{Y\}) = \{1,3,4\}$, $f^{-1}(\{a_2\}, \{Y\}) = \{\emptyset\}$ and $f^{-1}(X, Y) = Z$. This shows that the inverse image of every g-binary open sets in (X, Y, M_g) is g-pre open in (Z, τ). Hence f is g-binary pre-continuous but not binary g-α continuous because $\{1,3,4\}$ is not g-α open (Z, τ).

Example 3.10: Let $Z = \{1, 2, 3, 4\}$, $X = \{a_1, a_2\}, Y = \{b_1, b_2\}$ then $\tau = \{\emptyset, \{2\}, \{2,3\}, \{3,4\}, \{2,3,4\}, Z\}$ and $M_g = \{(\emptyset, \emptyset), (\{a_1\}, \{b_1\}), (\{a_1\}, \{Y\}), (\{a_2\}, \{Y\}), (X, Y)\}$. Clearly τ is a g-topology on Z and M_g is g-binary topology from X to Y. Define $f: Z \rightarrow X \times Y$ by $f(2) = f(3) = (a_1, b_1)$, $f(1) = f(4) = (b_1, b_2)$. Now $f^{-1}(\emptyset, \emptyset) = \emptyset$, $f^{-1}(\{a_1\}, \{b_1\}) = \{2,3\}$, $f^{-1}(\{a_1\}, \{Y\}) = \{2,3\}$, $f^{-1}(\{a_2\}, \{Y\}) = \{\emptyset\}$ and $f^{-1}(X, Y) = Z$. This shows that the inverse image of every g-binary open sets in (X, Y, M_g) is g-open in (Z, τ). Hence f is g-binary continuous but not binary g-regular continuous because $\{2,3\}$ is not g-regular open (Z, τ).

Conclusion:

g-binary continuity has been introduced and studied different types of continuities. Further relationships have been established between them.

References:

1. Jamal M. Mustafa, On Binary Generalized Topological Spaces, Refaad General Letters in Mathematics, 2(3), 111-116 (2017)
2. Norman Levine, Semi open sets and semi continuity in Topological spaces, Amer.Math.Monthly.70 36-41 (1963)
3. Norman Levine, Generalized closed sets in Topology, Rend. Cir. Mat. Palermo 2, 89-96 (1970)
4. Ryszard Engelking Generel Topology, Polish Scientific Publishers, Warszawa (1977).
5. S. Nithyanantha Jothi and P.Thangavelu, Topology between two sets, Journal of Mathematical Sciences & Computer Applications, 1(3), 95-107 (2011)

6. S. Nithyanantha Jothi and P.Thangavelu, On binary topological spaces, Pacific-Asian Journal Of Mathematics 5(2), 133-138 (2011)
7. S. Nithyanantha Jothi and P.Thangavelu, On binary continuity and binary separation axioms, Ultra Scientist of Physical Sciences Vol., 24(1) 121-126 (2012)
8. S. Nithyanantha Jothi, Binary semi continuous functions, International Journal of Mathematics Trends and Technology (IJMTI) 49(2), 152-155 (2017)