On βg^* - closed sets in Topological Spaces

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ABSTRACT

The aim of this paper is to define a new class of set namely βg^* - closed sets in topological spaces. Also we investigate about βg^* - continuous functions and βg^* - irresolute functions in topological spaces.
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1. Introduction

In 1970, Levine[3] introduced the concept of generalized closed sets in topological spaces. Many mathematicians started generalizing closed sets in recent years. M.K.R.S. Veerakumar[8] and S. P. Arya and T. Nour[2] introduced g* - closed sets and gs-closed sets respectively in topological spaces. In this paper we introduce a new class of sets namely βg^* - closed sets in topological spaces and study some of its basic properties.This class was obtained by generalizing closed sets via g^* - closed sets which was introduced by M.K.R.S. Veerakumar[8].

2. Preliminaries

Definition 2.1. *A subset A of a topological space (X, τ) is called*

1. *a pre - open set* [5] *if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$.*
2. *a semi - open set* [4] *if $A \subseteq cl(int(A))$ and a semi - closed set if $int(cl(A)) \subseteq A$.*
3. *an α - open set* [6]*if $A \subseteq int(cl(int(A)))$ and an α - closed set if $cl(int(cl(A))) \subseteq A$.*
4. *a semi - pre open set(= β-open)* [1] *if $A \subseteq cl(int(cl(A)))$ and a semi-pre closed set(= β - closed) if $int(cl(int(A))) \subseteq A$.*
5. *a regular open set* [7] *if $A = int(cl(A))$ and a regular closed set if $A = cl(int(A))$.*
6. *π - open* [9] *if A is the union of regular open sets.*

The intersection of all semi - closed (resp.pre-closed, semi-preclosed, regular-closed and α - closed) sets containing a subset A of (X,τ) is called the semi-closure (resp.pre-closure, semi-pre-closure, regular-closure and α-closure) of A and is denoted by scl(A) (resp. pcl(A),spcl(A), rcl(A) and αcl(A)).

Definition 2.2. *A subset A of a topological space(X,τ) is called*

(1) *a generalized closed set(briefly g - closed)* [3] *if cl$(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).*
(2) *a strongly generalized closed set(briefly g^* - closed)* [8] *if cl$(A) \subseteq U$ whenever $A \subseteq U$ and U is g-open in (X,τ).*
(3) *a generalized semi - closed set(briefly gs - closed)* [2] *if scl$(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X,τ).*

Definition 2.3. *A function f:X$\rightarrow$Y from a topological spaces X into a topological space Y is called*

(1) *continuous* [3] *if the inverse image of a closed set in Y is closed in X.*
(2) *pre - continuous* [5] *if the inverse image of a closed set in Y is pre - closed set in X.*
(3) *semi - continuous* [3] *if the inverse image of a closed set in Y is semi - closed set in X.*
(4) *α - continuous* [6] *if the inverse image of a closed set in Y is α - closed set in X.*
(5) *β - continuous* [1] *if the inverse image of a closed set in Y is β - closed set in X.*
(6) *r - continuous* [7] *if the inverse image of a closed set in Y is r - closed set in X.*
(7) *π - continuous* [9] *if the inverse image of a closed set in Y is π - closed set in X.*
(8) *g - continuous* [3] *if the inverse image of a closed set in Y is g - closed set in X.*
(9) *$g*$ - continuous* [8] *if the inverse image of a closed set in Y is $g*$ - closed set in X.*
(10) *gs - continuous* [2] *if the inverse image of a closed set in Y is gs - closed set in X.*

3. βg^* - closed sets

Definition 3.1. *A subset A of (X,τ) is called βg^*- closed set if $\beta cl(A) \subseteq U$ whenever $A \subseteq U$, U is g^*-open in (X,τ).*

Theorem 3.2. *Every closed set is βg^* - closed.*
Proof*: Let A be any closed set in X. Let U be a g^* - open set such that $A \subseteq U$. Since A is closed. We have cl(A) $= A \subseteq U$. But, $\beta cl(A) \subseteq$ cl$(A) \subseteq U$. Therefore $\beta cl(A) \subseteq U$. Hence A is a βg^* - closed set in X.*
The converse of the above theorem need not be true as seen from the following example.

Example 3.3. *Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ and $\tau^c = \{\phi, \{c\}, \{b, c\}, X\}$. βg^* - closed set $= \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$. Let $A = \{a, c\}$. Then the subset A is βg^* - closed but not a closed set.*

Theorem 3.4. *Every pre - closed set is βg^* - closed.*
Proof: *Let A be any pre - closed set in X. Let U be a g^* - open set such that $A \subseteq U$. Since A is pre-closed. We have pcl(A) = $A \subseteq U$. But, $\beta cl(A) \subseteq pcl(A) \subseteq U$. Therefore $\beta cl(A) \subseteq U$. Hence A is a βg^* - closed set in X.*
The converse of the above theorem need not be true as seen from the following example.

Example 3.5. *Let $X = \{a, b, c\}$, $\tau = \{\phi, \{b\}, \{b, c\}, X\}$ and pre - closed set = $\{\phi, \{a\}, \{c\}, \{a, c\}, X\}$. βg^* - closed set = $\{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, X\}$. Let A = $\{a, b\}$. Then the subset A is βg^* - closed but not a pre - closed set.*

Theorem 3.6. *Every semi - closed set is βg^*- closed.*
Proof*: Let A be any semi - closed set in X. Let U be a g^* - open set such that $A \subseteq U$. Since A is semi - closed. We have scl(A) = $A \subseteq U$. But, $\beta cl(A) \subseteq scl(A) \subseteq U$. Therefore $\beta cl(A) \subseteq U$. Hence A is a βg* - closed set in X.*
The converse of the above theorem need not be true as seen from the following example.

Example 3.7. *Let $X = \{a, b, c\}$, $\tau = \{\phi, \{c\}, \{a, c\}, X\}$ and semi - closed set = $\{\phi, \{a\}, \{b\}, \{a, b\}, X\}$. βg^* - closed set=$\{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, X\}$. Let A = $\{b, c\}$. Then the subset A is βg^* - closed but not a semi - closed set.*

Theorem 3.8. *Every α - closed set is βg^*- closed.*
Proof*: Let A be any α - closed set in X. Let U be a g^* - open set such that $A \subseteq U$. Since A is α - closed. We have αcl(A) = $A \subseteq U$. But, $\beta cl(A) \subseteq \alpha cl(A) \subseteq U$. Therefore $\beta cl(A) \subseteq U$. Hence A is a βg* - closed set in X.*
The converse of the above theorem need not be true as seen from the following example.

Example 3.9. *Let $X = \{a, b, c\}$, $\tau = \{\phi, \{b\}, \{a, c\}, X\}$ and α - closed set = $\{\phi, \{b\}, \{a, c\}, X\}$. βg^* - closed set = $\{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$. Let A = {a,b}. Then the subset A is βg^* - closed but not an α - closed set.*

Theorem 3.10. *Every β - closed set is a βg^*- closed.*
Proof*: Let A be any β - closed set in X. Let U be a g^* - open set such that $A \subseteq U$. Since A is β-closed. We have βcl(A) = $A \subseteq U$. Therefore $\beta cl(A) \subseteq U$. Hence A is a βg^* - closed set in X.*
The converse of the above theorem need not be true as seen from the following example.

Example 3.11. *Let $X = \{a, b, c\}$, $\tau = \{\phi, \{c\}, \{a, b\}, X\}$ and β - closed set = $\{\phi, \{b\}, \{c\}, \{b, c\}, X\}$. βg^* - closed set = $\{\phi, \{b\}, \{c\}, \{b, c\}\{a, c\}, X\}$. Let A = $\{a, c\}$. Then the subset A is βg^* - closed but not a β-closed set.*

Theorem 3.12. *Every r - closed set is βg^*- closed.*
Proof*: Let A be r - closed set in X. Let U be a g^* - open set such that $A \subseteq U$. Since A is r-closed. We have rcl(A) = $A \subseteq U$. But, $\beta cl(A) \subseteq rcl(A) \subseteq U$. Therefore $\beta cl(A) \subseteq U$. Hence A is a βg^* - closed set in X.*
The converse of the above theorem need not be true as seen from the following example.

Example 3.13. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ *and r - closed set* $= \{\phi, \{b, c\}, \{a, b\}, X\}$. βg^* *- closed set* $= \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$. *Let* $A = \{b\}$. *Then the subset A is* βg^* *- closed but not a r - closed set.*

Theorem 3.14. *Every* π *- closed set is* βg^* *- closed.*
Proof*: Let A be* π *- closed set in X. Let U be a* g^* *- open set such that* $A \subseteq U$. *Since A is* π *- closed. We have* $\pi cl(A) = A \subseteq U$. *But,* $\beta cl(A) \subseteq \pi cl(A) \subseteq U$. *Therefore* $\beta cl(A) \subseteq U$. *Hence A is a* βg^* *- closed set in X.*
The converse of the above theorem need not be true as seen from the following example.

Example 3.15. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, \{b\}, \{a, b\}, \{b, c\}, X\}$ *and* π *- closed set* $= \{\phi, X\}$. βg^* *- closed set* $= \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$. *Let* $A = \{a\}$. *Then the subset A is* βg^**-closed but not a* π *- closed set.*

Theorem 3.16. *Every g - closed set is* βg^* *- closed.*
Proof*: Let A be g - closed set in X. Let U be a* g^* *- open set such that* $A \subseteq U$. *Since A is g-closed. We have* $cl(A) \subseteq U$. *But,* $\beta cl(A) \subseteq cl(A) \subseteq U$. *Therefore* $\beta cl(A) \subseteq U$. *Hence A is a* βg^* *- closed set in X.*
The converse of the above theorem need not be true as seen from the following example.

Example 3.17. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ *and g - closed set* $= \{\phi, \{c\}, \{b, c\}, \{a, c\}, X\}$. βg^**-closed set* $= \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$. *Let* $A = \{b\}$. *Then the subset A is* βg^**-closed but not a g-closed set.*

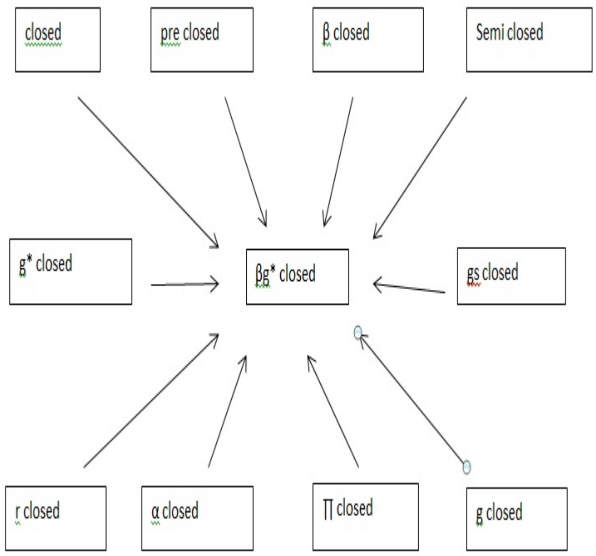
Theorem 3.18. *Every* g^* *- closed set is* βg^* *- closed.*
Proof*: Let A be* g^**-closed set in X. Let U be a* g^* *- open set such that* $A \subseteq U$. *Since A is* g^* *- closed. We have* $cl(A) \subseteq U$. *But,* $\beta cl(A) \subseteq cl(A) \subseteq U$. *Therefore* $\beta cl(A) \subseteq U$. *Hence A is a* βg^* *- closed set in X.*
The converse of the above theorem need not be true as seen from the following example.

Example 3.19. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, \{b\}, \{a, b\}, X\}$ *and* g^* *- closed set* $= \{\phi, \{c\}, \{b, c\}, \{a, c\}, X\}$. βg^**-closed set* $= \{\phi, \{a\}, \{c\}, \{b, c\}, \{a, c\}, X\}$. *Let* $A = \{a\}$. *Then the subset A is* βg^**-closed but not a* g^* *- closed set.*

Theorem 3.20. *Every gs - closed set is* βg^* *- closed.*
Proof*: Let A be gs-closed set in X. Let U be a* g^* *- open set such that* $A \subseteq U$. *Since A is gs - closed. We have* $scl(A) \subseteq U$. *But,* $\beta cl(A) \subseteq scl(A) \subseteq U$. *Therefore* $\beta cl(A) \subseteq U$. *Hence A is a* βg^* *- closed set in X.*
The converse of the above theorem need not be true as seen from the following example.

Example 3.21. *Let* $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ *and gs - closed set=* $\{\phi, \{a\}, \{a, b\}, \{a, c\}, X\}$. βg^* *- closed set* $= \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, X\}$. *Let* $A = \{b\}$. *Then the subset A is* βg^**-closed but not a gs - closed set.*

Remark 3.22. *The following diagram shows that the relations ship between* βg^* *- closed sets and known existing sets. None of the implication is reversible.*



Theorem 3.23. *If A is an βg^* - closed subset of X such that $A \subset B \subset \beta cl(A)$, then B is also βg^* - closed set in X.*
Proof: *Let A be an βg^*-closed set of X such that $\beta cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^* - open in X. To prove : B is βg^* - closed. Let $B \subseteq U$. now $\beta cl(B) \subseteq \beta cl(A) \subseteq U$ implise $\beta cl(B) \subseteq U$ whenever $B \subseteq U$ and U is g^* - open implise B is βg^* - closed*

Definition 3.24. *A subset A of a topological space (X,τ) is called βg^* - open set if and only if A^c is βg^* - closed in (X,τ).*

Theorem 3.25. *For a topological space (X,τ), the following hold.*

(1) *Every open set is βg^*-open.*
(2) *Every pre - open set is βg^* - open.*
(3) *Every semi - open set is βg^* - open.*
(4) *Every g - open set is βg^* - open.*
(5) *Every $g*$ - open set is βg^* - open.*
(6) *Every gs - open set is βg^* - open.*

Proof: *Obvious.*

Theorem 3.26. *If $\beta int(A) \subseteq B \subseteq A$ and if A is βg^* - open in X, then B is βg^* - open in X.*
Proof: *$B \subseteq A$ implies $X - A \subseteq X - B$, $\beta int(A) \subseteq B$ implies $X - B \subseteq X - \beta int(A)$. That is $X - A \subseteq X - B \subseteq X - \beta int(A) = \beta cl(X - A)$. Since X-A is βg^* - closed, by Theorem(3.23) X-B is βg^* - closed which implies B is βg^* - open.*

4. βg^* - continuous functions

Definition 4.1. *: A function f:$(X,\tau)\rightarrow(Y,\sigma)$ is said to be βg^*-continuous, if every f $^{-1}(V)$ is βg^* - closed in (X,τ) for every V in (Y,σ).*

Theorem 4.2. *A function f:$(X,\tau)\rightarrow(Y,\sigma)$, the following hold*

(1) *Every continuous function is βg^* - continuous.*
(2) *Every pre - continuous function is βg^* - continuous.*
(3) *Every semi - continuous function is βg^* - continuous.*
(4) *Every α - continuous function is βg^* - continuous.*
(5) *Every β - continuous function βg^* - continuous.*
(6) *Every r - continuous function is βg^* - continuous.*
(7) *Every π - continuous function is βg^* - continuous.*
(8) *Every g - continuous function is βg^* - continuous.*
(9) *Every g$*$ - continuous function is βg^* - continuous.*
(10) *Every gs - continuous function is βg^* - continuous.*

Proof*: Let V be a closed set in Y. Since f is continuous, then f $^{-1}$(V) is closed in X. Since every closed set is βg^* - closed, then f $^{-1}$(V) is βg^* - closed in X. Hence f is βg^* - continuous.*
Proof of (2) to (10) is obvious.

Example 4.3. *Let X=Y=$\{$a,b,c$\}$, $\tau=\{\phi,\{c\},\{a,c\},X\}$. $\sigma=\{\phi,\{a\},\{b,c\},Y\}$. $\sigma^c=\{\phi,\{b,c\},\{a\},Y\}$ and closed set $=\{\phi,\{b\},\{a,b\},X\}$. Then βg^* - closed set $=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},X\}$. Let f:$(X,\tau)\rightarrow(Y,\sigma)$ define a map f(a)=a, f(b)=b, f(c)=c, then f $^{-1}$(b,c)=$\{b,c\}$, f $^{-1}$(a)=$\{a\}$, which is in βg^* - closed set in X. Therefore f is βg^* - continuous function.*

Example 4.4. *Let X=Y=$\{$a,b,c$\}$, $\tau=\{\phi,\{b\},\{c\},\{b,c\},X\}$. $\sigma=\{\phi,\{b\},\{b,c\},Y\}$. $\sigma^c=\{\phi,\{a,c\},\{a\},Y\}$ and pre-closed set $=\{\phi,\{a\},\{a,b\},\{a,c\},X\}$. Then βg^* - closed set $=\{\phi,\{a\},\{b\},\{c\},\{a,b\},\{a,c\},X\}$. Let f:$(X,\tau)\rightarrow(Y,\sigma)$ define a map f(a)=c, f(b)=b, f(c)=a, then f $^{-1}$(a,c)=$\{b,c\}$, f $^{-1}$(a)=$\{c\}$, which is in βg^* - closed set in X. Therefore f is βg^* - continuous function.*

Example 4.5. *Let X=Y=$\{$a,b,c$\}$, $\tau=\{\phi,\{c\},\{a,c\},X\}$. $\sigma=\{\phi,\{b\},Y\}$. $\sigma^c=\{\phi,\{a,c\},Y\}$ and semi - closed and β - closed set $=\{\phi,\{a\},\{b\},\{a,b\},X\}$. Then βg^* - closed set $=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},X\}$. Let f:$(X,\tau)\rightarrow(Y,\sigma)$ define a map f(a)=b, f(b)=a, f(c)=c, then f $^{-1}$(a,c)=$\{b,c\}$, which is in βg^* - closed set in X. Therefore f is βg^* - continuous function.*

Example 4.6. *Let X=Y=$\{$a,b,c$\}$, $\tau=\{\phi,\{b\},\{a,c\},X\}$. $\sigma=\{\phi,\{a\},\{a,c\},Y\}$. $\sigma^c=\{\phi,\{b,c\},\{b\},Y\}$ and α - closed set $=\{\phi,\{b\},\{a,c\},X\}$. Then βg^* - closed set $=\{\phi,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},X\}$. Let f:$(X,\tau)\rightarrow(Y,\sigma)$ define a map f(a)=a, f(b)=c, f(c)=b, then f $^{-1}$(b,c)=$\{b,c\}$, f $^{-1}$(b)=$\{c\}$, which is in βg^* - closed set in X. Therefore f is βg^* - continuous function.*

Example 4.7. *Let X=Y=$\{$a,b,c$\}$, $\tau=\{\phi,\{b\},\{a,c\},X\}$. $\sigma=\{\phi,\{a\},\{a,c\},Y\}$. $\sigma^c=\{\phi,\{b,c\},\{b\},Y\}$ and π - closed and r - closed set $=\{\phi,X\}$. Then βg^* - closed set $=\{\phi,\{a\},\{b\},\{c\},\{a,b\},\{b,c\},\{a,c\},X\}$. Let f:$(X,\tau)\rightarrow(Y,\sigma)$ define* *a map f(a)=a, f(b)=c, f(c)=b, then* f^{-1}*(b,c)=*$\{b,c\}$*,* f^{-1}*(b)=*$\{c\}$*, which is in* βg^* *- closed set in X. Therefore f is* βg^* *- continuous function.*

Example 4.8. *Let X=Y=*$\{a,b,c\}$*,* $\tau = \{\phi, \{b\}, \{b,c\}, X\}$*.* $\sigma = \{\phi, \{a,c\}, Y\}$*.* $\sigma^c = \{\phi, \{b\}, Y\}$ *and g - closed and g∗ - closed set* $= \{\phi, \{a\}, \{a,b\}, \{a,c\}, X\}$*. Then* βg^* *- closed set* $= \{\phi, \{a\}, \{c\}, \{a,b\}, \{a,c\}, X\}$*. Let f:*$(X,\tau)\rightarrow(Y,\sigma)$ *define a map f(a)=a, f(b)=c, f(c)=b, then* f^{-1}*(b)=*$\{c\}$*, which is in* βg^* *- closed set in X. Therefore f is* βg^* *- continuous function.*

5. βg^* - irresolute functions

Definition 5.1. *: A function f:*$(X,\tau)\rightarrow(Y,\sigma)$ *is said to be* βg^**-irresolute, if every* $f^{-1}(V)$ *is* βg^* *- closed in* (X,τ) *for every V in* βg^* *- closed* (Y,σ)*.*

Theorem 5.2. *Every* βg^* *- irresolute function is* βg^* *- continuous.*
proof*: Let v be a* βg^* *- closed set in Y. Since every closed set is* βg^* *- closed, then*
f^{-1}*(v) is* βg^* *- closed in X. Hence f is* βg^**-irresolute.*

Example 5.3. *Let* $X = Y = \{a,b,c\}$*,* $\tau = \{\phi, \{b\}, \{b,c\}, X\}$*,* βg^* *- closed set* $= \{\phi, \{a\}, \{c\}, \{a,b\}, \{a,c\}, X\}$*.* $\sigma = \{\phi, \{a,c\}, Y\}$*,* βg^* *- closed set* $= \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, Y\}$*. Let f:*$(X,\tau)\rightarrow(Y,\sigma)$ *define a function f(a)=a, f(b)=c, f(c)=b, then* $f^{-1}(b,c) = \{b,c\}$*, which is in* βg^* *- closed set in X. Therefore f is* βg^* *- irresolute function.*

Remark 5.4. *The composition of two* βg^* *- continuous function need not be a* βg^* *- continuous function. It can be seen from the following example.*

Example 5.5. *Let* $X = Y = Z = \{a,b,c\}$ *with*
$\tau = \{\phi, \{c\}, \{a,c\}, X\}$*,* βg^* *- closed set* $= \{\phi, \{a\}, \{b\}, \{a,b\}, \{b,c\}, X\}$*.*
$\sigma = \{\phi, \{b\}, \{c\}, \{b,c\}, X\}$*,* βg^* *- closed set* $= \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, X\}$*.*
$\eta = \{\phi, \{b\}, \{a,c\}, X\}$*,* βg^* *- closed set* $= \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}, X\}$*.*
Define f:$(X,\tau) \rightarrow (Y,\sigma)$ *by f(a)=a, f(b)=c, f(c)=b.*
Define g:$(Y,\sigma) \rightarrow (Z,\eta)$ *by g(a)=a, g(b)=c, g(c)=b.*
Here $\{a,c\}$ *is a closed set in* (Z,η)*. But* $(g \circ f)^{-1}\{a,c\} = \{a,c\}$ *is not* βg^* *- closed set in* (X,τ)*. Therefore* $g \circ f$ *is not* βg^* *- continuous function.*

Theorem 5.6. *If f:*$(X,\tau)\rightarrow(Y,\sigma)$ *is* βg^* *- irresolute and g:*$(Y,\sigma)\rightarrow(Z,\eta)$ *is* βg^* *- continuous then* $g \circ f$ *is* βg^* *- continuous.*
proof: *Let V be closed set in Z. since g is* βg^* *- continuous then* g^{-1}*(V) is* βg^* *- closed in Y. Since f is* βg^* *- irresolute then* f^{-1}*(* g^{-1}*(V))* βg^* *- closed in X. Hence* $g \circ f$ *is* βg^* *- continuous.*

Theorem 5.7. *If f:*$(X,\tau)\rightarrow(Y,\sigma)$ *is* βg^* *- irresolute and g:*$(Y,\sigma)\rightarrow(Z,\eta)$ *is* βg^* *- irresolute then* $g \circ f$ *is* βg^* *- irresolute.*
proof: *Let V be* βg^* *- closed set in Z. since g is* βg^* *- irresolute then* g^{-1}*(V) is* βg^* *- closed in Y. Since f is* βg^* *- irresolute then* f^{-1}*(* g^{-1}*(V))* βg^* *- closed in X. Hence* $g \circ f$ *is* βg^* *- irresolute.*

6. Conclusion

In this paper we have defined βg^* - closed sets in topological spaces and studied its properties by comparing it with some of the existing closed sets and also we investigated βg^* - continuous functions and βg^* - irresolute functions. From the comparision we see that βg^* - closed sets is weaker than the other existing sets.

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