SOLVING NONLINEAR FRACTIONAL DIFFERENTIAL EQUATION USING MAHGOUB ADOMIAN DECOMPOSITION METHOD

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Abstract: In this article, Mahgoub Adomian Decomposition method (MADM) is proposed to determine the approximate solution of the nonlinear Fractional Differential Equations (FDEs). This method is a combined form of Mahgoub transform with Adomian decomposition method. The fractional derivatives are described in the Caputo sense. The results of numerical experiment show the efficiency of our newly developed method.

Key Words **- Mahgoub Adomian Decomposition method, Fractional Differential Equations, Caputo derivative.**

I. INTRODUCTION

Fractional calculus is an important tool of mathematical analysis applied to the study of integrals and derivatives of arbitrary order. Nowadays, the fractional calculus has found extensive application in various fields such as rheology, quantitative electrochemistry, probability, potential theory, scattering theory, elasticity, diffusion, biology and transport theory [1]. Many researchers have shown their interest in finding the numerical solution of linear and nonlinear FDEs. Some methods are Differential Transform Method [2], Homotophy Analysis Method [3] and Adomian Decomposition Method [4].

Integral transform methods have been proposed to find the analytical solution of linear FDE. Some of them are Laplace [5], Natural [6], Sumudu [7], Elzaki [8] and Mahgoub [9]. For solving nonlinear FDEs, the Adomian decomposition method was combined with Laplace transform method [10], with natural transform method [11], with Sumudu transform method [12] and with Elzaki transform method [13].

In this paper, the Mahgoub Adomian Decomposition method have been proposed for finding the numerical solution of nonlinear FDEs with Caputo derivatives. This paper has been organized as follows: Section 2 consists of basic definitions of fractional calculus and Mahgoub transform of fractional derivatives. Section 3 constructs the MADM for finding the numerical solutions for nonlinear fractional differential equations. Section 4 provides examples of FDE to illustrate the efficiency of this method.

II. PRELIMINARIES AND NOTATIONS

In this section, we give some basic definitions and properties of fractional calculus and Mahgoub transform.

Definition 1:

A real function $f(x), x > 0$ is said to be in the space $C_\mu,\ \mu \epsilon \mathbb{R}$ if there exists a real number $p > \mu$ such that $f(x) = x^p f_1(x)$ where $f_1(x) \epsilon C[0,\infty)$ and it is said to be in the space C_μ^m if and only if $f^{(m)} \epsilon C_\mu,\ m \epsilon \mathbb{N}$.

Definition 2:

The Riemann-Liouville fractional integral operator of order $\alpha \geq 0$, of a function $f \epsilon C_\mu, \mu \geq -1$, is defined as

$$J^\alpha f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t)dt, \alpha > 0, x > 0,$$

$$J^0 f(x) = f(x) \qquad (1)$$

Properties of the operator J^α is given by

i. $J^\alpha J^\beta f(x) = J^{\alpha+\beta} f(x)$
ii. $J^\alpha J^\beta f(x) = J^\beta J^\alpha f(x)$
iii. $J^\alpha x^\gamma = \frac{\Gamma(\gamma+1)}{\Gamma(\alpha+\gamma+1)} x^{\alpha+\gamma}$

Definition 3:

The fractional derivative of $f(x)$ in the Caputo sense is defined as

$${}^cD^\alpha f(x) = \frac{1}{\Gamma(m-\alpha)} \int_0^x (x-t)^{m-\alpha-1} f^m(t)dt. \qquad (2)$$

for $m-1 < \alpha \leq m, m \epsilon \mathbb{N}, x > 0$ and $f \epsilon C_{-1}^m$.

Definition 4:

Mahgoub transform is defined on the set of continuous functions and exponential order. We consider functions in the set A defined by

$$A = \left\{ f(t): |f(t)| < Pe^{\frac{|t|}{\epsilon_i}} if t \in (-1)^i \times [0,\infty), i = 1,2;\ \epsilon_i > 0 \right\}$$

where ϵ_1, ϵ_2 may be finite or infinite and the constant P must be finite.
Let $f \in A$, then Mahgoub transform is defined as

$$M[f(t)] = H(u) = u \int_0^{\infty} f(t) e^{-ut} dt,\ t \geq 0, \epsilon_1 \leq u \leq \epsilon_2 \qquad (3)$$

Mahgoub transform of simple functions are given below:

(i) $M[1] = 1$

(ii) $M[t] = \frac{1}{u}$

(iii) $M[t^2] = \frac{2}{u^2}$

(iv) $M[t^n] = \frac{n!}{u^n} = \frac{\Gamma(n+1)}{u^n}$

Mahgoub Transform for derivatives are:

(i) $M[f'(t)] = uH(u) - uf(0)$

$(ii) M[f''(t)] = u^2 H(u) - u^2 f(0) - uf'(0)$

$(iii) M[f^n(t)] = u^n H(u) - \sum_{k=0}^{n-1} u^{n-k} f^k(0)$

Lemma 5: [9]

If $H(u)$ is Mahgoub transform of $y(x)$, then Mahgoub transform of Caputo derivative, for $m-1 < \alpha \leq m, m \epsilon \mathbb{N}$ is

$$M[{}^c D^{\alpha} y(x)] = u^{\alpha} H(u) - \sum_{k=0}^{m-1} u^{\alpha-k} y^k(0) \qquad (4)$$

III. CONSTRUCTION OF MADM METHOD

Consider the following fractional differential equation

$${}^c D^{\alpha} y(x) + a_m y^{(m)}(x) + a_{m-1} y^{(m-1)}(x) + \cdots + a_1 y'(x) + a_0 y(x) + N(y(x), y'(x), \dots, y^{m-1}(x)) = f(x), x \geq 0, \qquad (5)$$

for $m-1 < \alpha \leq m, m \epsilon \mathbb{N}$.
Subject to the initial conditions

$$y^{(i)}(0) = b_i, \qquad i = 0,1,2, \dots, m-1 \qquad (6)$$

where a_i, b_i are known real constants. N is a nonlinear operator and $f(x)$ is known function. Let $[0, \mathrm{T}]$ be the interval over which we need to find the solution of the above initial value problem.

Applying the Mahgoub transform to both sides of equation (5) and by using the linearity of Mahgoub transforms, the result is

$$M({}^c D^{\alpha} y(x)) + a_m M(y^{(m)}(x)) + a_{m-1} M(y^{(m-1)}(x)) + \cdots + a_1 M(y'(x)) + a_0 M(y(x)) + M\left(N\left(y(x), y'(x), \dots, y^{(m-1)}(x)\right)\right) = M(f(x))$$

Using above Lemma and applying the formulas of Mahgoub transform, we get

$$u^{\alpha} M(y(x)) = \sum_{k=0}^{m-1} u^{\alpha-k} y^k(0) + M(f(x)) - a_m M(y^{(m)}(x)) - a_{m-1} M(y^{(m-1)}(x)) - \cdots - a_1 M(y'(x))$$
$$-a_0 M(y(x)) - M\left(N\left(y(x), y'(x), \dots, y^{(m-1)}(x)\right)\right)$$

$$M(y(x)) = \frac{1}{u^{\alpha}} \sum_{k=0}^{m-1} u^{\alpha-k} y^k(0) + \frac{1}{u^{\alpha}} M(f(x)) - \frac{1}{u^{\alpha}} \left[a_m M(y^{(m)}(x)) + a_{m-1} M(y^{(m-1)}(x)) + \cdots + a_1 M(y'(x)) + a_0 M(y(x))\right]$$
$$-\frac{1}{u^{\alpha}} M\left(N\left(y(x), y'(x), \dots, y^{(m-1)}(x)\right)\right) \qquad (7)$$

The MADM represents the solution as an infinite series

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \qquad (8)$$

and the nonlinear term $N\left(y(x), y'(x), \dots, y^{(m-1)}(x)\right)$ is decomposed in Adomian polynomials as

$$M\left(N\left(y(x), y'(x), \dots, y^{(m-1)}(x)\right)\right) = \sum_{n=0}^{\infty} A_n \qquad (9)$$

For the nonlinear function $Ny = f(y)$ the first five Adomian polynomials are given by

$$A_0 = f(y_0),$$
$$A_1 = f^{(1)}(y_0) y_1,$$
$$A_2 = f^{(1)}(y_0) y_2 + \frac{1}{2!} f^{(2)}(y_0) {y_1}^2$$
$$A_3 = f^{(1)}(y_0) y_3 + f^{(2)}(y_0) y_1 y_2 + \frac{1}{3!} f^{(3)}(y_0) {y_1}^3,$$
$$A_4 = f^{(1)}(y_0) y_4 + f^{(2)}(y_0) \left[\frac{1}{3!} {y_2}^2 + y_1 y_3\right] + f^{(3)}(y_0) \frac{1}{2} {y_1}^2 y_2 + f^{(4)}(y_0) \frac{1}{4!} {y_1}^4,$$

Substituting Eqns. (8) and (9) into (7), we have

$$M\left(\sum_{n=0}^{\infty} y_n(x)\right) = \frac{1}{u^{\alpha}} \left[\sum_{k=0}^{m-1} u^{\alpha-k} f^k(0) + M(f(x))\right] - \frac{1}{u^{\alpha}} \left[a_m M\left(\sum_{n=0}^{\infty} y_n^{(m)}(x)\right) + \cdots + a_1 M\left(\sum_{n=0}^{\infty} y_n'(x)\right) + a_0 M\left(\sum_{n=0}^{\infty} y_n(x)\right)\right]$$
$$-\frac{1}{u^{\alpha}} M\left(\sum_{n=0}^{\infty} A_n\right) \qquad (10)$$

Hence the iterations are defined by the following recursive algorithm

$$M(y_0(x)) = \frac{1}{u^\alpha}\sum_{k=0}^{m-1} u^{\alpha-k} f^k(0) + \frac{1}{u^\alpha} M\big(f(x)\big) \quad (11)$$

$$M\big(y_n(x)\big) = -\frac{1}{u^\alpha}\left[a_m M\left(y_{n-1}^{(m)}(x)\right) + a_{m-1} M\left(y_{n-1}^{(m-1)}(x)\right) + \cdots + a_1 M\left(y'_{n-1}(x)\right) + a_0 M\big(y_{n-1}(x)\big)\right] - \frac{1}{u^\alpha} M(A_{n-1}),$$

$$\text{for } n = 1,2,\ldots \quad (12)$$

Using the initial conditions (6) and applying the inverse Mahgoub transform to Eqns. (11) and (12) we obtain the values $y_0(x), y_1(x), y_2(x), \ldots, y_n(x)$ recursively.

IV. NUMERICAL EXAMPLES

Example 1

Consider the nonlinear fractional differential equation

$$^cD^\alpha y(x) = y^2 + 1, \quad m-1 < \alpha \le m, \; 0 < x \le 1, \quad (13)$$

Subject to the initial conditions

$$y^{(i)}(0) = 0, \qquad i = 0,1,2,\ldots,m-1 \quad (14)$$

Applying the Mahgoub transform in the Eqn. (13), then

$$M(^cD^\alpha y(x)) = M(y^2+1)$$

Use the initial conditions (14), then we have

$$M\big(y(x)\big) = \frac{1}{u^\alpha}(M(y^2)) + \frac{1}{u^\alpha}$$

In the view of (10), we have

$$M\left(\sum_{n=0}^{\infty} y_n(x)\right) = \frac{1}{u^\alpha} M\left(\sum_{n=0}^{\infty} A_n\right) + \frac{1}{u^\alpha} \quad (15)$$

The Mahgoub Adomian decomposition series has the form

$$M(y_0) = \frac{1}{u^\alpha}$$

$$M(y_n) = \frac{1}{u^\alpha} M\big(A_{n-1}(x)\big), \qquad n = 1,2,3,\ldots$$

where the Adomian polynomials for the nonlinearity $f(y) = y^2$ are

$$A_0 = {y_0}^2,$$
$$A_1 = 2y_0y_1,$$
$$A_2 = 2y_0y_2 + {y_1}^2,$$
$$A_3 = 2y_0y_3 + 2y_1y_2,$$
$$A_4 = 2y_0y_4 + 2y_1y_3 + {y_2}^2. \quad (16)$$

Using the above recursive relation, the first few terms of the Mahgoub Adomian decomposition series are derived as follows:

$$y_0 = \frac{x^\alpha}{\Gamma(\alpha+1)}$$

$$y_1 = \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} x^{3\alpha}$$

$$y_2 = \frac{2\Gamma(2\alpha+1)\Gamma(4\alpha+1)}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)} x^{5\alpha}$$

$$y_3 = \frac{\Gamma^2(2\alpha+1)\Gamma(6\alpha+1)}{\Gamma^4(\alpha+1)\Gamma^2(3\alpha+1)\Gamma(7\alpha+1)} x^{7\alpha} + \frac{4\Gamma(2\alpha+1)\Gamma(4\alpha+1)\Gamma(6\alpha+1)}{\Gamma^4(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)\Gamma(7\alpha+1)} x^{7\alpha}$$

etc.

The approximate solution is given by

$$y = y_0 + y_1 + y_2 + y_3 + \cdots$$

$$\text{i.e. } y(x) = \frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)} x^{3\alpha} + \frac{2\Gamma(2\alpha+1)\Gamma(4\alpha+1)}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)} x^{5\alpha} + \frac{\Gamma^2(2\alpha+1)\Gamma(6\alpha+1)}{\Gamma^4(\alpha+1)\Gamma^2(3\alpha+1)\Gamma(7\alpha+1)} x^{7\alpha} + \cdots$$

Table 1 shows the solution of Example 1 for different values of α. The value $\alpha = 1$ (ODE) is the only case for which we know the exact solution $y = tanx$ and our approximate solution is in good agreement with the exact values. The accuracy can be improved by computing more terms of approximate solution.

Table 1: Solution of Example 1

x	$\alpha = 1$		$\alpha = 0.5$	$\alpha = 1.5$	$\alpha = 2.5$	$\alpha = 3.5$
	Exact	MADM				
0.1	0.1003	0.100335	0.391973	0.023790	0.000952	0.000027
0.2	0.2027	0.202710	0.623723	0.067330	0.005383	0.000308
0.3	0.3093	0.309336	0.890186	0.123896	0.014833	0.001271
0.4	0.4228	0.422793	1.254167	0.191362	0.030450	0.003480
0.5	0.5463	0.546302	1.800674	0.268856	0.053197	0.007599
0.6	0.6841	0.684131	2.654936	0.356238	0.083925	0.014384
0.7	0.8423	0.842245	3.996807	0.453950	0.123412	0.024672
0.8	1.0296	1.029372	6.075247	0.563007	0.172391	0.039371
0.9	1.2602	1.258799	9.223523	0.685056	0.231574	0.059458
1.0	1.5574	1.551368	13.875249	0.822511	0.301676	0.085975

Example 2

Consider the nonlinear fractional differential equation

$$ {}^{c}D^{\alpha}y(x) = 2y - y^2 + 1, \quad 0 < \alpha \le 1, \ 0 < x \le 1, \tag{17}$$

Subject to the initial condition

$$y(0) = 0 \tag{18}$$

Applying the Mahgoub transform in the Eqn. (17), then

$$M({}^{c}D^{\alpha}y(x)) = M(2y - y^2 + 1)$$

Use the initial conditions (18), then we have

$$M\big(y(x)\big) = \frac{2}{u^\alpha}M(y(x)) - \frac{1}{u^\alpha}(M(y^2)) + \frac{1}{u^\alpha}$$

In the view of (10), we have

$$M(\sum_{n=0}^{\infty} y_n(x)) = \frac{2}{u^\alpha}M(\sum_{n=0}^{\infty} y_n(x)) - \frac{1}{u^\alpha}M(\sum_{n=0}^{\infty} A_n(x)) + \frac{1}{u^\alpha} \tag{19}$$

The Mahgoub Adomian decomposition series has the form

$$M(y_0) = \frac{1}{u^\alpha}$$

$$M(y_n) = \frac{2}{u^\alpha}M(y_{n-1}) - \frac{1}{u^\alpha}M\big(A_{n-1}(x)\big), \qquad n = 1,2,3,\ldots$$

where the Adomian polynomials for the nonlinearity $f(y) = y^2$ are given in (16)

Using the above recursive relation, the first few terms of the Mahgoub Adomian decomposition series are derived as follows:

$$y_0 = \frac{x^\alpha}{\Gamma(\alpha+1)}$$

$$y_1 = \frac{2x^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}x^{3\alpha}$$

$$y_2 = \frac{4x^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{2\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)}x^{4\alpha} - \frac{4\Gamma(3\alpha+1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)}x^{4\alpha} + \frac{2\Gamma(2\alpha+1)\Gamma(4\alpha+1)}{\Gamma^3(\alpha+1)\Gamma(3\alpha+1)\Gamma(5\alpha+1)}x^{5\alpha}$$

etc.,

The approximate solution is

$$y = \frac{x^\alpha}{\Gamma(\alpha+1)} + \frac{2x^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+1)}x^{3\alpha} + \frac{4x^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{2\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(4\alpha+1)}x^{4\alpha} - \frac{4\Gamma(3\alpha+1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha+1)}x^{4\alpha} + \cdots$$

Table 2 shows the approximate solution of Example 2 for $0 < x \le 1$ and for different values of α.

The Exact solution of Eqn. (17) for $\alpha = 1$ (ODE) is

$$y = 1 + \sqrt{2}tanh\left(\sqrt{2}x + \frac{1}{2}ln\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right)$$

Table 2: Solution of Example 2

x	$\alpha = 0.5$	$\alpha = 0.75$	$\alpha = 0.95$	$\alpha = 1$	
				Exact	MADM
0.1	0.594392	0.245432	0.128805	0.1103	0.110295
0.2	0.942711	0.475268	0.275732	0.2420	0.241977
0.3	1.184161	0.711247	0.443633	0.3951	0.395122
0.4	1.334191	0.942106	0.629672	0.5678	0.567934
0.5	1.411321	1.154790	0.828257	0.7560	0.756482
0.6	1.437452	1.338353	1.031670	0.9536	0.954756
0.7	1.436222	1.486138	1.230948	1.1529	1.155089
0.8	1.431643	1.597212	1.417081	1.3464	1.348968
0.9	1.447121	1.677276	1.582473	1.5269	1.528179
1	1.504798	1.739135	1.722571	1.6895	1.676254

Example 3

Consider the nonlinear fractional differential equation

$$ {}^{c}D^{\alpha}y(x) = 1 + y(x) + y'^{2}(x) - y^{2}(x), \quad 1 < \alpha \le 2,\ 0 < x < 1, \tag{20}$$

Subject to the initial condition

$$y(0) = 1,\ \ y'(0) = 0 \tag{21}$$

Applying the Mahgoub transform in the Eqn. (20), then

$$M({}^{c}D^{\alpha}y(x)) = M(1 + y(x) + y'^{2}(x) - y^{2}(x))$$

$$u^{\alpha}M(y(x)) - u^{\alpha}y(0) - u^{\alpha-1}y'(0) = 1 + M(y(x)) + M(y'^{2}(x) - y^{2}(x))$$

Use the initial conditions (21), then we have

$$M\big(y(x)\big) = \left(1 + \frac{1}{u^{\alpha}}\right) + \frac{1}{u^{\alpha}}\big(M\big(y(x)\big) + M(y'^{2}(x) - y^{2}(x))\big)$$

In the view of (10), we have

$$M\left(\sum_{n=0}^{\infty} y_n(x)\right) = \left(1 + \frac{1}{u^{\alpha}}\right) + \frac{1}{u^{\alpha}}\left(\sum_{n=0}^{\infty} y_n(x)\right) + M\left(\sum_{n=0}^{\infty} A_n(x)\right) \tag{22}$$

The Mahgoub Adomian decomposition series has the form

$$M(y_0) = \left(1 + \frac{1}{u^{\alpha}}\right)$$

$$M(y_n) = \frac{1}{u^{\alpha}}M(y_{n-1}) + \frac{1}{u^{\alpha}}M\big(A_{n-1}(x)\big), \qquad n = 1,2,3, \ldots$$

where the Adomian polynomials for the nonlinearity $f(y) = y'^{2} - y^{2}$ are

$$A_0 = {y_0'}^{2} - {y_0}^{2},$$

$$A_1 = 2y_0'y_1' - 2y_0y_1,$$

$$A_2 = (2y_0'y_2' + {y_1'}^{2}) - (2y_0y_2 + {y_1}^{2})$$

$$A_3 = (2y_0'y_3' + 2y_1'y_2') - (2y_0y_3 + 2y_1y_2),$$

$$A_4 = (2y_0'y_4' + 2y_1'y_3' + {y_2'}^{2}) - (2y_0y_4 + 2y_1y_3 + {y_2}^{2}).$$

Using the above recursive relation, the first few terms of the Mahgoub Adomian decomposition series are derived as follows:

$$y_0 = 1 + \frac{x^{\alpha}}{\Gamma(\alpha+1)}$$

$$y_1 = -\frac{x^{2\alpha}}{\Gamma(2\alpha+1)} + \frac{\alpha^2\Gamma(2\alpha-1)}{\Gamma^2(\alpha+1)\Gamma(2\alpha)}x^{2\alpha-1} - \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(2\alpha+2)}x^{2\alpha+1}$$

$$y_2 = -\frac{x^{3\alpha}}{\Gamma(3\alpha+1)} + \frac{\alpha^2\Gamma(2\alpha-1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha)}x^{3\alpha-1} - \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+2)}x^{3\alpha+1} - \frac{4\alpha^2\Gamma(3\alpha-1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha-1)}x^{4\alpha-2}$$
$$+ \frac{2\alpha^3(2\alpha-1)\Gamma(2\alpha-1)\Gamma(3\alpha-2)}{\Gamma^3(\alpha+1)\Gamma(2\alpha)\Gamma(4\alpha-2)}x^{4\alpha-3}$$

etc.

The approximate solution is

$$y = 1 + \frac{x^{\alpha}}{\Gamma(\alpha+1)} - \frac{x^{2\alpha}}{\Gamma(2\alpha+1)} - \frac{\alpha^2\Gamma(2\alpha-1)}{\Gamma^2(\alpha+1)\Gamma(2\alpha)}x^{2\alpha-1} - \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(2\alpha+2)}x^{2\alpha+1} - \frac{x^{3\alpha}}{\Gamma(3\alpha+1)} - \frac{\alpha^2\Gamma(2\alpha-1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha)}x^{3\alpha-1} - \frac{\Gamma(2\alpha+1)}{\Gamma^2(\alpha+1)\Gamma(3\alpha+2)}x^{3\alpha+1} - \frac{4\alpha^2\Gamma(3\alpha-1)}{\Gamma(\alpha+1)\Gamma(2\alpha+1)\Gamma(4\alpha-1)}x^{4\alpha-2} + \cdots$$

Table 3: Solution of Example 3

x	$\alpha = 1.5$	$\alpha = 1.75$	$\alpha = 1.95$	$\alpha = 2$
0.1	1.030605	1.012547	1.006319	1.005329
0.2	1.096489	1.045703	1.025998	1.022611
0.3	1.193160	1.099420	1.060707	1.053754
0.4	1.319540	1.174820	1.112238	1.100662
0.5	1.474069	1.273234	1.182449	1.165276
0.6	1.653487	1.395788	1.273231	1.249572
0.7	1.851824	1.542923	1.386405	1.355510
0.8	2.059272	1.713739	1.523488	1.484876
0.9	2.260871	1.905098	1.685303	1.638985
1	2.434909	2.110408	1.871333	1.818163

V. CONCLUSION

The main purpose of this paper is to find the solution of nonlinear fractional differential equation. The Adomian decomposition method is a powerful device for solving many functional equations. Our goal is to evaluate the nonlinear term in the fractional derivative using Adomian polynomials. In this paper, the Mahgoub Adomian decomposition method is proposed. Illustrative examples have been demonstrated the applicability of the presented new method.

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