wgr-continuous and wgr-irresolute functions in topology

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Abstract

In this paper we define and study the concept of weakly generalized regular (brifly.wgr-) continuous functions, contra wgr-continuous functions , strongly wgr-continuous functions, spwgr-continuous functions and wgr-irresolute functionsand wgr-connected spaces.

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1. Introduction

For the first time , N.Levine [8] has introduced the notion g-closed sets and g-open sets in topology. In 1993, N.Palaniappan [11] has defined and studied the notions of rg-closed sets , rg-continuity and rg-irresoluteness in topological spaces.In 1995, 1997 , 2009, 2011 , resp., Dontchev [6] , Arokirani et al[4] , Al-Omari et al [1] , S.Bhattacharya [5] and S.I.Mahmood [9] have defined and studied the concepts of gsp-closed sets , rg-open functions ,gpr-closed sets , gb-closed sets ,gr-closed sets and gr-continuity and gr-irresoluteness in toplogical spaces.In this paper , we define and study the concept of weakly generalized regular (brifly.wgr-) continuous functions, contra wgr-continuous functions , strongly wgr-continuous functions, spwgr-continuous functions and wgr-irresolute functionsand wgr-connected spaces.

2. Preliminaries

Throughout this paper (X , τ) and (Y, σ) (or simply X and Y) denote topological spaces on which no separation axioms are assumed unless explicitly stated . If A be a subset of X, the Closure of A and Interior of A denoted by Cl(A) and Int(A)respectivly.

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We, recall the following definitions and results hich are useful in the sequel.

2 Preliminaries

Definition 2.1: The subset of A of X is said to be.

(i) semi-pre open[2]set , if A $\subset Cl(Int(Cl(A)))$

(ii) regular open set [11] , if A =Int Cl(A).

(iii)regular closed set [11] , if A = Cl Int(A).

(iv)b-open [3] if A $\subset$ ClInt(A) $\cup$ IntCl(A).

The complement of a semipre-open (resp. b-open) set of a space X is called semipreclosed (resp. b-closed) in X .

Definition 2.2[2]: The intersection of all semipre-closed sets of X containing subset A is called the semipre-closure of A and is denoted by spCl(A).

Definition 2.3[11]: The intersection of all regular closed sets containing set A is called the regular closure of A and is denoted by rCl(A).

Definition 2.4[3]: The intersection of all b-closed sets containing set A is called the b- closure of A and is denoted by bCl(A).

Similarly , spInt(A) ,pInt(A), rInt(A) ,bInt(A),δInt(A) can be defined.

Definition 2.5[7]: A function f:X→Y is called contra-continuous if $f^{-1}(U)$ is closed in X for each open set U in Y.

Definition 2. 7: A subset A of a space (X, τ) is called:

(i) generalized closed (briefly, g- closed) **[8]** set if Cl(A) $\subseteq U$ whenever A $\subseteq U$ and U is open set in X

(ii) generalized regular -closed (briefly, gr- closed) **[5]** set if rCl(A) $\subseteq U$ whenever A $\subseteq U$ and U is semi-open set in X

(iii) regular generalized (briefly, rg- closed) [**11**] set if Cl(A) $\subseteq U$ whenever A $\subseteq U$ and U is r-open set in X

(iv) generalized semi-preclosed (briefly, gsp- closed) [**6**] set if spCl(A) $\subseteq U$ whenever A $\subseteq U$ and U is open in X

(v) generalized b-closed (brifly, gb-closed) [1] if bCl(A) ⊂U whenever A ⊂ U and U is open in X

The complement of a g-closed (resp, gr-closed, rg-closed ,gb-closed) set in X is called g-open (resp. gr-open, rg- open, gb-open) set in X.

3.Properties of continuous functions

We, recall the following

Definition 3.1[10]: A subset A of space X is called weakly generaralized regular closed (brifly,wgr-closed) set if rCl(A)⊆U whenever A⊆U and U is semipreopen in X

The complement of a wgr-closed set of X is called wgr-open set in X. The family of all wgr-open (resp,wgr-closed) sets a space X is denoted by WGRO(X) (resp, WGRF(X))

Clearly, every regular closed set is wgr-closed set, every gr-closed set is wgr-closed set

Definition 3.2: A function f:X→Y is called wgr-continuous if f^{-1} (V) is wgr -closed in X for every closed subset V of Y

Clearly, every gr-continuous functions is wgr- continuous**.**

Theorem 3.3: Let f:X→Y be a function. Then the following are equivalent.

(i) f is wgr-continuous

(ii) The inverse image of each open set Y is wgr-open in X

(iii) The inverse image of each closed set in Y is wgr-closed in X

Proof: (i)⟹(ii): Let G be open in Y. Then Y-G is closed in Y. By (i) f^{-1}(Y-G) wgr-closed in X. But f^{-1}(Y-G) = X-- f^{-1}(G) which is wgr-closed in X. Therefore f^{-1}(G) is wgr-open in X.

(ii)⟹(iii) and

(iii)⟹(i) follows easily.

We, recall the following

Definition 3.4 [10]: The intersection of all wgr-closed set containing set A is called the wgr-closure of A and is denoted wgrCl(A)

Lemma 3.5: Let x∊X, then x∊ wgr-Cl(A) if and only if V∩A≠ ϕ for every wgr-open set V containing x

Theorem 3.6: If a function f:X→Y is wgr-continuous then f(wgr-Cl(A)) ⊆Cl(f(A)) for every subset A of X.

Proof: Let f:X→Y be wgr-continuous. Let A⊆X. Then Cl(f(A)) is closed in Y. Since f is wgr-continuous, f^{-1}(Cl(A)) is wgr-closed in X.Suppose y∊ f(x), x∊wgr-Cl(A) Let G be an open set containing y = f(x). Since f is wgr-continuous. by Theorem 3.3, $f^{-1}(G)$ is wgr-open containing x so that $f^{-1}(G) \cap A \neq \phi$ by Lemma 3.5,. Therefore $f(f^{-1}(G) \cap A) \neq \phi$ which implies $f(f^{-1}(G) \cap f(A)) \neq \phi$.Since $f(f^{-1}(G) \subseteq G$, $G \cap f(A) \neq \phi$.This proves that y∊Cl(f(A)) that implies f(wgr-Cl(A)) ⊆Cl(f(A)).

Lemma 3.7: A subset A of space X is called wgr-open set if U ⊆rInt(A) whenever U⊆A and U is semipre-closed set in X

We, recall the following

Definition 3.8[10] : The union of all wgr-open sets which contained in A is called the wgr-interior of A and is denoted by wgrInt(A)

Theorem 3.9: Let X be a space in which every singleton set is rg-closed. Then f : X→Y is wgr-continuous if and only if x∊rInt(f^{-1}(V)) for every open subset V of Y containing f(x)

Proof: Suppose f:X→Y is wgr-continuous. Fix x∊X and an open set V in Y such that f(x)∊V. Then f^{-1}(V) is wgr-open. Since x∊ f^{-1}(V) and since {x} is rg-closed, x∊rInt(f^{-1}(V)) by lemma 3.7

Conversely, assume that x∈ rInt(f^{-1}(V)) for every open subset V of Y containing f(x). Let V be an open set in Y. Suppose F⊆ f^{-1}(V) and F is rg-closed. Let x∈F. Then f(x)∈V so that x∈rInt(f^{-1}(V)) that implies F⊆rInt(f^{-1}(V)). Therefore by lemma 3.8. f^{-1}(V) is wgr-open. This proves that f is wgr-continuous.

Theore 3.10: Let f:X→Y be wgr-continuous and g : Y→Z be continuous, then gof: X →Z be wgr-continuous.

Proof: Let V be any open subset of Z. Then g^{-1}(V) is open in Y. Since g is continuous function. Again , f is wgr- continuous and g^{-1}(V) is open set in Y then $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$ is wgr-open in X. This shows that gof is wgr-continuous.

We, define the following

Definition 3.11: A function f:X→Y is called contra wgr-continuous if f^{-1}(V) is wgr-closed in X for each open set V in Y

Definition 3.12: A space X is called wgr-$T_{1/2}$ if every wgr-closed set is regular -closed.

Theorem 3.13: Let f : X→Y be wgr-continuous and g : Y→Z be contra-continuous, then gof: X →Z be contra wgr-continuous.

Proof: Obvious.

We, define the following

Definition 3.14: **A** function f : X→Y is called strongly wgr-continuous if the inverse image of each wgr-open set of Y is open in X.

Definition 3.15: A set U⊂X is said to be a wgr-neighbourhood of a point x∈X if and only if there exists a wgr-open set A in X such that x∈A⊂U

Theorem 3.16: The following statement are equivalent for a function f : X→Y :

(i) f is strongly wgr-continuous

(ii) For each point x of X and each wgr-neighborhood V of f(x),there exist a open-neighborhood Uof x such that f(U) ⊆V

(iii) For each x in X and each V∈WGRO(f(x)), there exists U∈τ such that f(U) ⊆V

Proof: (i)⟹(ii): Assume x∈X and V is wgr-open set in Y containing f(x). Since , f is strongly wgr-continuous and let W=f^{-1}(V) be a open set in X containing x and hence f(W)= f(f^{-1}(V))⊂V.

(ii)⟹(iii): Assume that V⊂Y is a wgr-open set containing f(x), Then by (ii) there exists a open set U such that x∈U⊂ f^{-1}(V). Therefore, x∈ f^{-1}(V) ⊂ Cl(f^{-1}(V)). This shows that Cl(f^{-1}(V)) is a open-neighborhood of x.

(iii)⟹(i): Let V be a wgr-open set in Y, then Cl(f^{-1}(V)) is a open neighborhood of each x∈ f^{-1}(V). Thus, for each x is a interior point of Cl(f^{-1}(V)) which implies that f^{-1}(V) ⊂U. Therefore, f^{-1}(V) is a open set in X and hence f is a strongly wgr-continuous function.

We, define the following.

Definition 3.17 : A function f : X→Y is called spwgr-continuous if the inverse image of each semipreopen set of Y is wgr-open in X.

Clearly, every spwgr-continuous function is wgr-continuous function.

Theorem 3.18. Let f : X→Y be spwgr-continuous function and g :Y→Z be semipreirresolute then gof :X→Z is spwgr-continuous function.

Proof : Obvious.

Theorem 3.19 : Let f : X→Y be strongly-wgr-continuous function and g :Y→Z be spwgr-continuous then gof :X→Z is strongly semiprecontinuous function.

Proof : Obvious.

Definition 3.20: A function f:X→Y is called wgr-irresolute if f^{-1}(V) is wgr-closed in X for every wgr-closed subset V of Y

Theorem 3.21: Every wgr-irresolute function is wgr-continuous

Proof: Suppose f:X→Y is wgr-irresolute. Let V be any closed subset of Y. Then V is semi-pre-closed in Y. Then using lemma (Every semi-pre closed set is wgr-closed). V is wgr-closed in Y. Since f is wgr-irresolute, f^{-1}(V) is wgr-closed in X. This proves the theorem.

We, recall the following

Definition 3.22: A function f:X→Y is called r-closed if be image of each closed set of X is regular closed in Y

Theorem 3.23: Let f:X→Y be rg-irresolute and r-closed. Then f maps a wgr-closed set in X into a wgr-closed set in Y

Proof: Let A be wgr-closed in X. let f(A) ⊆ U, where U is rg-open in Y. Then A ⊆ f^{-1}(U)

Since f is rg-irresolute, f^{-1}(U) is rg-open in X. Since A is wgr-closed,

rCl(A) ⊆ f^{-1}(U) that implies f(rCl(A) ⊆ U.

Since f is r- closed f(rCl(A) is r-closed that implies rCl(f (A)) ⊆ rCl(f(rCl(A))=f(rCl(A) ⊆ U.

By using Definition 2.2 .f(A) is wgr-closed in Y

Theorem 3.24. Let f:X→Y and g : Y→Z be any two function. Let h=g.f Then

(i) h is wgr-continuous if f is wgr-irresolute and g is wgr- continuous

(ii) h is wgr-irresolute if both f abd g are both wgr-irresolute and

(iii) h is wgr-continuous if g is continuous and f is wgr-continuous

Proof: Let V be closed in Z. Suppose f is wgr-irresolute and g is wgr- continuous. Since g is wgr-continuous ,g^{-1}(V) is wgr-closed in Y. Since f is wgr-irresolute, using definition 3.23,

f^{-1} (g^{-1}(V)) is wgr-closed in X. This proves (i). To prove (ii), let f and g be both wgr-irresolute. Then g^{-1}(V) is wgr-closed in y. Since f is wgr-irresolute, using definition 3.23

We, define the following.

Definition 3.27 : A Topological space X is said to be wgr-connected if X cannot be written as the disjoint union of to non empty wgr-open sets in X.

Theorem 3.25: Let f:X→Y be a function

(i) If X is wgr-connected and if f is wgr-continuous, surjective, then Y is connected

(ii) If X is wgr-connected and if f is wgr-irresolute ,surjective, then Y is wgr-connected.

Proof: Let X be wgr-connected and f be wgr-continuous, surjective. Suppose Y is disconnected. Then Y = A∪B, where A and B are disjiont non empty open subset of Y. Since f is wgr-continuous surjective by using Theorem 3.3, X=f^{-1}(A)∪f^{-1}(B) where f^{-1}(A) and f^{-1}(B) are disjoint non empty wgr-open subsets of X. This contradicts the fact that X is wgr-connected. Therefore Y is connected. This proves (i)

Let X be wgr-connected and f be wgr-irresolute surjective. Suppose Y is not wgr-connected. Then Y = A∪B where A and B are disjoint non empty wgr-open subsets of Y. Since f is wgr-irresolute surjective by Theorem **X=** f^{-1}(A)∪f^{-1}(B) where f^{-1}(A) and f^{-1}(B) are disjoint non empty wgr-open subsets of X. This implies X is not wgr-connected a contradiction. Therefore Y is wgr-connected. This prove (ii)

$f^{-1}(g^{-1}(V))$ is wgr-closed in X. This proves (ii). Finally to prove (iii), let g be continuous and f be wgr-continuous. Then $g^{-1}(V)$ is closed in Y. Since f is wgr-continuous, using definition 3. $f^{-1}(g^{-1}(V))$ is wgr-closed in X. This proves (iii).

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