EOQ Model For Linearly Time Dependent Deteriorating Items With Variable Demand Under Trade Credits

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Abstract:

In this paper economic ordering policies of linear time-dependent deteriorating items and variable demand rateis presented. Trade credits are also used for benefits of buyer and supplier. In the inventory model, it is assumed that shortages are not allowed. In the present study mathematical models are derived under two different cases i.e. Case 1: $T > t_1$, Case 2: $t_1 > T$ in which the customer earns permissible delay period on sales revenue up to the permissible delay period. Theoretical Mathematical formulation is derived for an inventory system. Examples are given for two cases to validate the formulation.

Key words: Inventory, credit, linear time- dependent deterioration , unit purchase cost dependent demand rate.

1. Introduction:

Many economic order quantity (EOQ) inventory models were developed during last decades under the assumptions of constant deterioration and constant demand rate. Later on, many researchers developed EOQ models by taking variable demand rate. In deriving the EOQ formula, it is assumed that the customer must pay for the items as soon as he receives items from a seller. However in practice, a seller will allow a certain fixed period (credit period) for settling the amount the retailer owes to him for the items supplied.
The study of inventory system comes into effect in 1915. Harris (1915) studied inventory problems. He developed an EOQ model that was also derived independently by Wilson (1934).
Demand of inventory may vary with time, price or with the inventory level displayed in a market. In recent years, many inventory models are working for the economic replenishment policy for an inventory system having time dependent demand rate. Berotoni (1962) inspired Covert and Philip (1973) to develop an inventory model for deteriorating items with Weibull distribution by using two parameters. Silver and Meal (1969) developed an EOQ model by using time varying-demand rate. Donaldson (1977) developed an inventory replenishment model with a linear demand rate over a finite - time horizon.
Dave and Patel (1983) developed an inventory model for deteriorating items with time dependent demand rate. Chung (1989) presented the analysis of the effect of the trade credit on inventory discussions based on the accepted principles of financial analysis. Data and Pal (1988) developed an EOQ model by introducing a variable deterioration rate and power demand pattern. Chung and Tang (1994) determined the replenishment schedules for deteriorating items with time proportional demand.
Teng et al. (2012) developed an EOQ model with trade credit financing for non–decreasing demand and fundamental theoretical results obtained. Sarkar (2011) developed an EOQ model with delay in payment for time varying deterioration rate and obtained a function for maximization of profit. Pricing and lot sizing policies for deteriorating items with partial backlogging under inflation was presented by Hsieh and Dye (2010) by considering pricing and lot sizing policies for deteriorating items with partial backlogging under inflation. This study is related to an EOQ model for a deteriorating item demand rate is unit purchase cost dependent and variable deterioration under permissible delay in payment. The proposed inventory model is

based on deteriorating items like fruits and vegetables whose deterioration rate increases with time. Among the various time-varying demands in EOQ models, the more realistic demand approach is to consider a quadratic demand rate along with variable rate of deterioration. For setting the account, the model is developed under two circumstances: case-1: The credit period is less than or equal to the cycle time and case-2: the credit period is greater than the cycle time. Main emphasis is laid on working out on exact solution for the model. An example is provided which stands in support of the developed model. The sensitivity of the solution with the changes of the values of the parameters associated with the model is discussed.

2. Assumptions and Notations:

The following assumptions are made for developing the model

(i) The demand rate is unit purchase cost dependent
(ii) The deterioration is linearly time dependent.
(iii) Shortages are not allowed and lead time is zero.
(iv) During the permissible delay period the sales revenue generated is deposited in an interest bearing account. At the end of the trade credit period the customer pays off all units ordered and begins paying for the interest charged on the items in stock.

The following notations are used for developing the model.

(i)	p	:	The unit purchase cost
(ii)	$D = ap^{-\gamma}$	:	The unit purchase cost dependent demand rate, $\gamma > 0$
(iii)	h_p	:	The inventory holding cost (excluding interest charges) per rupee of unit purchase cost
(iv)	A	:	The replenishment cost
(v)	I_p	:	The interest charges per rupee investment per year
(vi)	I_e	:	The interest earned per rupee per year
(vii)	I_c	:	The interest charged per rupee per year
(viii)	t_1	:	The permissible period (in year) of delay in settling the account with the supplier
(ix)	T	:	Time period interval (in year between two successive years)
(x)	$H.C.$	:	Holding cost
(xi)	IP_1	:	Interest is payable during the time $(T - t_1)$
(xii)	IE_1	:	Interest earn in the cycle $[0, T]$
(xiii)	$Z_1(\text{p}, T)$	:	Profit when $T > t_1$

(xiv) $Z_2(\mathrm{p},T)$: Profit when $t_1 > T$

3. Mathematical Formulation:

$$\frac{dI(t)}{dt}+(\alpha+\beta t)I(t)=-D \qquad \dots (1)$$

Where $0<\alpha<1,\ \beta>0$

Deterioration rate $=\alpha+\beta t$

Demand rate $D=ap^{-\gamma},\ \gamma>0$

With boundary conditions $I(0)=I_0$ and $I(T)=0$

Solution of equation (1) is given by

$$I(t)=De^{-\left(\alpha t+\frac{1}{2}\beta t^2\right)}\left\{(T-t)+\frac{1}{2}\alpha\left(T^2-t^2\right)+\frac{1}{6}\beta\left(T^3-t^3\right)\right\} \qquad \dots (2)$$

Holding cost: For the cycle $[0,T]$

$$H.C.=h\int_0^T I(t)\,dt$$

$$=h\int_0^T De^{-\left(\alpha t+\frac{1}{2}\beta t^2\right)}\left\{(T-t)+\frac{1}{2}\alpha\left(T^2-t^2\right)+\frac{1}{6}\beta\left(T^3-t^3\right)\right\}dt$$

$$=hD\left\{\frac{1}{2}T^2+\frac{1}{6}T^3+\frac{1}{24}\left(2\beta-3\alpha^2\right)T^4-\frac{1}{12}\alpha\beta T^5-\frac{1}{72}\beta^2T^6\right\}$$

Where $h=h_p p$

Neglecting the terms of degree more than four

$$H.C.=\frac{1}{2}hDT^2\left\{1+\frac{1}{3}\alpha T+\frac{1}{12}\left(2\beta-3\alpha^2\right)T^2\right\} \qquad \dots (3)$$

Case I: When $T>t_1$

The interest is payable during the time $(T-t_1)$.The interest payable in any cycle $[0,T]$ is

$$IP_1=pI_p\int_{t_1}^T I(t)\,dt$$

$$=pI_p\int_{t_1}^T De^{-\left(\alpha t+\frac{1}{2}\beta t^2\right)}\left\{(T-t)+\frac{1}{2}\alpha\left(T^2-t^2\right)+\frac{1}{6}\left(T^3-t^3\right)\right\}dt$$

$$=pDI_p\left\{\begin{array}{l}\left(-t_1+\frac{1}{2}t_1^2+\frac{1}{6}\beta t_1^3\right)T+\left(\frac{1}{2}-\frac{1}{2}\alpha t_1+\frac{1}{4}\alpha^2t_1^2\right)T^2+\frac{1}{6}\left(\alpha-\beta t_1\right)T^3+\\ \frac{1}{24}\left(2\beta-3\alpha^2\right)T^4+\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}\left(2\beta+3\alpha^2\right)t_1^4\end{array}\right\} \qquad \dots (4)$$

Interest earn in the cycle $[0,T]$ is

$$IE_1=pI_e\int_0^T tD\,dt$$

$$IE_1=\frac{1}{2}pI_eDT^2 \qquad \dots (5)$$

Profit = Sales revenue – Cost of placing order (O.C.) – Purchasing cost – Carrying cost – Interest payable per year + Interest earned per cycle.

$$Z_1(\mathrm{p},T)=pD-\frac{A}{T}-\frac{CQ}{T}-\frac{Dh}{T}\left\{\frac{1}{2}T^2+\frac{1}{6}\alpha T^3+\frac{1}{24}\left(2\beta-3\alpha^2\right)T^4\right\}$$

$$-\frac{pDI_p}{T}\left\{\begin{array}{l}\left(-t_1+\frac{1}{2}t_1^2+\frac{1}{6}\beta t_1^3\right)T+\left(\frac{1}{2}-\frac{1}{2}\alpha t_1+\frac{1}{4}\alpha^2 t_1^2\right)T^2+\frac{1}{6}(\alpha-\beta t_1)T^3+\\ \frac{1}{24}(2\beta-3\alpha^2)T^4+\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}(2\beta+3\alpha^2)t_1^4\end{array}\right\}+\frac{1}{2}\frac{pI_eDT^2}{T}$$

$$Z_1(\mathrm{p},T)=pD-\frac{A}{T}-CD\left(1+\frac{1}{2}\alpha T+\frac{1}{6}\beta T^2\right)-Dh\left\{\frac{1}{2}T+\frac{1}{6}\alpha T^2+\frac{1}{24}(2\beta-3\alpha^2)T^3\right\}$$

$$-pDI_p\left[\begin{array}{l}\left(-t_1+\frac{1}{2}t_1^2+\frac{1}{6}\beta t_1^3\right)+\left(\frac{1}{2}-\frac{1}{2}\alpha t_1+\frac{1}{4}\alpha^2 t_1^2\right)T+\frac{1}{6}(\alpha-\beta t_1)T^2+\frac{1}{24}(2\beta-3\alpha^2)T^3\\ +\frac{1}{T}\left\{\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}(2\beta+3\alpha^2)t_1^4\right\}\end{array}\right]+\frac{1}{2}pDI_eT$$

Put $D=ap^{-\gamma}$ and $h=h_p p$

$$Z_1(\mathrm{p},T)=ap^{-\gamma+1}-\frac{A}{T}-Cap^{-\gamma}\left(1+\frac{1}{2}\alpha T+\frac{1}{6}\beta T^2\right)-ap^{-\gamma+1}h_p\left\{\frac{1}{2}T+\frac{1}{6}\alpha T^2+\frac{1}{24}(2\beta-3\alpha^2)T^3\right\}$$

$$-ap^{-\gamma+1}I_p\left[\left(-t_1+\frac{1}{2}t_1^2+\frac{1}{6}\beta t_1^3\right)+\left(\frac{1}{2}-\frac{1}{2}\alpha t_1+\frac{1}{4}\alpha^2 t_1^2\right)T+\frac{1}{6}(\alpha-\beta t_1)T^2\right.$$

$$\left.+\frac{1}{24}(2\beta-3\alpha^2)T^3+\frac{1}{T}\left\{\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}(2\beta+3\alpha^2)t_1^4\right\}\right]+\frac{1}{2}ap^{-\gamma+1}I_eT \quad \ldots(6)$$

Differentiating (6) w.r.t 'p'

$$\frac{\partial Z_1(p,T)}{\partial p}=a(1-\gamma)p^{-\gamma}+Ca\gamma p^{-\gamma-1}\left(1+\frac{1}{2}\alpha T+\frac{1}{6}\beta T^2\right)$$

$$-a(1-\gamma)p^{-\gamma}h_p\left\{\frac{1}{2}T+\frac{1}{6}\alpha T^2+\frac{1}{24}(2\beta-3\alpha^2)T^3\right\}$$

$$-a(1-\gamma)p^{-\gamma}I_p\left[\begin{array}{l}\left(-t_1+\frac{1}{2}t_1^2+\frac{1}{6}\beta t_1^3\right)+\left(\frac{1}{2}-\frac{1}{2}\alpha t_1+\frac{1}{4}\alpha^2 t_1^2\right)T+\frac{1}{6}(\alpha-\beta t_1)T^2\\ +\frac{1}{24}(2\beta-3\alpha^2)T^3+\frac{1}{T}\left\{\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}(2\beta+3\alpha^2)t_1^4\right\}\end{array}\right]+\frac{1}{2}a(1-\gamma)p^{-\gamma}I_eT \quad \ldots(7)$$

$$\frac{\partial^2 Z_1(p,T)}{\partial p^2}=-\gamma a(1-\gamma)p^{-\gamma-1}-(1+\gamma)Ca\gamma p^{-\gamma-2}\left(1+\frac{1}{2}\alpha T+\frac{1}{6}\beta T^2\right)$$

$$+a\gamma(1-\gamma)p^{-\gamma-1}h_p\left\{\frac{1}{2}T+\frac{1}{6}\alpha T^2+\frac{1}{24}(2\beta-3\alpha^2)T^3\right\}$$

$$+\gamma a(1-\gamma)p^{-\gamma-1}I_p\left[\begin{array}{l}\left(-t_1+\frac{1}{2}t_1^2+\frac{1}{6}\beta t_1^3\right)+\left(\frac{1}{2}-\frac{1}{2}\alpha t_1+\frac{1}{4}\alpha^2 t_1^2\right)T+\frac{1}{6}(\alpha-\beta t_1)T^2\\ +\frac{1}{24}(2\beta-3\alpha^2)T^3+\frac{1}{T}\left\{\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}(2\beta+3\alpha^2)t_1^4\right\}\end{array}\right]-\frac{1}{2}\gamma a(1-\gamma)p^{-\gamma-1}I_eT \ldots(8)$$

Differentiating (6) w.r.t 'T'

$$\frac{\partial Z_1(p,T)}{\partial T}=\frac{A}{T^2}-Cap^{-\gamma}\left(\frac{1}{2}\alpha+\frac{1}{3}\beta T\right)-ap^{-\gamma+1}h_p\left\{\frac{1}{2}+\frac{1}{3}\alpha T+\frac{1}{8}(2\beta-3\alpha^2)T^2\right\}$$

$$-ap^{-\gamma+1}I_p\left[\begin{array}{l}\left(\frac{1}{2}-\frac{1}{2}\alpha t_1+\frac{1}{4}\alpha^2 t_1^2\right)+\frac{1}{3}(\alpha-\beta t_1)T+\frac{1}{8}(2\beta-3\alpha^2)T^2\\ -\frac{1}{T^2}\left\{\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}(2\beta+3\alpha^2)t_1^4\right\}\end{array}\right]+\frac{1}{2}ap^{-\gamma+1}I_e \quad \ldots(9)$$

Differentiating (9) w.r.t 'p'

$$\frac{\partial Z_1(p,T)}{\partial p\partial T}=Ca\gamma p^{-\gamma-1}\left(\frac{1}{2}\alpha+\frac{1}{3}\beta T\right)-a(1-\gamma)p^{-\gamma}h_p\left\{\frac{1}{2}+\frac{1}{3}\alpha T+\frac{1}{8}(2\beta-3\alpha^2)T^2\right\}$$
$$-a(1-\gamma)p^{-\gamma}I_p\begin{bmatrix}\left(\frac{1}{2}-\frac{1}{2}\alpha t_1+\frac{1}{4}\alpha^2t_1^2\right)+\frac{1}{3}(\alpha-\beta t_1)T+\frac{1}{8}(2\beta-3\alpha^2)T^2\\-\frac{1}{T^2}\left\{\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}(2\beta+3\alpha^2)t_1^4\right\}\end{bmatrix}+\frac{1}{2}a(1-\gamma)p^{-\gamma}I_e \quad \ldots (10)$$

$$\frac{\partial^2 Z_1(p,T)}{\partial T^2}=-\frac{2A}{T^3}-\frac{1}{3}Cap^{-\gamma}\beta-ap^{-\gamma+1}h_p\left\{\frac{1}{3}\alpha+\frac{1}{4}(2\beta-3\alpha^2)T\right\}$$
$$-ap^{-\gamma+1}I_p\left[\frac{1}{3}(\alpha-\beta t_1)+\frac{1}{4}(2\beta-3\alpha^2)T+\frac{2}{T^3}\left\{\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}(2\beta+3\alpha^2)t_1^4\right\}\right] \quad \ldots (11)$$

Put $\frac{\partial Z_1(p,T)}{\partial p}=0$

$$a(1-\gamma)p^{-\gamma}+Ca\gamma p^{-\gamma-1}\left(1+\frac{1}{2}\alpha T+\frac{1}{6}\beta T^2\right)+a(1-\gamma)p^{-\gamma}h_p\left\{\frac{1}{2}T+\frac{1}{6}\alpha T^2+\frac{1}{24}(2\beta-3\alpha^2)T^3\right\}$$
$$-a(1-\gamma)p^{-\gamma}I_p\begin{bmatrix}\left(-t_1+\frac{1}{2}t_1^2+\frac{1}{6}\beta t_1^3\right)+\left(\frac{1}{2}-\frac{1}{2}\alpha t_1+\frac{1}{4}\alpha^2t_1^2\right)T+\frac{1}{6}(\alpha-\beta t_1)T^2+\frac{1}{24}(2\beta-3\alpha^2)T^3\\+\frac{1}{T}\left\{\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}(2\beta+3\alpha^2)t_1^4\right\}\end{bmatrix}$$
$$+\frac{1}{2}a(1-\gamma)p^{-\gamma}I_eT=0$$

Multiplying by $24p^{\gamma+1}T$

$$24a(1-\gamma)pT+Ca\gamma\left(24T+12\alpha T^2+4\beta T^3\right)+(1-\gamma)a\gamma h_pp\left\{12T^2+4\alpha T^3+(2\beta-3\alpha^2)T^4\right\}$$
$$-a(1-\gamma)I_p\begin{bmatrix}\left(-24t_1+12t_1^2+4\beta t_1^3\right)T+\left(12-12\alpha t_1+6\alpha^2t_1^2\right)T^2+4(\alpha-\beta t_1)T^3+(2\beta-3\alpha^2)T^4\\+\left\{12t_1^2-4\alpha t_1^3-(2\beta+3\alpha^2)t_1^4\right\}\end{bmatrix}$$
$$+12a(1-\gamma)I_epT^2=0$$

$$\left\{(1-\gamma)a\gamma h_pp(2\beta-3\alpha^2)\right\}T^4+\left\{4\alpha(1-\gamma)a\gamma h_pp-a(1-\gamma)I_p+4(\alpha-\beta t_1)+4\beta Ca\gamma\right\}T^3$$
$$+\left\{12(1-\gamma)a\gamma h_pp-a(1-\gamma)I_p+\left(12-12\alpha t_1+6\alpha^2t_1^2\right)+12a(1-\gamma)I_ep+12Ca\alpha\gamma\right\}T^2$$
$$+\left\{24a(1-\gamma)p+24Ca\gamma-a(1-\gamma)I_p\left(-24t_1+12t_1^2+4\beta t_1^3\right)\right\}T$$
$$-a(1-\gamma)I_p\left\{12t_1^2-4\alpha t_1^3-(2\beta+3\alpha^2)t_1^4\right\}=0 \quad \ldots(12)$$

Put $\frac{\partial Z_1(p,T)}{\partial T}=0$

$$\frac{A}{T^2}-ap^{-\gamma+1}h_p\left\{\frac{1}{2}+\frac{1}{3}\alpha T+\frac{1}{8}(2\beta-3\alpha^2)T^2\right\}$$
$$-ap^{-\gamma+1}I_p\begin{bmatrix}\left(\frac{1}{2}-\frac{1}{2}\alpha t_1+\frac{1}{4}\alpha^2t_1^2\right)+\frac{1}{3}(\alpha-\beta t_1)T+\frac{1}{8}(2\beta-3\alpha^2)T^2\\-\frac{1}{T^2}\left\{\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}(2\beta+3\alpha^2)t_1^4\right\}\end{bmatrix}+\frac{1}{2}a(1-\gamma)p^{-\gamma}I_e=0$$

Multiplying by $24T^2$

$$24A-ap^{-\gamma+1}h_p\left\{12T^2+8\alpha T^3+3\left(2\beta-3\alpha^2\right)T^4\right\}$$
$$-ap^{-\gamma+1}I_p\begin{bmatrix}\left(12-12\alpha t_1+6\alpha^2t_1^2\right)T^2+8\left(\alpha-\beta t_1\right)T^3+3\left(2\beta-3\alpha^2\right)T^4\\-\left\{12t_1^2-4\alpha t_1^3-\left(2\beta+3\alpha^2\right)t_1^4\right\}\end{bmatrix}+12a\left(1-\gamma\right)p^{-\gamma}I_eT^2=0$$

$$\left\{-3ap^{-\gamma+1}h_p\left(2\beta-3\alpha^2\right)-ap^{-\gamma+1}I_p3\left(2\beta-3\alpha^2\right)\right\}T^4+\left\{-ap^{-\gamma+1}h_p8\alpha-ap^{-\gamma+1}I_p8\left(\alpha-\beta t_1\right)\right\}$$
$$+\left\{-12ap^{-\gamma+1}h_p-ap^{-\gamma+1}I_p\left(12-12\alpha t_1+6\alpha^2t_1^2\right)+12a\left(1-\gamma\right)p^{-\gamma}I_e\right\}T^2+24A$$
$$+ap^{-\gamma+1}I_p\left\{12t_1^2-4\alpha t_1^3-\left(2\beta+3\alpha^2\right)t_1^4\right\}=0 \qquad ...(13)$$

$$\frac{\partial^2Z_1\left(p,T\right)}{\partial p\partial T}=Ca\gamma p^{-\gamma-1}\left(\frac{1}{2}\alpha+\frac{1}{3}\beta T\right)-a\left(1-\gamma\right)p^{-\gamma}h_p\left\{\frac{1}{2}+\frac{1}{3}\alpha T+\frac{1}{8}\left(2\beta-3\alpha^2\right)T^2\right\}$$
$$-a\left(1-\gamma\right)p^{-\gamma}I_p\begin{bmatrix}\left(\frac{1}{2}-\frac{1}{2}\alpha t_1+\frac{1}{4}\alpha^2t_1^2\right)+\frac{1}{3}\left(\alpha-\beta t_1\right)T+\frac{1}{8}\left(2\beta-3\alpha^2\right)T^2\\-\frac{1}{T^2}\left\{\frac{1}{2}t_1^2-\frac{1}{6}\alpha t_1^3-\frac{1}{24}\left(2\beta+3\alpha^2\right)t_1^4\right\}\end{bmatrix}$$

For $rt-s^2$ put $r=\frac{\partial^2Z_1\left(p,T\right)}{\partial p^2},t=\frac{\partial^2Z_1\left(p,T\right)}{\partial T^2}$ and $s=\frac{\partial^2Z_1\left(p,T\right)}{\partial p\partial T}$

$$\frac{\partial^2Z_1\left(p,T\right)}{\partial p^2}\frac{\partial^2Z_1\left(p,T\right)}{\partial T^2}-\left(\frac{\partial^2Z_1\left(p,T\right)}{\partial p\partial T}\right)^2>0 \text{ And } r=\frac{\partial^2Z_1\left(p,T\right)}{\partial p^2}<0$$

Case II: $t_1>T$

In this case the customer earns permissible delay period on sales revenue; up to the permissible delay period and no interest is payable during the period for the item kept in stock.

Interest earned for the time period $\left[0,\mathrm{T}\right]$ is

$$pI_e\int_0^T tDdt=DI_ep\int_0^T tdt$$
$$=\frac{1}{2}DT^2I_ep$$

Interest earned for the permissible delay period $\left[T,t_1\right]$

$$pI_e\left(t_1-T\right)\int_0^T Ddt=pI_e\left(t_1-T\right)DT$$

Total interest earned during the cycle is

$$IE_2=\frac{1}{2}DT^2I_ep+pI_e\left(t_1-T\right)DT$$
$$=DTI_ep\left(t_1-\frac{1}{2}T\right)$$

Profit = Sales revenue – Cost of placing order (O.C.) – Purchasing cost – Carrying cost + Interest earned per cycle.

$$Z_2\left(\mathrm{p},T\right)=ap^{-\gamma+1}-\frac{A}{T}-Cap^{-\gamma}\left(1+\frac{1}{2}\alpha T+\frac{1}{6}\beta T^2\right)-ap^{-\gamma+1}h_p\left\{\frac{1}{2}T+\frac{1}{6}\alpha T^2+\frac{1}{24}\left(2\beta-3\alpha^2\right)T^3\right\}$$
$$Z_2\left(\mathrm{p},T\right)=pD-\frac{A}{T}-\frac{CQ}{T}-\frac{Dh}{T}\left\{\frac{1}{2}T^2+\frac{1}{6}\alpha T^3+\frac{1}{24}\left(2\beta-3\alpha^2\right)T^4\right\}+\frac{pI_eDT}{T}\left(t_1-\frac{T}{2}\right)+pI_eap^{-\gamma}\left(t_1-\frac{T}{2}\right)$$

$$Z_2(\mathrm{p},T) = ap^{-\gamma+1} - \frac{A}{T} - Cap^{-\gamma}\left(1+\frac{1}{2}\alpha T+\frac{1}{6}\beta T^2\right) - ap^{-\gamma+1}h_p\left\{\frac{1}{2}T+\frac{1}{6}\alpha T^2+\frac{1}{24}\left(2\beta-3\alpha^2\right)T^3\right\}$$
$$+pI_e ap^{-\gamma}\left(t_1-\frac{T}{2}\right) \qquad \text{...(14)}$$

Differentiating (11) w.r.t 'p'

$$\frac{\partial Z_2(p,T)}{\partial p} = a(1-\gamma)p^{-\gamma} + Ca\gamma p^{-\gamma-1}\left(1+\frac{1}{2}\alpha T+\frac{1}{6}\beta T^2\right) + a(1-\gamma)p^{-\gamma-1}h_p\left\{\frac{1}{2}T+\frac{1}{6}\alpha T^2+\frac{1}{24}\left(2\beta-3\alpha^2\right)T^3\right\}$$
$$-\frac{1}{2}a\gamma(1-\gamma)p^{-\gamma-1}I_eT$$

Differentiating (11) w.r.t 'T'

$$\frac{\partial Z_2(p,T)}{\partial T} = \frac{A}{T^2} - Cap^{-\gamma}\left(\frac{1}{2}\alpha+\frac{1}{3}\beta T\right) - ap^{-\gamma+1}h_p\left\{\frac{1}{2}+\frac{1}{3}\alpha T+\frac{1}{8}\left(2\beta-3\alpha^2\right)T^2\right\} - \frac{1}{2}pI_eap^{-\gamma}$$

Put $\frac{\partial Z_2(p,T)}{\partial p} = 0$

$$a(1-\gamma)p^{-\gamma} + Ca\gamma p^{-\gamma-1}\left(1+\frac{1}{2}\alpha T+\frac{1}{6}\beta T^2\right) + a(1-\gamma)p^{-\gamma-1}h_p\left\{\frac{1}{2}T+\frac{1}{6}\alpha T^2+\frac{1}{24}\left(2\beta-3\alpha^2\right)T^3\right\}$$
$$-\frac{1}{2}a\gamma(1-\gamma)p^{-\gamma-1}I_eT = 0$$

Multiplying by $24p^{\gamma+1}$

$$24a(1-\gamma)p + Ca\gamma\left(24+12\alpha T+4\beta T^2\right) + a(1-\gamma)h\left\{12T+4\alpha T^2+\left(2\beta-3\alpha^2\right)T^3\right\}$$
$$-12a\gamma(1-\gamma)I_eT = 0$$
$$a(1-\gamma)h\left(2\beta-3\alpha^2\right)T^3 + \left\{4\beta Ca\gamma+4\alpha a\gamma h\right\}T^2 + \left\{12\alpha Ca\gamma+12a\gamma h-12a\gamma(1-\gamma)I_e\right\}T$$
$$+24a(1-\gamma)p + 24Ca\gamma = 0 \qquad \text{...(15)}$$

Put $\frac{\partial Z_2(p,T)}{\partial T} = 0$

$$\frac{A}{T^2} - Cap^{-\gamma}\left(\frac{1}{2}\alpha+\frac{1}{3}\beta T\right) - ap^{-\gamma+1}h_p\left\{\frac{1}{2}+\frac{1}{3}\alpha T+\frac{1}{8}\left(2\beta-3\alpha^2\right)T^2\right\} - \frac{1}{2}pI_eap^{-\gamma} = 0$$

Multiplying by $24p^{\gamma}T^2$

$$24Ap^{\gamma} - Ca\left(12\alpha T^2+8\beta T^3\right) - ah_pp\left\{12T^2+8\alpha T^3+3\left(2\beta-3\alpha^2\right)T^4\right\} - 12pI_eaT^2 = 0$$
$$3ah_p\left(2\beta-3\alpha^2\right)T^4 + 8a\left(C\beta+h_p\alpha\right)T^3 + 12a\left(C\alpha+h_p+pI_e\right)T^2 - 24Ap^{\gamma} = 0 \qquad \text{...(16)}$$

To minimize the total cost $Z_1(p,T)$ per unit time the optimal value of p and T can be obtained by solving the following equations

$$\frac{\partial Z_1(p,T)}{\partial p} = 0 \text{ and } \frac{\partial Z_1(p,T)}{\partial T} = 0 \qquad \text{...(17)}$$

Providing the Equation (6) satisfies the following conditions:

$$\frac{\partial^2 Z_1(p,T)}{\partial p^2}\frac{\partial^2 Z_1(p,T)}{\partial T^2} - \left(\frac{\partial^2 Z_1(p,T)}{\partial p\partial T}\right)^2 > 0 \text{ And } r = \frac{\partial^2 Z_1(p,T)}{\partial p^2} < 0$$

By solving the set of non – linear equations (17) the optimal values of p and T can be obtained and hence the total maximum profit $Z_1(p,T)$.

4. Numerical examples:

For case I:

Example 1: Select the following numerical values $A = 250$ units/year, $I_p = 0.15$ per year $I_c = 0.20$ per year, $I_e = 0.12$ per year, $h = 0.12$ per year, $p = 20.5$ per year, $t_1 = 0.35$ year, $\alpha = 0.1$, $T = 0.3$ year. The value of the model is obtained by the software Mathematica.

Total profit $Z_1(p,T) = 2124.25$

For case II:

Example 2: Select the following numerical values $A = 200$ units/year, $I_p = 0.12$ per year $I_c = 0.18$ per year, $I_e = 0.10$ per year, $h = 0.12$ per year, $p = 18$ per year, $t_1 = 0.38$ year, $\alpha = 0.1$, $T = 0.4$ year. The value of the model is obtained by the software Mathematica.

Total profit $Z_2(p,T) = 2101.27$.

5. Sensitivity Analysis:

For validation of the model parameters α, β, γ and A are changed and total profit is calculated.

The sensitivity analysis is performed by changing each of parameters by $-10\%, -5\%, +5\%$ and $+10\%$ taking one parameter at a time and keeping the remaining parameters unchanged.

The analysis is based on the example-1 and the results are shown in the Table-1. The following points are observed.

Parameter	% Change	p	T	$Z_1(p,T)$.
α	–10	20.0102	0.3230	2180.59
	–5	20.0191	0.3236	2180.83
	5	20.1382	0.3251	2180.90
	10	20.1393	0.3402	2192.83
β	–10	20.0102	0.3230	2180.59
	–5	20.1501	0.3425	2182.61
	5	20.1499	0.3432	2184.01
	10	20.1418	0.3451	2189.22
γ	–10	20.0102	0.3230	2180.59
	–5	20.2232	0.3239	2180.92
	5	20.4321	0.3432	2189.01
	10	20.4522	0.3535	2192.42
A	–10	20.0122	0.3230	1828.31
	–5	20.3881	0.3383	2032.62
	5	18.4561	0.1234	2562.83
	10	18.4561	0.1023	2683.24

6. Conclusion and future Research:

The ordering policies of linear time-dependent deteriorating items and variable demand rate either with or without instantaneous replenishment are important problems. In this paper, the effect of total inventory cost with the help of dependent parameters is discussed. Total inventory cost for case – I and case – II are calculated with the help of software Mathematica. The above model can be extended in several ways like constant demand and constantly deteriorating perishable items.

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