

Experimental Analysis of Tribological Behaviors of Submersible Pump Thrust Bearing using Taguchi Method

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Abstract: Pumps are widely used in domestic, agricultural, commercial and industrial application for lifting, transporting and handling of water. Majority of electricity is consumed by the pump and as bearing have clear effect on performance of the pump, so it quite necessary to increase the performance and efficiency of pump by reducing friction and wear of the thrust bearing of submersible type of pump. The entire work deals with the analysis of friction and wear of thrust bearing. A disk of SS 410 and pin of Teflon and polyamide 6 material was prepared and experimental investigation is done with the help of pin on disk tribometer at room temperature based on Taguchi's L9 orthogonal array (OA) considering load, speed and sliding distance as the design parameter and coefficient of friction and wear rate as the responses under wet condition. The obtained results are analyzed using DOE and Minitab software which shows the average wear of Teflon is 30.09 micron and Polyamide 6 is 22.00 micron and COF is 0.16 and 0.10 respectively.

Index Terms -friction, wear, Minitab, pins on disk, Taguchi, thrust bearing, tribometer etc.

I. INTRODUCTION

Pumps are widely used to lift and transfer water in the domestic, agricultural, commercial, industrial and municipal sectors. As now a day's submersible pumps are widely used in agricultural application and they consumed approximately 10-15% of total electricity produced in the world. Therefore it is necessary to increase the efficiency of pumps by reducing the faults occurred in different parts of the pump like shaft, bearings, thrust etc. Bearing faults increase vibration level 85%, where power consumption increases 14% and pump efficiency decreases 18% [1]. Many polymers are used as a bearing material like acetal, nylon etc. Polymer based materials are widely used as they possesses good combination of mechanical and tribological properties [1]. Polyamide 6 (PA6) is a major class of engineering plastics with a well balance of chemical resistance, wear resistance, mechanical and thermal properties.

In this work V4 submersible pump thrust bearing is considered for investigation purpose. Bearing size used in V4 pump is 60 mm and made up of thrust material like SS 410, SS 304 and bearing material Teflon, fiber, graphite, carbon graphite etc. Pump bearings in the arid regions are greatly affected by temperature, water quality, lubricants, and maintenance operations. The thrust bearing of submersible pumps are getting worn out because of friction due to continuous rotation and shock loading which is mainly due to cavitations, which may result in damage of the fixed pad, impeller shaft. The thrust bearing will fail very rapidly without efficient fluid lubrication. Bearing faults increase vibration level, power consumption and decreases pump efficiency. As pump efficiency decreases, water power decreases and/or consumed power increases affecting water distribution and management system. So it is important to minimize its wear by conducting the tribological testing.

II. LITERATURE REVIEW

Wherever Bin-Bin Jia et al [1] analyses the friction and wear properties of polyamide 66 (PA66), polyphenylene sulfide (PPS) and polytetrafluoroethylene (PTFE) sliding against themselves under dry sliding and oil-lubricated conditions using a pin-on-disc tribometer. HuLin Li et al [2] study the tribological behaviour of PTFE composites by preparing PTFE/copper composites by compression molding at room temperature. Gregory F. Simmons et al [3] presented Dynamic characteristics of polymer face tilting pad journal bearings and investigates using a single pad, load on pad configuration over a range of shaft speeds and loads. A. Mesgarnejad et al [4] investigate the lubrication mechanism of MoS₂-coated thrust ball bearings operating over extended time the results of extensive set of tests are presented for various loads and oscillation frequencies to assess the friction and wear characteristics as function of time. Wojciech Litwin [5] states that water lubricated bearings have been in industrial use for well over a century. Despite that fact, certain solutions continue to employ the standard rubber bearing with lubricating grooves located along entire bush circumference. Martsinkovsky et al [6] describes the problem of increasing the bearing capacity of the thrust bearings is a subject of numerous scientific and technological researches. J.M. Fildes et al [7] presented the engineered surfaces can provide superior resistance to abrasive and adhesive wear. Ramdziah Md. Nasir [8] chooses paddy straws and cockle shells due to their availability and potential as green composites after being harvested. Ling Zhou et al [9] investigated a multistage deep-well centrifugal pump (DCP) with different impeller rear shroud radius have been investigated both numerically and experimentally under multi conditons.

III. DESIGN OF EXPERIMENT AND REGRESSION ANALYSIS

It is methodology based on statistics and other discipline for arriving at an efficient and effective planning of experiments with a view to obtain valid conclusion from the analysis of experimental data. Design of experiments determines the pattern of observations to be made with a minimum of experimental efforts. To be specific Design of experiments (DOE) offers a systematic approach to study the effects of multiple variables / factors on products / process performance by providing a structural set of analysis in a design matrix. More specifically, the use of orthogonal Arrays (OA) for DOE provides an efficient and effective method for determining the most significant factors and interactions in a given design problem.

Orthogonal Array

While there are many standard orthogonal arrays available, each of the arrays is meant for a specific number of independent design variables and levels. Standard notation for orthogonal Arrays is, $L_n (X^m)$

Where,

n =Number of experiments to be conducted

X =Number of levels

m = Number of factors

Common Orthogonal Arrays are as follows

(2- Level arrays)--- $L_4 (2^3)$, $L_8 (2^7)$, $L_{12} (2^{11})$, $L_{16} (2^{15})$, $L_{32} (2^{31})$, $L_{64} (2^{63})$ etc.

(3- Level arrays)--- $L_9 (3^4)$, $L_{18} (2^1*3^7)$, $L_{27} (3^{13})$, $L_{54} (2^1*3^{25})$, $L_{81} (3^{40})$ etc.

(4-Level arrays)--- $L_{16} (4^5)$, $L_{32} (2^1*4^9)$ etc.

Note: Arrays $L_{18} (2^1*3^7)$, $L_{54} (2^1*3^{25})$, $L_{32} (2^1*4^9)$ etc. are for mixed level factors.

For example, if one wants to conduct an experiment to understand the influence of 4 different independent variables with each variable having 3 set values (levels), then an L_9 orthogonal array might be the right choice. The L_9 OA is meant for understanding the effect of 4 independent factors each having 3 factor level values. This array assumes that there is no interaction between any two factors, while in many cases, no interaction model assumption is valid, and there are some cases where there is a clear evidence of interaction.

In this investigation work, which is carried out for 3 factors (load, speed and sliding distance), each factor at 3 levels, an $L_9 (3^4)$ orthogonal array is chosen for conducting the experiments. The fourth column is kept vacant. There are totally 9 trials to be conducted and each trial is based on the combination of level values.

Table 1 Layout of $L_9 (3^4)$ Orthogonal Array

Trial No.	Factor A	Factor B	Factor C	Response Y
1	1	1	1	Y1
2	1	2	2	Y2
3	1	3	3	Y3
4	2	1	2	Y4
5	2	2	3	Y5
6	2	3	1	Y6
7	3	1	3	Y7
8	3	2	1	Y8
9	3	3	2	Y9

Statistical Regression Analysis

Statistical regression analysis is the study of the relationship between two or more variables, used to establish the empirical equation relating input-output parameters, by utilizing least square method. Moreover, it is the most commonly used statistical modeling technique developed based on experimental data. The following steps are to be considered for carrying out statistical regression analysis of a process.

- Identifying the important process control variables and finding their upper and lower limits,
- Conducting the experiments as per the design matrix and recording the response parameters,
- Process modeling,
- Uses of regression analysis,
- Assumptions of linear regression analysis,
- Checking the adequacy of regression models (Analysis of Variance (ANOVA)).

Identifying the important Process Control Variables and finding their upper and low limits

Identification of important process variables includes both inputs and outputs- i.e., factors and responses. In the present study, initially the list of factors responsible for wear process including their upper and lower levels of settings, are identified based on the actual experimentation carried out after consulting the related literature. From the list of factors identified, a detailed review of each of the factors is done to ensure whether these factors are independent in nature.

The design of L_9 OA provides three levels for each factor at a different level combination. If these 3 Levels are normalized between '-1' and '+1', then the weighing factors for level 1, level 2, and level 3 are -1, 0, and +1 respectively.

Table 2 Model of Experimentations

Trail No.	A	B	C	Y=W
1	-1	-1	-1	Y1
2	-1	0	0	Y2
3	-1	+1	+1	Y3
4	0	-1	0	Y4
5	0	0	+1	Y5
6	0	+1	-1	Y6
7	+1	-1	+1	Y7
8	+1	0	-1	Y8
9	+1	+1	0	Y9

Process Modeling

A process generally consists of several discrete and / or continuous input factors and some of these factors (may not all) can be controlled or varied during experimentation. Each response (output) is to be expressed, as a function of the input factors. The output responses are generally assumed to be continuous. After the completion of an experimental plan and conducting the experiment, the experimental data are used to derive an empirical (approximation) model linking the outputs and inputs and to find out which factors influence a response most. Usually, this is done with the help of fitting a polynomial model to the data. These empirical models could be either linear or non-linear in nature.

The linear relationship of input variables (independent) and output response (dependent) of a model with three factors X_1, X_2, X_3 , and their interaction terms can be expressed as follows:

$$Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \alpha_3 X_3 + \alpha_4 X_1 X_2 + \alpha_5 X_1 X_3 + \alpha_6 X_2 X_3 + \alpha_7 X_1 X_2 X_3 + \text{Error} \quad (1)$$

Where Y is the response, $\alpha_0, \alpha_1, \dots, \alpha_7$ are the coefficients of the process model, to be computed using a least square regression method. The unknown coefficients (in coded form) of the model, based on the results of an experiment, are computed using the following mathematical relationship

$$\alpha_j = \frac{\sum_{i=1}^n X_{ji} Y_i}{n} \quad (2)$$

Where $j = 0, 1 \dots 7$ and $i = 1, 2 \dots n$

Checking the Adequacy of Regression Models Analysis of Variance (ANOVA)

The adequacy of the models is tested using the analysis of variance (ANOVA) technique. It is a statistical tool for testing null hypothesis for designed experimentation, where a number of different variables are being studied simultaneously. ANOVA is used to quickly analyze the variances present in the experiment with the help of Fisher test (F test). The results of F test indicate whether there are differences in the means due to varying the test conditions. If the estimates are similar, the changes of the subgroup averages being detectably different are small. If the estimates are significantly different, then the subgroup averages may be significantly different.

IV Experimental Set Up and Procedure

The Experiment is to be conducted on Pin on Disk machine supplied by ducom instrumentation, Bangalore which having a rotating disk in contact with a fixed pin with a spherical top as shown in figure 1. In these experiments, the user typically has the ability to control and measure the applied normal load, unidirectional speed or oscillation frequency. Both the normal and friction forces are measured with transducers. The pin holder is attached to a fixture that is allowed to deflect slightly; the transducer measures this deflection and converts it to a force. Performance is generally characterized by friction coefficient and wear rates (wear per unit time) determined by mass or volume loss with the aid of a profilometer.

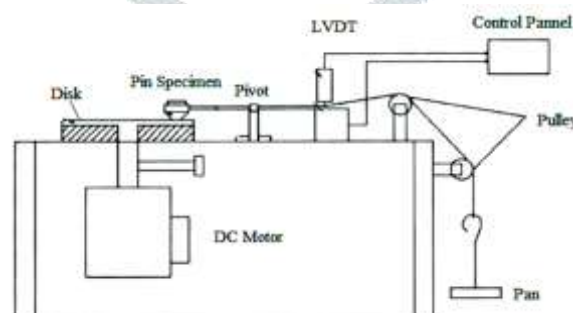


Figure 1. Pin on Disk Schematic Drawing

For the pin-on-disk wear test, two specimens are required. One, a pin with a radiused tip, is positioned perpendicular to the other, usually a flat circular disk. A ball, rigidly held, is often used as the pin specimen. The test machine causes either the disk specimen or the pin specimen to revolve about the disk center. In either case, the sliding path is a circle on the disk surface. The plane of the disk may be oriented either horizontally or vertically. The pin specimen is pressed against the disk at a specified load usually by means of an arm or lever and attached weights. Wear results are usually obtained by conducting a test for a selected sliding distance and for selected values of load and speed. Wear results may in some cases be reported as plots of wear volume versus sliding distance using different specimens for different distances.

Table 3 Specifications of Pin on Disk Tribometer

Parameter	Value
Wear Disc Size	165 mm Dia. 8 mm Thick
Specimen Pin Size	3,6,8,10,12 mm dia. 25-30 mm long
Disk Rotation	Min 200 rpm Max.2000 rpm
Sliding speed	0.5 to 10 m/sec
Wear track Dia.	Min.50 mm, Max. 200 mm
Load Range	Min 5N,Max 200N
Frictional Force	Min 0N Max 200N
Wear	Min -2000 Micrometer,Max 2000 Micrometer
Motor	1.5 hp,1415 rpm,50 hz,3.3A

Specimen Preparation

The typical pin specimen is cylindrical or spherical in shape. Cylindrical or spherical pin specimen diameter ranges from 3 to 12 mm. The disk specimen diameter is 165 mm and thickness is 8 mm. The sample used in the experiment is a Teflon and Polyamide 6 for pin specimens and SS410 for disk specimen. The substrate is first machined and then finished to a pin of dia. 8 mm and 32 mm length. The Disk is made up of SS410 material having diameter 165 mm and thickness of 8 mm. The Pin and Disc specimens are shown in figure 2 and properties are tabulated in table 4



Figure2. Specimen of Teflon, PA6 pin specimen and SS410 Disk

Table 4 Thermal and mechanical properties of the Materials

Properties	SS 410	Teflon	Polyamide 6 (PA6)
Density (gm/cm ³)	7.8	2.18	1.91
Tensile strength (MPa)	1225	33	50
Hardness	422 BHN	205 BHN	175 BHN
Melting point (°C)	1480-1530	317-337	221

Table 5 Assigning of Levels to the Independent Variable

Level	Low	Medium	High
Load (N)	10	30	50
Speed (rpm)	1300	1400	1500
Sliding Velocity (m/s)	3.40	3.66	3.93
Sliding Distance(meter)	2000	3000	4000
Code	-1	0	+1

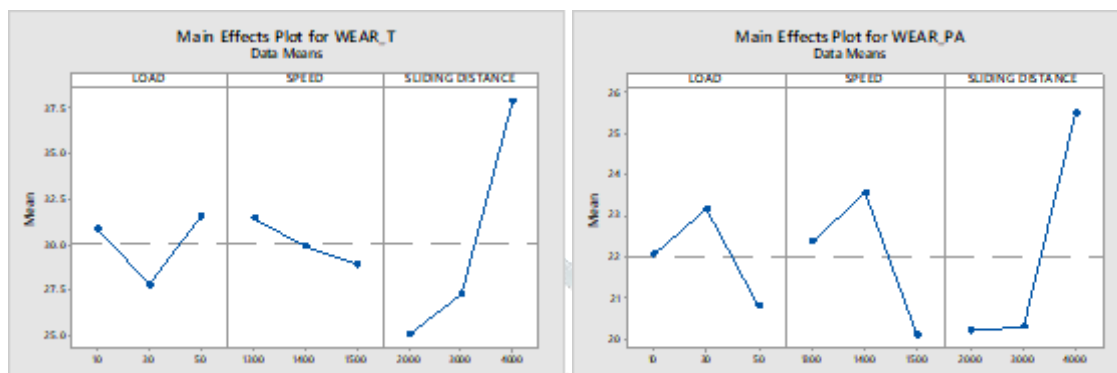
V Result And Discussion

The Table 6 shows the design parameters and output results obtained from experimentation conducted using pin on disk setup at room temperature.

Table 6 Design Parameters and Responses

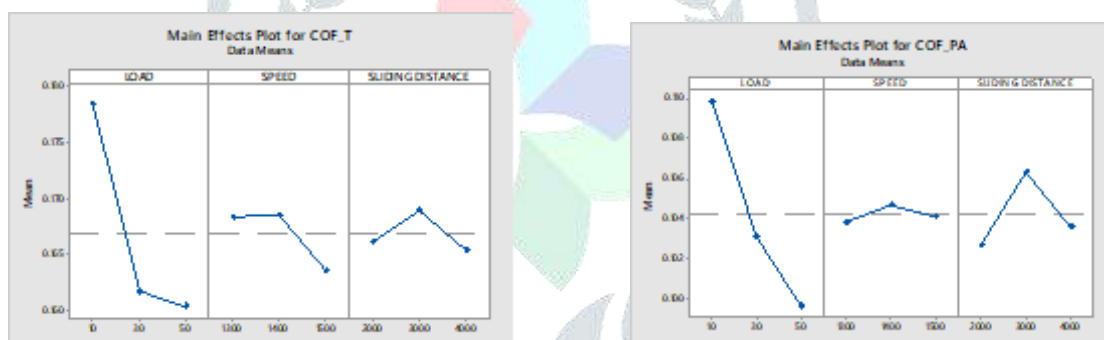
Sr. No.	Load (N)	Speed (rpm)	Sliding Dist. (m)	Sliding Vel. (m/s)	Time (min)	Teflon		Polyamide 6 (PA6)	
						Wear (micrometer)	COF	Wear (micrometer)	COF
1.	10	1300	2000	3.40	9.80	28.46	0.1789	21.6	0.1075
2.	10	1400	3000	3.66	13.65	25.78	0.1786	23.13	0.1132

3.	10	1500	4000	3.93	16.99	38.45	0.1777	21.48	0.1089
4.	30	1300	3000	3.40	14.70	27.31	0.1693	19.63	0.1046
5.	30	1400	4000	3.66	18.20	36.65	0.1617	29.18	0.1025
6.	30	1500	2000	3.93	08.49	19.44	0.1542	20.65	0.1022
7.	50	1300	4000	3.40	19.60	38.59	0.1569	25.88	0.0994
8.	50	1400	2000	3.66	09.10	27.28	0.1653	18.37	0.0984
9.	50	1500	3000	3.93	12.74	28.85	0.1589	18.15	0.1012



Graph1. Main effect plot for wear of Teflon and Polyamide 6

The above figure shows the effect of Load, Speed and Sliding distance on wear of Teflon and Polyamide 6 material, the analysis is made using the popular software specifically used for design of experiment applications known as MINITAB 17. The average wear of Teflon is 30 micron while as Polyamide 6 is 22 micron. The low wear is observed at load 10 N, speed 1500 rpm and at sliding distance 2000 m on Teflon material and at load 50 N, speed 1500 rpm and at sliding distance 2000 m on Polyamide 6 material.



Graph2. Main effect plot for COF of Teflon and Polyamide 6

The above figure shows the effect of Load, Speed and Sliding distance on COF of Teflon and Polyamide 6 material. The lowest value of COF of Teflon material observed at load 50 N, speed 1500 rpm, sliding distance 4000 m and at load 50 N, speed 1300 rpm and sliding distance 2000 m of Polyamide 6 material.

ANOVA and Regression Analysis

In order to find out statistical significance of various factors like load, speed, and sliding distance on wear and COF, analysis of variance (ANOVA) is performed on experimental data.

Wear

Table 8 shows the results of the ANOVA with the wear. This analysis is undertaken for a level of confidence of significance of 5% i.e. the level of confidence 95%. The last column of the table indicates that the main effects are highly significant (all have very small p-values). From Table 8, one can observe that sliding distance (p = 0.011) for Teflon and (p = 0.098) for Polyamide 6 have great influence on wear but the factor load (p = 0.844) for Teflon and (p = 0.648) for Polyamide 6 has relatively less significant contribution on wear.

Table 8 ANNOVA Analysis and Model Summary for Teflon and Polyamide 6 (wear)

Source	Teflon					Polyamide 6				
	DF	Adj SS	Adj MS	F-Value	P-Value	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	257.534	85.845	5.4	0.05	3	52.435	17.478	1.7	0.281

Load	1	0.687	0.687	0.04	0.844	1	2.419	2.419	0.24	0.648
Speed	1	9.677	9.677	0.61	0.47	1	7.775	7.775	0.76	0.424
Sliding Distance	1	247.17	247.17	15.55	0.011	1	42.241	42.241	4.12	0.098
Error	5	79.457	15.891			5	51.317	10.263		
Total	8	336.991				8	103.752			

Regression Equation for wear of Teflon

$$\text{Wear (T)} = 28.1 + 0.0169 * \text{Load} - 0.0127 * \text{Speed} + 0.00642 * \text{Sliding Distance} \dots (4)$$

Regression Equation for wear of Polyamide 6

$$\text{Wear (PA)} = 30.9 - 0.0318 * \text{Load} - 0.0114 * \text{Speed} + 0.00265 * \text{Sliding Distance} \dots (5)$$

Substituting values of the variables for the above equation, the sliding wear of both materials can be calculated. The positive value of the coefficient suggests that the sliding wear of material increases with their associated variables. Whereas the negative value of the coefficient suggest that the sliding wear of the material will decreases with the increase in associated variables. Same procedure can be used for regression analysis of COF.

Coefficient of Friction

Table 9 shows the results of the ANOVA with the COF. This analysis is undertaken for a level of confidence of significance of 5% i.e. the level of confidence 95%. The last column of the table indicates that the main effects are highly significant (all have very small p-values). From Table 9, one can observe that load (p = 0.024) for Teflon and (p = 0.0048) for Polyamide 6 have great influence on wear but the factor sliding distance (p = 0.908) for Teflon and speed (p = 0.898) for Polyamide 6 has relatively less significant contribution on COF.

Table 9 ANNOVA Analysis and Model Summary for Teflon and Polyamide 6 (COF)

Source	Teflon					Polyamide 6				
	D F	Adj SS	Adj MS	F-Value	P-Value	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	3	0.00052	0.00017	3.7	0.097	3	0.000157	0.000052	8.94	0.019
Load	1	0.00048	0.00048	10.36	0.024	1	0.000156	0.000156	26.61	0.004
Speed	1	0.00003	0.00003	0.72	0.434	1	0	0	0.02	0.898
Sliding Distance	1	0.000001	0.000001	0.02	0.905	1	0.000001	0.000001	0.21	0.668
Error	5	0.00023	0.00004			5	0.000029	0.000006		
Total	8	0.00075				8	0.000187			

Regression Equation for COF of Teflon

$$\text{COF (T)} = 0.2148 - 0.000451 * \text{Load} - 0.000024 * \text{Speed} - 0.000012 * \text{Sliding Distance} \dots (6)$$

Regression Equation for COF of Polyamide 6

$$\text{COF (PA)} = 0.1086 - 0.000255 * \text{Load} + 0.000001 * \text{Speed} + 0.000000 * \text{sliding Distance} \dots (7)$$

Confirmation Test

The confirmation test is the final test in the design of experiment process. The purpose of the confirmation test is to validate the conclusions drawn during the analysis phase. This test is conducted to verify the improvement of results and to predict the optimum performance at the selected levels of significant parameters. Table 10 shows the comparison of confirmation test.

Table 10 Actual and Predicted wear

Trial No.	Teflon			Polyamide 6		
	Actual Wear (micrometer)	Predicted Wear (micrometer)	Error	Actual Wear (micrometer)	Predicted Wear (micrometer)	Error
1.	28.46	24.599	3.861	21.6	21.062	0.538
2.	25.78	29.749	-3.969	23.13	22.572	0.558
3.	38.45	34.899	3.551	21.48	24.082	-2.602
4.	27.31	31.357	-4.047	19.63	23.076	-3.446
5.	36.65	36.507	0.143	29.18	24.586	4.594
6.	19.44	22.397	-2.957	20.65	18.146	2.504
7.	38.59	38.115	0.475	25.88	25.09	0.79

8.	27.28	24.005	3.275	18.37	18.65	-0.28
9.	28.85	29.155	-0.305	18.15	20.16	-2.01

Table 11 Actual and Predicted Coefficient of friction

Trial No.	Teflon			Polyamide 6		
	Actual COF	Predicted COF	Error	Actual COF	Predicted COF	Error
1.	0.1789	0.18317	-0.0043	0.1075	0.10735	0.00015
2.	0.1786	0.17093	0.0077	0.1132	0.10745	0.00575
3.	0.1777	0.15869	0.0190	0.1089	0.10755	0.00135
4.	0.1693	0.16215	0.0072	0.1046	0.10225	0.00235
5.	0.1617	0.14991	0.0118	0.1025	0.10235	0.00015
6.	0.1542	0.17367	-0.0195	0.1022	0.10245	-0.0003
7.	0.1569	0.14113	0.0158	0.0994	0.09715	0.00225
8.	0.1653	0.16489	0.0004	0.0984	0.09725	0.00115
9.	0.1589	0.15265	0.0063	0.1012	0.09735	0.00385

The above results, table 10 and 11 shows the error in actual wear and predicted wear as well as the error in actual and predicted COF for the Teflon and Polyamide 6 material under wet condition, as the error between actual and predicted results are within the acceptable limits which confirmed the validity of Taguchi method for enhancing the wear performance and optimizing the wear parameters under wet sliding conditions.

VI. CONCLUSION

The experiment is carried out with the help of pin on disk setup using Teflon which is the existing material used in thrust bearing and Polyamide 6 polymer material according to Taguchi technique at room temperature and at same design factors, which shows that the wear of Teflon material is more than polyamide 6 material. Therefore it may be better choice to use polyamide 6 material in thrust bearing application. The average wear observed for Teflon under wet condition is 30.09 micron while for polyamide 6 is 22.00 micron which shows polyamide may be a better choice for the application. The average Coefficient of Friction observed for Teflon under wet condition is 0.16 while for polyamide 6 is 0.10. The sliding distance is most controllable factor for both wear and COF for Teflon as well as Polyamide 6 material.

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