IMPLEMENTATION OF HILBERT TRANSFORM IN ANALYTICAL ANALYSIS

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ABSTRACT: Hilbert Transform faces several challenges in dealing with closely-spaced frequency components, short-time and weak disturbances, and interrelationships between two time-varying modes of nonlinear vibration due to its mixed mode problem associated with empirical mode decomposition (EMD). To address these challenges, analytical mode decomposition (AMD) based on Hilbert Transform is proposed and developed for an adaptive data analysis of both stationary and non-stationary responses. With a suite of predetermined bisecting frequencies, AMD can analytically extract the individual components of a structural response between any two bisecting frequencies and function like an adaptive bandpass filter that can deal with frequency-modulated responses with significant frequency overlapping. It is simple in concept, rigorous in mathematics, and reliable in signal processing.The Hilbert transform is a widely used transform in signal processing. In this thesis we explore its use for different applications: electrocardiography, modulation etc. The Hilbert- transform is a very popular method for spectral analysis for nonlinear and/or non-stationary processes. We examine its connection with the Hilbert transform and show limitations of the method. Lastly, the connection between the Hilbert transform and single-sideband modulation is investigated.

Index Terms- AMD, adaptive, EMD, Hilbert Transform, modulation, spectral.

I. INTRODUCTION

The Hilbert transform is named after David Hilbert (1862-1943). Its first usedates back to 1905 in Hilbert's work concerning analytical functions in connection to the Riemann problem. In 1928 it was proved by Marcel Riesz (1886-1969) that the Hilbert transform is a bounded linear operator on Lp(R) for 1 < p <1.This result was generalized for the Hilbert transform in several dimensions (andsingular integral operators in general) by Antonin Sigmund (1900-1992) and Alberto Calderon (1920-1998).Mainly, the importance of the transform is due to its property to extend real functions into analytic functions. This property certainly induces a vast numberof applications, especially in signal theory, and obviously the Hilbert transformis not merely of interest for mathematicians.

It is well-known that modal analysis of apparently linear structures frequently generates inconsistent modal parameters and that transfer functions often lack the property of reciprocity. These inconsistencies are usually attributed to non- linearities in the structure under test. However, methods for assessing the deviation of frequency response data from the linear ideal have eluded the practising test engineer. The fact that in modal analysis the theoretical complex frequency response functions are linear analytic functions means that a unique relationship exists between their real and imaginary parts. This relationship has been successfully used in many areas of signal processing and for any linear complex analytic function from which the real part of the function can be generated

from its imaginary part (and vice ***versa)*** the relationship is known as the Hilbert transform.The Hilbert transform, an integral transform which is a relative of the Fourier transform,was described by Titchmarsh [l] in 1937, who presented formal proofs of the derivation of the Hilbert transform by this means and by consideration of a general analytic functionG(z), whose real and imaginary parts form a Hilbert transform pair.

1.1 Frequency Domain. With the development of digital signal processing

Techniques such as Fast Fourier Transform (FFT), modal tests and analysis become competitive in modal property characterization of structures (Alvin et al., 2003). In order to determine modal parameters, the frequency response function of a structure between its excitation and structural response is estimated from the available vibration measurements.

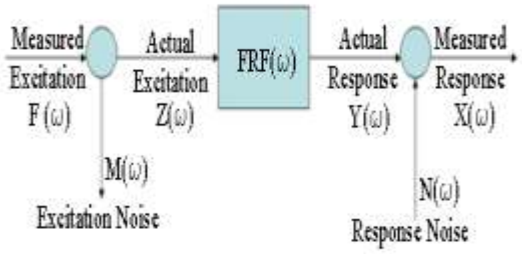


Fig. 1. relationship between excitation and response

In Figure 1, F(ω) and X(ω) represent the Fourier transforms of the measured excitation and the measured response, respectively; Z(ω) = F(ω) - M(ω) is the Fourier transform of the actual excitation and Y(ω) = X(ω) - N(ω) is the Fourier transform of the actual response; M(ω) and N(ω) represent the mechanical and measurement noises to the input and structural response, respectively; FRF(ω) is the frequency response function of the structural system. Mathematically, the response Y(ω) can be related to the excitation Z(ω) .

According to Trendafilova (1998) and Monaco et al. (2000), frequency response functions can be used to quantify and localize minor damage. However, they face difficulties when the input excitation is unknown with ambient vibration of structures. In this case, Brincker et al.

(2000; 2001) developed a frequency domain decomposition method under two assumptions: (1) white noise input, and (2) lightly structural damping. Singular value decomposition (SVD) can then be applied to expand the power spectrum density matrix of output responses into the same form as conventional matrix decomposition in modal analysis. Consequently, a first-order linear approximation of the output power spectrum density matrix is used for the estimation of mode shapes and damping coefficient. Although powerful for closed-spaced natural frequency identification, SVD requires the availability of pre-selected natural frequencies and is applicable only when the assumptions are valid.

II. HILBERT TRANSFORM

The Hilbert transform of a function f(x) is defined by:

$$F(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{f(x)}{t-x}dx$$

- Theoretically, the integral is evaluated as a Cauchy principal value. Computationally one can write the Hilbert transform as the convolution:

$$F(t) = \frac{1}{\pi t} \cdot f(t)$$

- which by the convolution theorem of Fourier transforms, may be evaluated as the product of the transform of f(x) with -i*sgn(x), where:

$$\operatorname{sgn}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases}$$

- The Hilbert transform can be considered to be a filter which simply shifts phases of all frequency components of its input by $-\pi/2$ radians.

An “analytic” (complex time) signal Y(t) can be constructed from a real-valued input signal y(t):

Y(t) = y(t) + j h(t)

where,

- Y(t) is the analytic signal constructed from y(t) and its Hilbert transform
- y(t) is the input signal
- h(t) is the Hilbert Transform of the input signal

The real and imaginary parts can be expressed in polar coordinates as:

- $Y(t) = A(t) \exp[j\psi(t)]$

where,

- A(t) is the “envelope” or amplitude of the analytic signal
- ψ is the phase of the analytic signal (the derivative of ψ is called the “instantaneous frequency”)

III. RESULT AND DISCUSSION

3.1 Single Sideband Modulation Via The Hilbert Transform

This demo shows the use of the discrete Hilbert Transform in Single Sideband Modulation.

The Hilbert Transform finds applications in modulators and demodulators, speech processing, medical imaging, direction of arrival (DOA) measurements, essentially anywhere complex-signal (quadrature) processing simplifies the design.

3.2 Introduction

Single Sideband (SSB) Modulation is an efficient form of Amplitude Modulation (AM) that uses half the bandwidth used by AM. This technique is most popular in applications such as telephony, HAM radio, and HF communications, i.e., voice-based communications. This demo shows how to implement SSB Modulation using a Hilbert Transformer.

To motivate the need to use a Hilbert Transformer in SSB modulation, it's helpful to first quickly review double sideband modulation.

3.2.1 Double Sideband Modulation

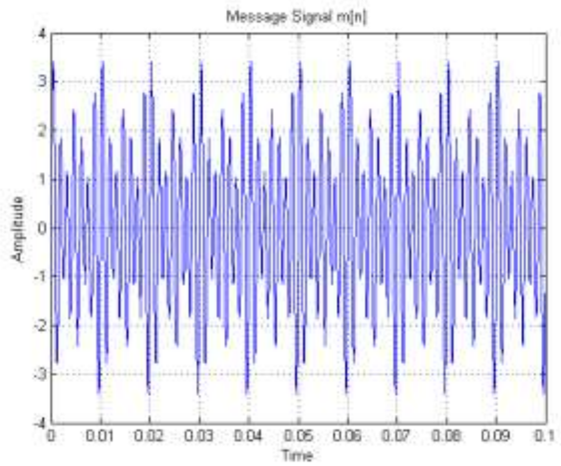
A simple form of AM is the Double Sideband (DSB) Modulation, which typically consists of two frequency-shifted copies of a modulated signal on either side of a carrier frequency. More precisely this is referred to as a DSB Suppressed Carrier, and is defined as

$$f[n] = m[n]\cos(2\pi f_o n / f_s)$$

where m[n] is usually referred to as the message signal and fo is the carrier frequency. As shown in the equation above, DSB modulation consists of multiplying the message signal m[n] by the carrier cos(2*pi*fo*n/fs), therefore, we can use the modulation theorem of Fourier transforms to calculate the transform of f[n]

$$F(f) = \frac{1}{2}[M(f-f_o) + M(f+f_o)]$$

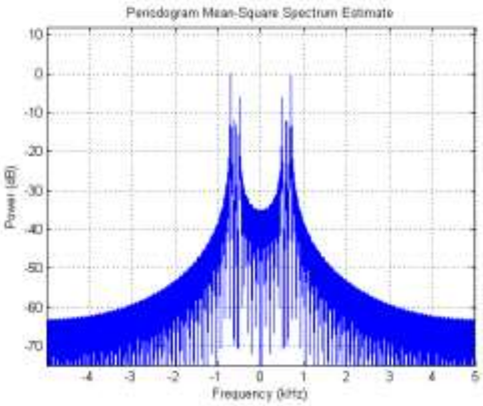
where M(f) is the Discrete-time Fourier Transform (DTFT) of m[n].



Below we calculate and plot the mean-square (power) spectrum of the message signal.

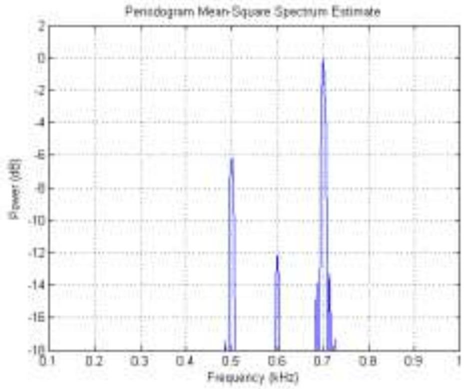
```
h = spectrum.periodogram;
opts = msspectrumopts(h,m);
opts.NFFT = 4096;
opts.Fs = Fs;
opts.CenterDC = true;
msspectrum(h,m,opts)
% Let's zoom into the area of interest.
```

```
Xlims = get(gca,'xlim');
set(gca,'xlim',Xlims,'ylim',[-75 12])
```



The double-sided power spectrum clearly shows the three tones near DC. If we zoom in further we'll be able to read the power of each component.

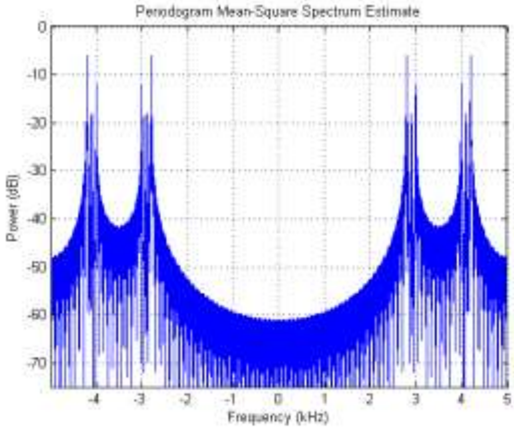
```
set(gca,'xlim',[0.1 1],'ylim',[-18 2])
```



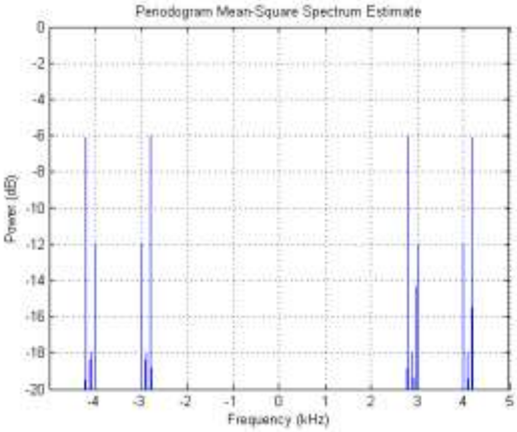
The blue solid line is the modulated message signal, and the red dotted line is the slow varying message signal. The power spectrum of our modulated signal is then

```
msspectrum(h,f,opts)

% Let's zoom into the area of interest.
Xlims = get(gca,'xlim');
set(gca,'xlim',Xlims,'ylim',[-75 0])
set(gcf,'color','white');
```

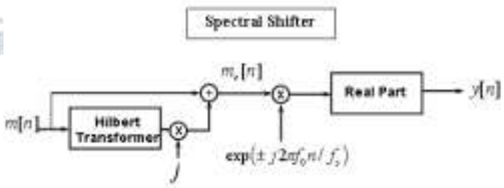


We can see that the message signal (three tones), has been shifted to the center frequency fo. Moreover, each component's power has been reduced to one quarter, due to the amplitudes being halved, as indicated by the DTFT of the modulated m[n].



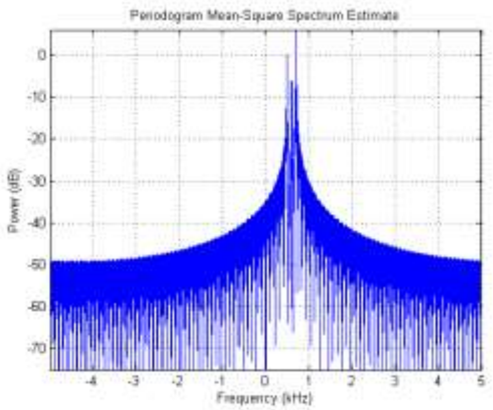
Our positive frequency components are now at -6, -18, and -12 dB.

Using the message signal m[n] defined above we'll create an analytic signal by employing the Hilbert Transform, which will then be modulated to the desired center frequency. The scheme is shown in the diagram below.

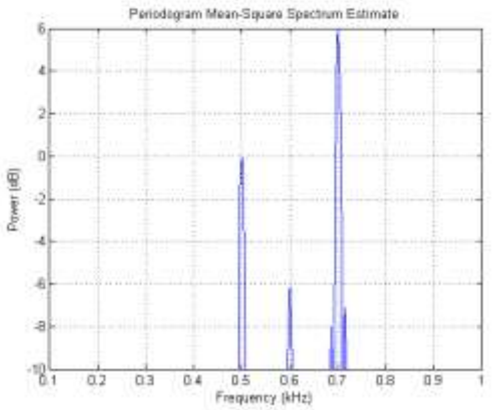


Using this method of spectral shifting will ensure that the power of our signal is shifted to the frequency of interest while maintaining a real-valued signal in the end.

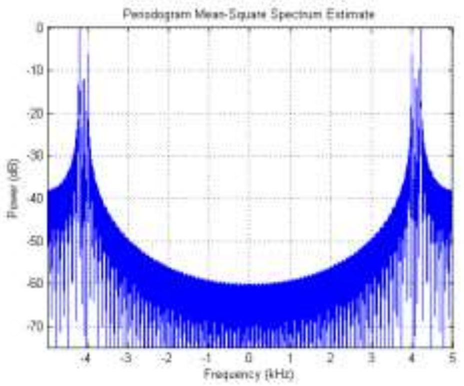
As we indicated earlier the analytic signal is made up of the original real-valued signal plus the Hilbert Transform of that real signal. Running the real signal by the hilbert function in the Signal Processing Toolbox™ will produce an analytic signal.



As shown in the spectrum plot, our analytic signal is complex and only contains positive frequency components. Moreover, if we measure the power, or zoom in our plot further at the positive frequency component we'll see that the power of the frequency components of the analytic signal is twice the total power of the positive (or negative) frequency component of the real signal, i.e., it's similar to a one-sided spectrum which contains the signal's total power. See zoomed-in plot below.



We see that the power of the analytic (complex) signal's frequency components 500, 600, and 700 Hz are roughly 0, -6, and 6 dB, respectively, which is the original signal's total power. These values correspond to our original real-valued signal which has three tones with amplitudes of 1, 0.5, and 2, respectively.



As shown in the plot above our signal has been modulated to a new center frequency of fo without creating the frequency pairs, i.e., it resulted in upper sideband.

If we compare the spectral plot above with that of the DSB modulation we can see that the Spectral Shifter accomplished the SSB modulation.

IV. CONCLUSION

This paper based on the literature survey of different literature papers. Hilbert Transform faces several challenges in dealing with closely-spaced frequency components, short-time and weak disturbances, and interrelationships between two time-varying modes of nonlinear vibration due to its mixed mode problem associated with empirical mode decomposition (EMD). To address these challenges, analytical mode decomposition (AMD) based on Hilbert Transform is proposed and developed for an adaptive data analysis of both stationary and non-stationary responses.

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