A STUDY ON PROPERTIES OF CONNECTEDNESS IN INTUITIONISTIC L-FUZZY SPECIAL TOPOLOGICAL SPACES

[1]S.YAHYA MOHAMED, [2]E. NAARGEES BEGUM
[1]PG & Research Department of Mathematics, Government Arts college,Trichy-22,
[2] Department of Mathematics, Sri Kailash womens college, Thalaivasal, Tamilnadu, India.

Abstract : Intuitionistic fuzzy set can be utilized as a proper tool for representing hesitancy concerning both membership and non-membership of an element to a set. Atanassov introduced and studied the concept of Intuitionistic fuzzy sets. Later this concept is used to define intuitionistic fuzzy special sets by coker and intuitionistic fuzzy topological spaces are introduced by coker. In this paper, a new concept of connectedness in Intuitionistic L-fuzzy special topological spaces is introduced and some properties and theorems about connectedness in Inuitionistic L-fuzzy special topological space are discussed.

Key words: Intuitionistic fuzzy special topological space, Intuitionistic L-fuzzy special topological space, C_5 – Connectedness.

1. Introduction

Fuzzy sets are sets whose elements have degrees of membership. Fuzzy sets are introduced by Lotti.A. Zadeh (1965) as an extension of the classical notion of sets.

The concept of an intuitionistic fuzzy set can be viewed as an alternative approach to define a fuzzy set in cases where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. In general, the theory of intuitionistic fuzzy sets is the generalization of fuzzy sets.

The idea of an intuitionistic fuzzy set was first published by Krassimir Atanassov. Later this concept is used to define intuitionistic fuzzy special sets by coker and intuitionistic fuzzy topological spaces are introduced by coker. In general, the theory of intuitionistic fuzzy sets is the generalization of fuzzy sets. Several researches have shown interest in the intuitionistic fuzz set theory and successfully applied in many other field. Fuzzy application in almost every direction of mathematics such as arithmetic, topology, graph theory, probability theory, logic etc.,

This paper is organized as follows. The definition of fuzzy Topological space, intuitionistic fuzzy Topological space, intuitionistic fuzzy special set and intuitionistic fuzzy special topological space are introduced in section 2. In section 3, we extend the fuzzy special Topological space to L-fuzzy special Topological space, In that, the range of the function will be a lattice. Also the definitions of intuitionistic L- Fuzzy special Topological space are introduced and also we discuss about the theorems on intuitionistic L- Fuzzy special topological space in Connectedness.

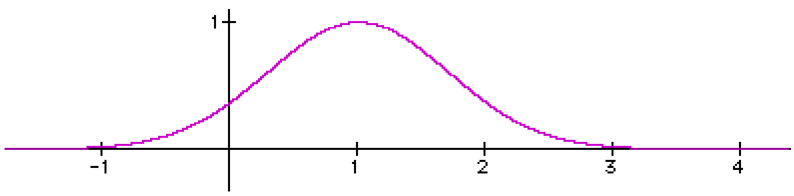
2. Preliminaries

Definition 2.1:

Let X be a nonempty set. A fuzzy set A in X is characterized by its membership function $\mu_A : X \to [0, 1]$ and $\mu_A(x)$ is interpreted as the degree of membership of element x in fuzzy set A for each $x \in X$. It is clear that A is completely determined by the set of tuples $A = \{(u, \mu_A(u))|u \in X\}$.

Example:

The membership function of the fuzzy set of real numbers "close to 1", is can be defined as $A(t) = \exp(-\beta(t - 1)2)$where β is a positive real number.



A membership function for "x is close to 1".

Definition 2.2:

An L-fuzzy set ϕ on U is a mapping ϕ:U → L, where L is a 'transitive partially ordered set'. In this work, we assume that (L,≤) is a preordered set. Notice that it is natural to assume that the relation ≤ is not antisymmetric; if $x, y \in L$ are synonyms, that is, words or expressions that are used with the same meaning, then $x \le y$ and $x \ge y$, but still x and y are distinct words.

Example:

Suppose that U consists of a group of people. The L-fuzzy set, whose membership function ϕ , describes how well the persons in U can ski. For instance, there exist people who can ski very well, some ski badly, and some are moderate skiers.

Definition 2.3:

Let a set E be fixed. An IFS A in E is an object of the following .

A = { (x,μ_A(x) , υ_A(x)), x∈E }, Where the functions μ_A (x) : E →[0, 1] and υ_A (x) : E →[0, 1] determine the degree of membership and the degree of non-membership of the element x∈E, respectively, and for every x∈E: $0 \le \mu_A(x)+ \upsilon_A (x) \le 1$ When υ_A (x) = 1 - μ_A (x) *for all* x∈E is ordinary fuzzy set. In addition, for each IFS A in E, if $\pi_A(x)= 1- \mu_A (x) - \upsilon_A (x)$ Then π_A(x) is called the degree of indeterminacy of X to A or called the degree of hesitancy of x to A. It is obvious that $0 \le \pi_A (x) \le 1$, for each x∈E

Example:

An IF-set is a pair of mappings μ : X → [0, 1], ʋ: X → [0, 1] such that μ (x) + ʋ (x) ≤ 1 for any x ∈ X. In our case X is the set of all pupils in the considered class. If A(x) is the number of acceptation of the pupil x (hence A(x) ∈{0,1,…n} where n is the number of pupils in the class), then we put μ(x) = A(x)/ n

Similarly μ(x) =N(x) /n , where N(x) is the numbers of non-acceptation of the pupil x. Since A(x) + N(x) ≤ n. We obtain μ(x) + ʋ(x) =A(x)/n+N(x)/n ≤ 1, hence the pair (μ,ʋ) is an example of an IF-set.

Definition:2.4

A ***topology*** on a set X is a collection τ of subsets of X having the following properties:

i. Ø and X are in τ.
ii. The union of the elements of any sub collection of τ is in τ.
iii. The intersection of the elements of any finite sub collection of τ is in τ.

A set X for which a topology τhas been specified is called a topological space.

Definition:2.5

Two non – empty open subsets of a topological space X are said to be ***separate*** X if they are disjoint and their union is X.

Definition:2.6

A topological space X are said to be which cannot be separated by such a pair is said to be ***connected.***

Definition 2.7

A family δ ⊆ I* of fuzzy sets is called a fuzzy topology for X if it satisfies the following three axioms

1.∀α∈ I, α∈ δ

2. ∀A, B ∈ δ ⇒ A ∧B∈ δ

3. $\forall (A_j)_j \in J \in \delta \Rightarrow \upsilon j \in J \; A_j \in \delta$

The pair (X, δ) is called a ***fuzzy topological space***.

Example:

Let X= {a,b}. Let A be a fuzzy set on X defined as A(a) =0.5, A (b)=0.4.

Then δ=$\{\bar{0}, A, \bar{1}\}$is a fuzzy topology.(X, δ) is a fuzzy topological space. $\bar{0}(a) = 0$∀a ∈ X, $\bar{1}$ (a) = 1∀a ∈ X.

Definition: 2.8

An intuitionistic fuzzy topology on a non-empty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms

(T_1)$0_\sim$, $1_\sim$ ∈ τ
(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
(T_3)$\cup G_i \in \tau$ for any arbitrary family $\{G_i : G_i \in \tau, i \in I\}$.

In this case the pair (X, τ) is called an ***intuitionistic fuzzy topological space*** and intuitionistic fuzzy set in τ is known as intuitionistic fuzzy open set in X.

Definition: 2.9

Let X be a non-empty fixed set. A ***intuitionistic fuzzy special set*** A is an object having the form $A=<X,A_1,A_2>$. Where $A_1$and A_2 are subsets of X satisfying $A_1 \cap A_2= \phi$.Theset A_1 is called the set of members of A. While A_2 is called the set of non - members of A.

Definition: 2.10

An intuitionistic fuzzy special topology on a non-empty set X is a family τ of intuitionistic fuzzy special sets in X containing $\underset{\sim}{\phi}$, $\underset{\sim}{X}$ and closed under finite infima and arbitrary suprema. In this case the pair (X, τ) is called an ***intuitionistic fuzzy special topological space*** and any intuitionistic fuzzy special set in τ is known as an intuitionistic fuzzy special open set in X.

Definition: 2.11

The complement $\bar{A}$of an intuitionistic fuzzy special open set A in an intuitionistic fuzzy special topological space (X, τ) is called an ***intuitionistic fuzzy special closed*** set in X.

Definition: 2.12

Let N be an intuitionistic fuzzy special sets in (X,τ).

(a) If there exist intuitionistic fuzzy special open sets M and W in X satisfying the following properties, then N is called ***C_i-disconnected***

(i = 1,2,3,4).

C_1: $N \subseteq M \cup W$, $M \cap W \subseteq \bar{N}$, $N \cap M \neq \underset{\sim}{\phi}$, $N \cap W \neq \underset{\sim}{\phi}$

C_2: $N \subseteq M \cup W$, $M \cap W \cap N = \underset{\sim}{\phi}$, $N \cap M \neq \underset{\sim}{\phi}$, $N \cap W \neq \underset{\sim}{\phi}$,

C_3: $N \subseteq M \cup W$, $M \cap W \subseteq \bar{N}$,$M \not\subseteq \bar{N}$, $W \not\subseteq \bar{N}$

C_4: $N \subseteq M \cup W$, $M \cap W \cap N = \underset{\sim}{\phi}$, $M \not\subseteq \bar{N}$, $W \not\subseteq \bar{N}$

(b) N is said to be C_i-connected (i = 1,2,3,4) if N is not C_i-disconnected (i =1,2,3,4).

3. Intuitionistic L- Fuzzy special Topological spaces

Definition: 3.1

An intuitionistic L- fuzzy topology on a non-empty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms

(T_1)$0_\sim$, $1_\sim \in \tau$

(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1 ,G_2 \in \tau$

(T_3)$\cup G_i \in \tau$ for any arbitrary family $\{G_i : G_i \in \tau, i \in I\}$

In this case the pair (X, τ) is called an ***intuitionistic L- fuzzy topological space*** and intuitionistic fuzzy set in τ is known as intuitionistic L- fuzzy open set in X.

Definition: 3.2

Let X be a non-empty fixed set. A ***Intuitionistic L- fuzzy special set*** A is an object having the form $A=<X,A_1,A_2>$. Where $A_1$and A_2 are subsets of X satisfying $A_1 \cap A_2= \phi$.Theset A_1 is called the set of members of A. While A_2 is called the set of non - members of A.

Definition: 3.3

An intuitionistic L- fuzzy special topology on a non-empty set X is a family τ of intuitionistic L- fuzzy special sets in X containing $\underset{\sim}{\phi}$, $\underset{\sim}{X}$ and closed under finite infima and arbitrary suprema. In this case the pair (X, τ) is called an ***intuitionistic L-fuzzy special topological space*** and any intuitionistic L-fuzzy special set in τ is known as an intuitionistic L- fuzzy special open set in X.

Definition: 3.4

The complement $\bar{A}$of an intuitionistic L-fuzzy special open set A in an intuitionistic L- fuzzy special topological space (X, τ) is called an ***intuitionistic L-fuzzy special closed*** set in X.

Definition: 3.5

Let X be a nonempty fixed set. An intuitionistic L-fuzzy special set. A is an object having the form A < X, A_1, A_2>, where $A_1$and A_2 are subsets of X satisfying $A_1 \cap A_2 = \phi$. The set A_1 is called the set of members of A, while A_2 is called the set of "nonmembers of" A.

Definition: 3.6

a) X is called ***C₅ - disconnected***, if there exists an intuitionistic L-fuzzy special set A which is both intuitionistic L-fuzzy special open and intuitionistic L-fuzzy special closed, such that $\underset{\sim}{\phi} \neq A \neq \underset{\sim}{X}$. Also, X is called C_5 - connected, if X is not C_5 - disconnected.

Definition: 3.7

The complement $\bar{A}$ of an intuitionistic L-fuzzy special open set A in an intuitionistic L- fuzzy special topological space (X, τ) is called an ***intuitionistic L-fuzzy special closed*** set in X.

Theorem: 3.8

C_5-connectedness implies connectedness.

Proof

Suppose that there exist non- empty intuitionistic L-fuzzy special open sets A and B such that

$$A \cup B = \underset{\sim}{X}, \quad A \cap B = \underset{\sim}{\phi}$$

from which we get,

$$A_1 \cup B_1 = X,\ A_2 \cap B_2 = \phi,\ A_1 \cap B_1 = \phi,\ A_2 \cup B_2 = X,$$

In other words $A = \overline{B}$.

Hence A is intuitionistic L-fuzzy special class open. i.e., (X, τ) is C_5- disconnected.

Theorem: 3.9

(X,τ) is C_5-connected iff there exist no nonempty intuitionistic L-fuzzy special Open sets A and B in X such that $A = \overline{B}$.

Proof

Suppose that A and B are Intuitionistic L-fuzzy special open sets in X such that $A \neq \underset{\sim}{\phi} \neq B$ and $A = \overline{B}$.

Since $A = \overline{B}$,B is an Intuitionistic L-fuzzy special closed set and

$$A \neq \underset{\sim}{\phi}, \quad B \neq \underset{\sim}{X}.$$

But this is a contradiction to the fact that X is C_5 -connected.

Conversely,

Let A be both an Intuitionistic L-fuzzy special open set and Intuitionistic L-fuzzy special closed set such that $\underset{\sim}{\phi} \neq A \neq \underset{\sim}{X}$.

Now take $B = \overline{A}$

In this case B is an Intuitionistic L-fuzzy special open set such that and$A \neq \underset{\sim}{X}$

$$\Rightarrow \quad B = \overline{A} \neq \underset{\sim}{\phi}$$

which is a contradiction.

Theorem: 3.11

If N is C_1-connected, then N is C_s-connected.

Proof

Let N be C_s-disconnected.

Then there exist intuitionistic L-fuzzy special sets A, B such that $N = A \cup B$,

$A, B \neq \underset{\sim}{\phi}$ follows and A, B are weakly separated.

Let P = cl(A) and Q = cl(B). Then P, Q are intuitionistic L-fuzzy special open sets. We have seen that

$$N \subseteq P \cup Q \text{ and}$$

$$P \cap Q \subseteq \overline{N}$$

If $P \cap N = \underset{\sim}{\phi}$, then $P \subseteq \overline{N}$

$$\Rightarrow N \subseteq \overline{p} \quad \Rightarrow N \subseteq cl(A) \subseteq \overline{B} \quad \Rightarrow N \subseteq \overline{B}.$$

Since $N = A \cup B$ and $A \cup B \subseteq \overline{B}$,

we obtain a contr adiction. Hence $P \cap N \neq \underset{\sim}{\phi}$ follows.

Similarly, it can be proved that $Q \cap N \neq \underset{\sim}{\phi}$.

Theorem: 3.11

If N is C_3-connected, then N is C_M-connected.

Proof.

Let N be C_M-disconnected.

Then there exist intuitionistic L-fuzzy special sets A, B such that $N = A \cup B$, $A, B \neq \underset{\sim}{\phi}$ and A, B are q-separated.

Let $P = \overline{Cl(A)}$, and $Q = \overline{Cl(B)}$,. Then P, Q are intuitionistic L-fuzzy special open sets.

Now,

$$cl(A) \cap cl(B) \subseteq \overline{A} \cap \overline{B} = A \cup B$$

$$cl(A) \cap cl(B) = \overline{N} \Rightarrow N$$

$$cl(A) \cap cl(B) \subseteq \overline{Cl(A) \cap Cl(B)} = \overline{Cl(A)} \cup \overline{Cl(B)}$$

$$cl(A) \cap cl(B) = P \cup Q$$

$\Rightarrow N \subseteq P \cup Q$,

$P \cap Q = \overline{Cl(A)} \cap \overline{Cl(B)}$

$= \overline{Cl(A) \cup Cl(B)}$

$= \overline{Cl(A \cup B)}$

$\subseteq \overline{(A \cup B)}$

$= \overline{N}$

$\Rightarrow P \cap Q \subset \overline{N}$.

If $P \subseteq \overline{N}$, then $N \subseteq cl(A) \Rightarrow N \cap B = \underset{\sim}{\phi}$

(since $cl(A) \cap B = \underset{\sim}{\phi}$) and $N \cap B = (A \cup B) \cap B$

$= B$

$N \cap B = \underset{\sim}{\phi}$.

This is a contradiction.

Hence $P \not\subseteq \overline{N}$ follows $Q \not\subseteq \overline{N}$ can be proved similarly.

4. Conclusion

The work in this paper is a step forward to strengthen the theoretical foundation of connectedness in topological space, fuzzy topological spaces and intuitionistic fuzzy special topological spaces. Also, we introduced the concept of connectedness in intuitionistic L-fuzzy special topological spaces.

In future from this concept can able to extend fuzzy concept for find out various solution in various spaces.

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