PRIME E-CORDIAL LABELING OF SOME SPECIAL GRAPHS

1 N.Sujatha, 2 Dr.K.Ameenal Bibi, 3 T.Ranjani, 4 S.Priyanka
1 Associate Professor, AAA, Walajapet,
2 Associate Professor,3,4 Research Scholars, Department of Mathematics,
1Department of Mathematics,
1AAA,Walajapet,Vellore,India.

Abstract: Let G be a simple (p,q) graph and let f:E(G)→{1,2,3,……,n} be a mapping. Then f is called a prime E-cordial labelling of a graph G, if there exists an induced labeling f*:V(G)→{0,1} defined by f*(V)={∑f(uv) / uv ∈ E(G) (mod 2)}. A graph G which admits prime E-cordial labeling is called a prime E-cordial graph.

Here, we have proved that Peterson graph, Fan graph, Flower graph admits prime E-cordial labeling.

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I. INTRODUCTION

We consider a finite, connected, undirected and simple graph G=(V(G), E(G)) with p-vertices and q-edges which is denoted by G(p,q). For standard terminology and notations we refer Gathon [4].

Definition 1.1: The graph labeling is an assignment of numbers to the vertices or edges or both subject to certain conditions.

Definition 1.2: A binary vertex labeling of a graph G with an induced edge labeling f*:E(G)→{0,1} defined by f*(e=uv) = │f(u)-f(v)│ is called a cordial labeling if │Vf(0)─Vf(1)│≤1 and │ef(0)─ef(1)│≤1. A graph G is a cordial graph if G admits a cordial labelling.

The concept of cordial labeling was introduced by Ebrahim Cahit (Turkey) as a weaker version of graceful and harmonious labelings. He also investigated several results on this newly defined concept.

Definition 1.3: Let G be a graph with vertex set V(G) and edge set E(G) and let f: E(G)→{0,1} define a mapping f* on V(G) by f*(V) = ∑f(uv) / uv ∈ E(G) (mod 2). The function f is called an E-cordial labeling of G if │Vf(0)─Vf(1)│≤1 and │ef(0)─ef(1)│≤1. A graph G is called E-cordial graph if G admits an E-cordial labeling.

In 1997, Yilmag and Cahit[3] introduced E-cordial labeling as a weaker version of edge-graceful labeling and with the blend of cordial labeling.

Definition 1.4: The Peterson graph is a 3-regular graph with 10 vertices and 15 edges.

Definition 1.5: The Fan graph is denoted by F_n and described as F_n=P_n+K_1, where P_n indicates the path graph with n vertices.

Definition 1.6: The Helm graph H_n is the graph obtained from a wheel graph W_n by attaching a pendant vertex through an edge tip end rim vertex of W_n.

Definition 1.7: The Flower graph Fl_n is the graph obtained from a helm H_n by joining each pendant vertices of the helm to the apex vertex. Here the pendant vertices of helm H_n are referred as extended vertices of Fl_n.

2. Main Results:

Theorem 2.1: Peterson graph P_n admits prime E-cordial labeling.

Proof: Peterson graph is a 3-regular graph with 10 vertices and 15 edges.

Let u_0, u_1,……..u_9 be the edges and let v_0, v_1,……..v_9 be the vertices of the graph.

Let e_1, e_2,……..e_5 be the inner edges.

We defined the labeling as follows f: E(G)→{1,2,3,5,……..15} then the induced function f*(V) = ∑f(uv) / uv∈E(G) (mod 2))

Thus the labeling defined above satisfies the conditions of prime E-cordial labeling.

Hence, the proof.
**Illustration 2.2:**  
Figure 1: Prime E-cordial labeling of Peterson $P_n$ graph

**Theorem 2.3:**  
The fan graph $F_n$ admits prime E-cordial labeling.

**Proof:**  
Let $F_n$ be a fan graph joining by a path $P_n$ of length $n-1$.  
Let $u_0, u_1, u_2, \ldots, u_{n-1}$ be the edges and let $v_0, v_1, v_{n-1}$ be the vertices of the graph.  
We defined the labeling function as follows $f: E(G) \rightarrow \{1, 2, 3, \ldots, n\}$ then the induced function $f'(v) = \sum f(uv) / (uv \in E(G) \mod 2)$  
Thus the labeling defined above satisfies the conditions of prime E-cordial labeling. Hence, $F_n$ is a prime E-cordial graph.

**Illustration 2.4:**  
Figure 2: Prime E-cordial labeling of $F_8$

**Theorem 2.5:**  
The flower graph $f_n$ admits prime E-cordial labeling.

**Proof:**  
Let $f_n$ be a flower graph.  
The flower graph $f_n$ joining by a path $P_n$ of length $n-1$.  
Let $u_0$ be the apex vertex $u_1, u_2, \ldots, u_{n-1}$ be the rim vertices and let $u^1_1, u^1_2, \ldots, u^1_{n-1}$ be the external vertices.  
We defined the labeling function as follows $f: E(G) \rightarrow \{1, 2, 3, \ldots, n\}$ then the induced function $f'(V) = \sum f(uv) / (uv \in E(G) \mod 2))$  
Thus the labeling defined above satisfies the conditions of prime E-cordial labeling. Thus, the flower graph $f_n$ is a prime E-cordial labeling.
Conclusion:

In this paper, we have obtained prime E-cordial labeling for Peterson graph, Fan graph and the Flower graph. We further motivated to verify the above labeling process for some more special classes of graphs.

References: